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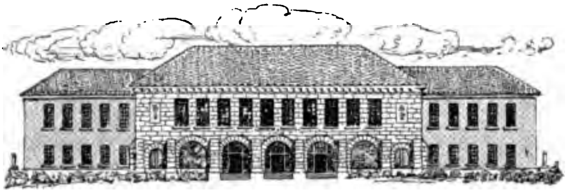

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GENERAL PHYSICS

AND ITS APPLICATION TO INDUSTRY AND EVERYDAY LIFE

BY

ERVIN S. FERRY

PROFESSOR OF PHYSICS IN PURDUE UNIVERSITY

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PREFACE

THE present book is designed for that large class of students who in the early part of their college career require a coordinated elementary course in the fundamental principles, the methods, and the industrial applications of physics. The purpose is not only to impart information, but also to give training in the methods by which facts are correlated in laws, and these laws applied to the affairs of life.

In order that the mind of the student may not be distracted from the physics by difficulties foreign to the subject, hypotheses still in controversy are not considered, and no mathematics is assumed beyond the elements of algebra and trigonometry. New ideas are first developed and then expressed by definitions or laws in physical terms. Definitions are carefully distinguished from defining equations. In making the questions at the end of the various sections, the object has been to develop in the student the power of connecting facts and laws with familiar phenomena rather than the ability to enunciate the various definitions, laws, and equations given in the text. It is hoped that from the study of the numerous solved problems the student will early acquire an ability in effective attack as well as in the orderly presentation of work.

The bases for the selection of the laws to be included in the text have been the frequency of their occurrence in the ordinary affairs of life and the wideness of their application in the arts. In the selection of the unusually large amount of illustrative material included, due regard has been given to the special interests of students of agriculture, engineering, and general science. The recent war has produced many highly important and interesting devices, some of which are here presented to students for the first time. Most of the purely illustrative material is printed in smaller type than the discussion of principles. Much of this may well be left for private reading and not assigned for recitation.

It is a pleasure to acknowledge my indebtedness to Professors R. G. Dukes, A. T. Jones, G. W. Sherman, L. V. Ludy and R. V. Achatz for criticism of the manuscript, to Miss Elizabeth Mitchell for the solution of many of the problems in the back of the book and to Professor D. C. Miller for the construction of several diagrams of compound wave forms.

ERVIN S. FERRY.

LAFAYETTE, INDIANA, U. S. A.
February 1, 1921.

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DYNAMICS

CHAPTER I

FUNDAMENTAL NOTIONS OF DYNAMICS

1. Stress and Force.—If a spiral spring be compressed and then released, the two ends will tend to recede from one another. If a magnet and a piece of iron, both free to move, be placed near one another, each will move toward the other. The distance between the earth and any body near it always tends to diminish. When two bodies interact so as either to produce or tend to produce a change in the motion of one body relative to the other, their mutual action is called a *stress*, and the medium connecting the bodies is said to be in a state of stress. When the attention is confined to the cause which tends to change the motion of one of the bodies, the other body being left out of consideration, the action of the stress on this body is called force. *Force* may be defined as any cause which either changes or tends to change the motion of a body relative to another. If the attention be limited to the motion of one of the bodies, the force acting is said to be an external or impressed force. If, on the other hand, the two bodies be considered as forming a single system, and no force from outside the system acts upon it, the force acting upon each body is called an internal force. Other names sometimes used for force are load, thrust, push, pull, pressure, tension, attraction, repulsion, effort, resistance.

Force is the *direct* cause of the change in the motion of a body. For instance, a wind is not a force, but the push exerted by wind against a sail or other body is a force. Falling rain is not a force, but the push exerted by it against an umbrella is a force. Fire is not a force, but the expansive push which makes hot coal break is a force. A stretched rubber band is not a force, but the pull which tends to draw the ends toward one another is a force.

Whenever there is a stress between two bodies there must be a medium extending from one to the other. In some cases this intervening medium is not obvious. In the first illustration above, the effect is due to a stress in the spiral spring between the two bodies. In the case of the magnet and the piece of iron the medium is an invisible, all-pervading, highly attenuated substance whose properties have been pretty thoroughly studied. It is known that this medium, called the ether, serves for the propagation of light, electric, and magnetic effects, and it is highly probable that gravitational attraction is also due to a stress of this medium.

2. The Gravitational Units of Force.—One of the facts first discovered in the history of science is that any body on the surface of the earth is pulled toward the earth. This force is called *weight* or the *force of gravity*. The weights of bodies at a given place can be compared by means of the distortions they produce in a spiral spring when suspended freely under the influence of the earth's attraction. For instance, if each of two bodies A and B when suspended from the end of a vertical spiral spring produces the same elongation of the spring, they are of equal weight. If a third body produces the same elongation as do A and B together, then the weight of the third body equals twice the weight of A or B. Proceeding in this manner, a spiral spring can be "calibrated" so as to give directly the weights of bodies, compared to any unit weight taken as a standard of comparison. This is the essence of the earliest and simplest method of comparing forces. It is a method still used in engineering and in ordinary life.

In English-speaking countries the gravitational unit of force is the weight at London of a certain piece of platinum preserved in the Office of the Exchequer. It is called the pound weight avoirdupois. Of course a force of k pounds weight need not be due to gravity nor act in a vertical direction. On the continent of Europe the gravitational unit of force is taken as the weight at

Paris of a certain lump of platinum deposited in the Archives of Paris. This unit of force is called the kilogram weight.

1 kilogram weight = 2.2046 pounds weight.
1 pound weight = 0.45359 kilogram weight.

The attraction exerted by the earth on a given body depends upon the latitude of the place at which the body is situated, upon its distance from the center of the earth, and upon such local conditions as the proximity of great mountains, etc. Consequently, the weight of a body is an indefinite quantity depending upon the position of the body. At sea level, the ratio of the weight of a given body situated at the equator to its weight at the pole is as 978 is to 983. It is useless to take account of this small difference in engineering work, but in scientific work a unit of force must be employed which is absolutely constant. The absolute unit of force will be defined in a later paragraph.

3. The Effects of Force.—The primary effects of force are two in number. A force can cause distortion, i.e., a displacement of one part of a body relative to another part. Also, if a body is free to move, a force can cause a change in the velocity of the body. This change of velocity may be either a change of direction or a change of magnitude. The ratio of the change of the velocity of a body to the time occupied by the change is called the *acceleration* of the body's motion. Acceleration is measured by the change of velocity occurring in unit time.

4. Newton's First Law of Motion.—A slight blow with the fist will impart to an ordinary punching bag a considerable speed; but if the bag were filled with sand, a much stronger blow would be required to give it the same speed. Little force is needed to stop a moving football, but a considerable force would be required to stop a cannon ball moving with the same speed. A small force is sufficient to deflect from a straight path a stream of water issuing from a garden hose, but to cause a railway train to go around a curve requires well-spiked rails.

Whatever can be changed in motion only by the application of force is called *matter*. That property which requires force to change either the magnitude or direction of the motion of a body is called *inertia.* Inertia is the distinguishing characteristic of matter.

This great law of Inertia, or Newton's First Law of Motion, has been enunciated in the form, *a body will continue to move with its present speed, in a straight line, until acted upon by an external force.*

Illustrations abound of the effect of inertia. If a passenger train suddenly stops, the passengers will be thrown forward on account of the inertia of their bodies preventing them from instantly changing their motion relative to the train. For the same reason, the front ends of loaded freight cars are sometimes pushed out when the train is stopped too suddenly.

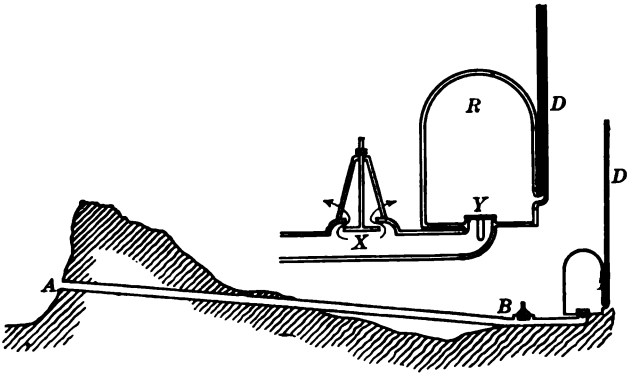


Fig. 1.

An impact-exploding shell contains a detonating cap in front of a small metal plunger. When the flight of the shell is suddenly arrested, the plunger continues to advance, thereby striking and exploding the detonating cap. The shock of this explosion sets off the main charge.

Water pipes are often burst due to the continuance of the motion of the water after the valve has been closed. To guard against this accident, the large water gates in city water mains are arranged so that it is impossible to close them quickly.

Locomotives are supplied with water while in motion by the simple device of dropping the forward end of an inclined pipe into a long trough of water extending parallel to the rails. As the water in the trough tends to remain at rest with reference to the ground, the moving inclined pipe slips under a portion of the water and raises it vertically until it falls into the water tank on the tender.

5. The Hydraulic Ram.—This is a common device by means of which the inertia of a long column of water falling through a short distance will cause a small part of the water to rise to a level above the source. In Fig. 1, the reservoir *A* is joined by a long pipe *AB* to a device consisting of two valves *X* and *Y* shown on a larger scale in the drawing above. The valve *X* opens downward and the valve *Y* upward. When water descends the drive pipe *AB*, some of it will escape about the valve *X*. In passing around this valve the water will exert upon it an upward force which will cause it to suddenly close. But on account of the inertia of the long column of water, the motion will tend to continue even after escape at *X* is stopped. When *X* closes, the water column strikes against the valve *Y*, enters the air chamber *R* and rises in the discharge pipe *D*. When the water comes to rest both valves fall, thereby causing water to again escape at *X* and preventing the water in the delivery pipe from flowing back. When water again escapes at *X*, the preceding action is repeated, thereby forcing more water into the delivery pipe.

With a drive pipe 30 feet long, a fall of 4 ft., and a supply of 120 gallons per hour, a commercial hydraulic ram will deliver 10 gallons per hour at a height of 30 ft. With a drive pipe 100 ft. long, a fall of 14 ft., and a supply of 1800 gallons per hour, a larger size machine will raise 150 gallons per hour to a height of 100 ft.

6. Newton's Second Law of Motion.—If forces of 1, 2, 3, etc., pounds' weight, respectively, be applied to a given body, it is found by experiment that the body will be given accelerations in the direction of the applied forces in the ratio of 1:2, 1:3, etc. In other words, *when a force acts upon a body, there is produced in the body's motion an acceleration whose direction is that of the force, and whose magnitude is directly proportional to that of the force applied.*

This law has been thoroughly tested and verified in the most diverse ways and is taken as one of the fundamental principles of dynamics. It is usually called Newton's Second Law of Motion.

7. Representation of Forces.—For the complete specification of a force three characteristics are necessary. They are magnitude, direction, and line of action. Since a straight line is also completely described by these same characteristics, a force can be represented by a straight line. For example, in the case of a body resting on a table as shown in Fig. 2, the weight of the body, i.e., the force acting downward on the body, is represented in direction and line of action by the line *AB*. In the same manner, the upward force exerted by the table on the body is represented by the line *CD*.

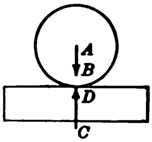


Fig. 2.

8. Newton's Third Law of Motion.—Imagine two boys pulling in opposite directions on a rope which passes through a hole in a fence. If one boy pulls as hard as the other, the rope will not move in either direction. If one boy should tie his end of the rope to a post, the other might be quite unaware of the fact. That is, the post offers the same resistance to the force exerted by the boy pulling the rope as did the other boy. In other words, the force exerted on the post develops on the boy pulling the rope another force—equal to the first and in the opposite direction.

No force can be exerted unless there is an opposing force. The driving wheels of an advancing automobile push backward on the ground. The ground pushes forward on the wheels. The motion forward of the car is due to this push exerted by the ground. If the car were on ice or other smooth surface, the wheels would slip instead of push on the ground and there would be no push exerted by the ground on the wheels.

A stress always has two ends—it acts upon two bodies or upon two parts of a body. The two ends of a stress are called its action (or force), and its reaction. Either end may be called the action (or force), and then the other end would be called the reaction of the force. A force and its reaction are developed at the same time, and one exists just as long as the other. It should be carefully noted that a force and its reaction always act upon different bodies.

This two-endedness of a stress is described by the statement that *every force has an equal and oppositely directed reaction acting upon a different body.* This is called Newton's Third Law of Motion.

9. Illustrations of Force and Reaction.—Consider a body falling toward the earth. If the earth pulls down upon the body, it follows that the body pulls up on the earth with an equal force. This force can be represented by the lines F_1 and F'_1 in Fig. 3.

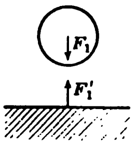


Fig. 3.

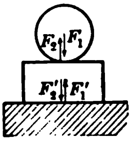


Fig. 4.

If, instead of falling, the body rests upon a table, there are the same action and reaction due to the earth's gravitational force as before (F_1 and F'_1, Fig. 4). But there is now an additional stress due to the passive resistance offered by the table, which exactly counteracts the tendency of the body and the earth to come together. The table pushes upward on the body with a force F_2, and the table also pushes downward upon the earth with an equal and oppositely directed force F'_2.

It must not be forgotten that the action and the reaction are always applied to different bodies. If two forces are applied to the same body so as to oppose one another's effect, one is the action and the other is the counteraction. Action and reaction are the two ends of a single stress; action and counteraction are two ends of different stresses. Action and reaction are equal: action and counteraction are not always equal.

Consider the case of the horse and cart. One often hears the query, " If a horse pulls a cart and the cart pulls backward on the horse to an equal extent, how can either the horse or cart move with reference to the earth? "

To answer this, consider first how it is that the horse can move over the earth when he is not pulling the cart. In this case he pushes backward on the earth. The earth, therefore, pushes forward upon him with an equal force thereby causing him to move. If anything is fastened to him, whether above, behind, or in any other position, it must also move if he moves. If the object fastened to him is a saddle, the saddle pushes down upon his back, and he pushes up against it, but the stress between the horse and the saddle does not prevent his moving along the earth. In the same way, if the object fastened to him is a cart, he pulls forward on the cart and the cart pulls back with equal force upon him. But the pull that the cart exerts on him and the push that the earth exerts on him are parts of different stresses, and if the push of the earth is greater than the pull of the cart, he starts the cart and makes it move.

In Fig. 5 let the rectangle A represent the horse, and the rectangle B, the cart. The horse's feet push backward against the earth with a certain force F_1, while the earth pushes against the horse's feet in the opposite direction with equal force. The horse pulls on the cart with a force F_2, while the cart pulls backward on the horse with equal force. Frictional and other passive resistances between the cart and the earth exert on the earth a force F_3 which develops an equal and oppositely directed reaction on the cart. Call forces acting in the direction of motion positive, and forces in the opposite direction negative.

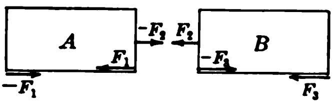


Fig. 5.

Then, if the road is level, the total force acting on the horse is (F_1-F_2), and the total force acting on the cart is (F_2-F_3). If the motion of the horse and cart is uniform, the total force acting on the horse, is

$$(F_1-F_2)=0,$$

and the total force acting on the cart is

$$(F_2-F_3)=0.$$

Therefore, in order to keep the cart moving uniformly, the horse needs to exert a force

$$F_1 = F_2 = F_3.$$

If, however, the horse is to increase the cart's motion,—for example, if he is to start it from rest,—then both $(F_1 - F_2)$ and $(F_2 - F_3)$ must be greater than zero. That is, we must have F_1 greater than F_2, and F_2 greater than F_3.

10. Hero's Engine.—A liquid or gas under pressure in a tube will exert a force against each point of the inner surface. Associated with each of these forces there is a reaction acting upon the liquid or gas, having the same magnitude as the force at the given point and acting in the opposite direction. In Fig. 6 forces acting upon the tube are represented by arrows directed away from the axis, and forces acting upon the contained fluid by arrows directed toward the axis. The fluid cannot push against the tube at any particular point unless the tube pushes against the fluid at that point.

If holes be made through the walls of the tube, for example at *a* and *b*, Fig. 7, there will be no wall at these points to exert force against the contained fluid, and consequently no forces at these points pushing against the tube.

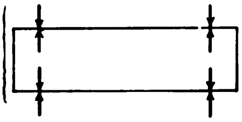


Fig. 6.

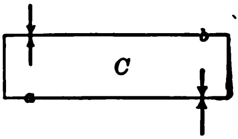


Fig. 7.

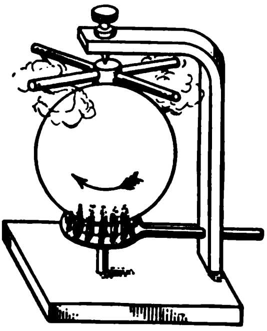


Fig. 8.

When all points of the inside of the tube are acted upon by equal forces, as in Fig. 6, there is no tendency for the tube to move. But when there are some points at which there are forces and some at which there are none, as in Fig. 7, the tube will tend to move. With the holes at *a* and *b*, the tube will tend to rotate in the clockwise direction about an axis perpendicular to the plane of the paper through *C*.

Hero's steam engine, devised about 120 B.C., consists of a boiler, Fig. 8, capable of rotation. When the steam generated in the boiler issues from holes near the ends of the tubes arranged as shown, the tubes will be pushed in the direction opposite to that of the issuing steam. Many lawn sprinklers operate on the same principle.

11. Independence of Forces.—As the result of experience and of observations extending through several centuries, it has been concluded that, *if several forces act simultaneously upon a body, each force produces its own effect independently of all the others.* This is called the Principle of the Independence of Forces. Whether a body be at rest or in motion, the effect of an applied force is the same.

A boat being rowed across a river is acted upon by two forces—one due to the current parallel to the banks and another due to the rower perpendicular to the banks. Each stroke of the oars causes the boat to advance perpendicularly to the banks by just the same amount that it would if the boat were not at the same time drifting downstream.

If a ball be projected horizontally, it will fall the same distance in a given time that it would fall if the horizontal motion were zero.

12. Equilibrium.—A system of forces applied to a body is in *equilibrium* if the motion of the body is unchanged in magnitude or direction by the action of the forces. A body is said to be in equilibrium when the forces acting upon it are in equilibrium. If the system of forces acting upon a body is in equilibrium, the combined effect of all the forces on the motion of the body is zero.

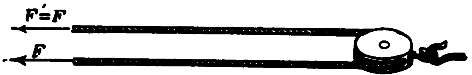
To be in equilibrium a body need not be at rest. In the case of the horse and cart, if the horse pushes directly backward on the ground, then as long as the pull of the cart backward on the horse equals the push of the ground forward on the horse, the horse goes on with the same speed in the same direction. That is, he is in equilibrium, although he is not at rest. If the horse should push backward on the ground and at the same time sidewise on it, so as to turn a corner, he might continue to move just as fast as he was going before, but since the direction in which he goes would be changed, he would no longer be in equilibrium.

13. Mechanical Advantage.—The ratio of the force developed by any machine, to the force applied, is called the *actual mechanical advantage* of the machine. The ratio of the distance through which the applied force acts, to the distance through which the opposing force acts is called the *theoretical mechanical advantage.*

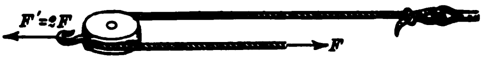
It is frequently required to move a given body through a certain distance in a given direction against a force that is much larger than that at our disposal.

For the solution of such a problem there are many riggings of blocks and tackle of different mechanical advantages that require various equipments of ropes and pulleys. The particular one selected will depend upon the requirements of the problem and also upon the equipment at one's disposal. Some of the standard riggings are represented in Fig. 9.

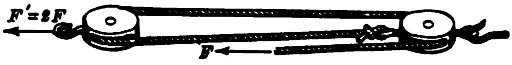
A pulley consists of one or more wheels with grooved rims, called sheaves, supported in a frame called a block. In the present discussion we shall assume the rope to be perfectly flexible and the axle without friction. In this case the tension in the rope on the two sides of a sheave will be equal. In the actual case, however, the tensions on the two sides of a sheave will differ by an amount depending upon the friction of the axle and the stiffness of the rope. For old rope of ordinary size the difference may be taken to be about one-tenth.



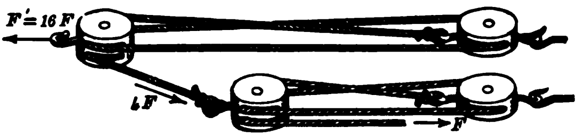
Single Whip



Running Tackle



Gun Tackle



Luff on Luff

Fig. 9.

In the diagrams, Fig. 9, the applied force is represented by the symbol *F*, and the force produced by *F'*. If it be desired to simply change the direction in which a force is applied without producing any mechanical advantage, the Single Whip may be used. In the Running Tackle, the rope, one end of which is fixed, is passed around a movable pulley. The movable pulley is pulled equally by the rope on each side thereby producing a mechanical advantage of two.

With a rope and two single pulleys, one of which is provided with two rings, we have the Gun Tackle, which has a mechanical advantage of either two or three, depending upon which is used as the running pulley. If the pulley *A* be provided with two sheaves we will have the Luff Tackle of mechanical advantage four. The Luff on Luff consists of one luff attached to another and has a mechanical advantage of sixteen.

When the friction of the pulleys and the stiffness of the rope are neglected, the actual mechanical advantage equals the theoretical. But when they are not neglected, the actual mechanical advantage is less than the theoretical.

QUESTIONS

1. Aside from any difference in value, would there be any advantage in buying silver in New Orleans and selling in New York, provided the same spring balance were used at each place?

2. A heavy body is suspended by a string, and has another string attached to its lower side. If the lower string is jerked suddenly downward it will break, while if a steady pull is applied to it, the upper string will break. Explain fully, stating the physical law involved in the explanation.

3. Explain the agitation of a liquid when hauled in a tank over a rough road. Explain the difficulty experienced by a dog in catching a dodging rabbit.

4. In splitting wood, a man will sometimes drive the axe into the chunk and then, while the axe is in the chunk, strike the chopping block with the back of the axe head. At other times, while the axe is in the chunk he will strike the chopping block with the chunk. When would each method be the proper one to employ?

5. Represent graphically the forces that act on each of the following bodies: (*a*) a man in a hammock at rest; (*b*) the pendulum of a clock at the end of its swing; (*c*) the piston of a steam engine; (*d*) the paddle of a canoe; (*e*) a spring board at rest when you are on one end of it.

6. A man standing on a platform scales fires a rifle bullet (*a*) into the platform; (*b*) vertically upward into the air; (*c*) horizontally into the air. What indication will be observed on the scale beam in each case?

7. If all railway trains, and the animals on the earth were simultaneously to move eastward, what would be the ultimate effect upon the earth's motion?

8. What is the cause of the " kick " of a rifle? Would a kick be produced if the gun were fired in a vacuum?

9. How could a man on a perfectly smooth horizontal table move himself in a horizontal direction?

10. A body resting on the floor is pulled downward by its weight and is pushed upward with an equal force. Hence the resultant force in the vertical direction is zero. Why will not any additional upward force, however small, cause the body to move upward?

11. Could a sailboat be propelled by the impact on the sail of compressed air escaping from a tank on the stern? Would it make any difference if the stream of air were directed backward over the stern? Explain.

12. With Hero's engine, rotation is produced when the steam escapes into the air. Show that the same effect would be produced if the engine were in a vacuum.

13. There is the same tension throughout the rope being pulled by two tug-of-war teams. That is, the winning team is pulling with the same force as the weaker team. Explain.

14. Discuss the following: "Suppose the wheel in Fig. 10 is not rigidly fastened to its axle, but connected by ball bearings, so you can lift the axle, with the wheel on it, out of the frame, and the wheel continues to rotate if once set in motion. If you make it move clockwise, by pushing it at the place of the arrow, your body will exert a stress on the earth, to the left, of, say, 10 units, and the push to the right will mostly go into the wheel; only a small share into the frame; say 9 units against 1 unit. When lifting axle and wheel out, putting the latter on the floor edgewise, it will run forward on the floor, to the right, until the 9 units are exhausted by friction. Then action and reaction will be alike, 10 units each way. But if instead of pushing the wheel's upper part, you push its lower part, making it rotate counterclockwise, and then put it on the floor, it will go forward *to the left*. Then the action on the ground will be 19 units to the left and but 1 unit to the right—action and reaction not being alike any more, and the reaction not opposite to the reaction."—N. Johannsen.

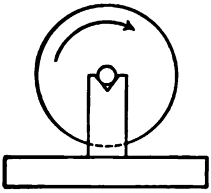


FIG. 10.

14. Units of Length.—By legislative enactment the various countries have legalized certain standards of length. There are but two that are extensively used. The foot is one-third of the British Standard Yard. This latter is by law defined to be the distance, at the temperature 62° F., between the centers of two transverse lines on a certain bronze bar deposited in the Office of the Exchequer in London. The foot is the unit of length employed in engineering and in ordinary life throughout all English-speaking countries.

The unit of length in common use throughout the continent of Europe is called the meter. The meter is the distance, at the temperature of melting ice, between two parallel lines engraved on a certain platinum bar preserved in the Archives in Paris. One thousand meters is called a kilometer; one-tenth of the meter is called a decimeter; one-hundredth is called a centimeter; and one-thousandth is called a millimeter. In scientific work, throughout the world, the centimeter is used as the unit of length.

The meter was intended to be one ten-millionth part of the distance from the equator to one of the poles of the earth. In 1799 very careful surveys were made for the purpose of determining this distance, and from this measurement the meter was constructed. More recent surveys have shown this measurement to be slightly in error. It is now known that a quadrant of the earth's surface, instead of measuring 10,000,000 meters, is more nearly equal to 10,000,880 meters.

The relation between the two units of length is shown in the following table:

1 centimeter = 0.39371 inch
1 meter = 3.2809 feet
1 kilometer = 0.62139 mile

1 inch = 2.5399 centimeters
1 foot = 0.30479 meter
1 mile = 1.6093 kilometers

15. Work.—When, in opposition to a resisting force, the position of a body is changed, work is said to be done against the force. *Work* may be defined as the accomplishment of a change in the position of a body against an opposing force. Work done by a force is called positive and work done against a force is called negative.

If a body is lifted, work is done against the force of gravity. If a spiral spring is compressed, work is done against the force of the spring. If a body is dragged along the ground, work is done against the frictional resistance to motion. If a man swims up-stream, he does work against the current, whether he is advancing or whether his position with reference to the earth does not change. Since in walking a man raises himself at each step, work is here done against the force of gravity. Since our muscles are constantly relaxing and recovering, work is done against the force of gravity when we attempt to stand erect for any considerable length of time, or when we hold a heavy body with outstretched arm.

Walking consists of a series of intercepted falls. At each step the body is raised against the force of gravity through a certain distance, and then allowed to fall through the same distance. This distance is about 1.25 in. for a man, and about 3 in. for a horse. Thus at each step a positive amount of work is done against the force of gravity.

The rider of a racing sulky is behind the axle, thus tending to lift the horse. By this arrangement the work done by the horse in lifting himself at each step is diminished.

No work is done when a body rests upon a table or when the pressure of the steam in an engine does not move the piston. No work is done when a piece of iron is supported either by a string or by a magnet.

It should be kept in mind that for the performance of work, the two bodies between which the stress exists must move with respect to one another.

16. Measure of Work.—From the definition of work, the quantity of work done against an opposing force depends upon both the magnitude of this force and the displacement effected against this force. As an arbitrary convention, if a body is moved in the line of action of an opposing force, the magnitude of the work done, for a given change of the distance between the bodies between which the stress exists, is taken to be directly proportional to the resisting force; and, for a given resisting force, the work done is taken to be directly proportional to the change of distance between the bodies between which the stress exists.

A combination of these two variations gives the law,* *if a body is moved in the direction of the line of action of a force, the work done is measured by the product of the force and the displacement of the body.*

Or, in symbols

$$W = Fx. \qquad (1)$$

If the angle between the direction of the force and that of the displacement be zero, the work is done *by* the force. If the angle be 180°, the work is done *against* the force. If the angle be 90°, no work is done either by or against the force.

If the direction of the force F makes an angle ϕ with that of the displacement x, the work done equals the product of the force and the projection of the displacement in the direction of the force. Thus, in general,

$$W = Fx \cos \phi. \qquad (2)$$

---

* If A varies as B when C is constant, and A varies as C when B is constant, then will A vary as BC when both B and C vary. Hall and Knight, "Higher Algebra," p. 23.

Frequently a freight car is moved by a locomotive on a parallel track by means of a pole AB, Fig. 11. Let the force, in the direction of the pole be F, the displacement of the car be x and the angle between the pole and the track be ϕ. Then the projection of the displacement in the direction of the pole will be [$CD =$]$x \cos \phi$, and the work done will be $W = Fx \cos \phi$.

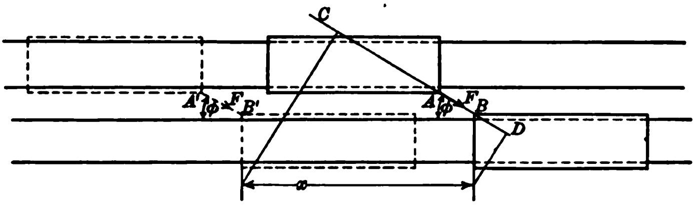


FIG. 11.

17. Efficiency.—The ratio of the work developed by any machine, to the work applied to it, is called the *efficiency* of the machine.

18. Energy.—If a spiral spring be compressed, a force must be exerted through a certain distance—that is, work must be done on the spring. If the compressed spring be released it can, in turn, do work on some other body. Similarly, if a body be raised from the earth against the force of gravity, work must be done upon the body. If the elevated body be allowed to fall, it can do work on some other body. Again, by applying force at the shaft of a fly-wheel, the fly-wheel can be set in rotation. After being set in rotation, the fly-wheel can do work on some other body in being brought to rest.

In the above examples of the distorted spring, the body raised against gravity, and the body set into rotation, all have the ability to do work on account of work having previously been done upon them in bringing them into their present condition. The ability to do work is called energy. *Energy* is stored work. The amount of energy possessed by a system of bodies is the amount of work it can do in passing from its present condition to some standard condition. Energy may appear in many different aspects. Thus we speak of mechanical energy, heat energy, electrical energy, etc. Energy in any form can be transformed into any other form.

19. The Principle of the Conservation of Energy.—Whenever work is done, one system of bodies loses energy and another gains energy. The results of most carefully planned and accurately executed experiments indicate that *the amount of energy lost by one system of bodies always equals the amount of energy gained by another.* In other words, energy can be neither created nor destroyed. All observed natural phenomena are in accord with this conclusion. This very important generalization is called the Principle of the Conservation of Energy.

It should be noted that there is no such thing as a law of the conservation of force. Force can be either created or destroyed at one part of a system of bodies without any corresponding change of force taking place in the remainder of the system. Force may appear or disappear without any change in the energy of the system. Work is not required to produce force, nor is energy necessarily liberated when force disappears. The failure to appreciate these facts has been a fruitful source of disappointment for inventors of "perpetual motion" machines.

20. The Branches of Physics.—Any great department of knowledge in which the results of investigation have been correlated and systematized is called a science. Physics is the science of energy. It investigates those phenomena of nature which depend either upon the transfer of energy from one body to another, or upon the transformation of energy from one aspect into another. For convenience of study, physics is frequently divided into the following branches: Dynamics, Sound, Heat, Electricity, Magnetism, and Light. Dynamics is that branch of physics which investigates force. Dynamics is divided into Statics and Kinetics. Statics treats of forces in equilibrium, while kinetics treats of the relation of force to motion.

Newton gave the name mechanics to the science of machines or the application of dynamics to machinery and engineering structures. In this he is followed by most logical modern writers, although it is not infrequent to see the terms dynamics and mechanics used interchangeably.

CHAPTER II

FORCES

§1. *Static Moments*

21. Translation and Rotation.—By translation is meant a displacement of a body from one position to another in such a manner that all points of the body traverse equal parallel paths. A displacement of a body such that all points describe circular paths about the same axis is called rotation. The axis of rotation may be outside the body or it may pass through the body.

Motion of translation is often called linear motion, and motion of rotation is often called angular motion.

Motions in opposite directions are distinguished by the signs plus and minus. Rotation in the direction of the hands of a clock may be termed positive, and rotation in the counterclockwise direction negative.

22. Torques. Moments of Forces.—When a force is applied to a body capable of angular motion, a change in the angular motion will be produced if the line of action of the force does not pass through the axis of rotation of the body. Whenever the angular velocity of a body about any axis is changing, there is said to be a "torque" acting about the given axis. A *torque* is anything capable of producing a change in the angular velocity of a body.

The numerical measure of the turning effect of a force in producing a change in angular velocity is termed the *moment of the force*. Thus a torque is measured by the moment of the force acting on the body.

The moment of a force equals the product of the force and the perpendicular distance from the axis of rotation of the body to the line of action of the force.

A moment tending to produce rotation about the given axis in the clockwise direction is termed positive; one tending to produce rotation in the counterclockwise direction is termed negative.

When a body is in equilibrium the motion is constant. If there is no change of angular motion about a given axis, the sum of the moments with respect to that axis of all the forces acting upon the body must equal zero. Hence *if a body is in equilibrium under the action of any number of coplanar forces, the algebraic sum of the moments of all the forces, about any axis normal to their plane, equals zero.*

This proposition is called the Theorem of Moments. In using the theorem of moments, due regard must be paid to the sign of each term. When distances are measured in feet, and forces in pounds weight, moments of forces are expressed in pound-feet. When distances are measured in meters and forces in kilograms weight, moments of forces are expressed in kilogram-meters.

The perpendicular distance from the axis of rotation of a body to the line of action of the force is called the *lever arm* of the force. From the fact that the moment of a force is measured by the product of the force and its lever arm, it follows that if the lever arm is long enough, the moment may be large even when the force is not very great, and that with a very small lever arm the moment of the force would not amount to much unless the force was a very great one. In fact, if the lever arm is zero, the moment of the force is zero, whatever finite value the force may have. This simply means that when the line of force passes through the axis, no force, however great, makes the body tend to turn in either direction.

23. Illustrations of Moments of Forces.—In Fig. 12, let *c* be the point of contact of the edge of a spool of thread with the table. The loose end of the thread is pulled with a force *F*. This force tends to make the spool turn about the point *c*.

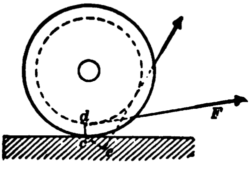


FIG. 12.

If the thread is pulled in the direction indicated, the line *cd*, which is the lever arm of the force, will lie above the table. The spool will roll to the right and wind up the thread. If, however, the thread is pulled in a direction more nearly straight up from the table, the lever arm will lie below the table top. In this case the spool will roll away to the left and unwind the thread.

The lever is a familiar application of the theorem of moments. It consists of a rigid rod, either straight or bent, capable of rotation about a fixed axis called the fulcrum. The forces applied to a lever are in a plane perpendicular to this axis. There is always one force at the fulcrum. If there are but two other forces, the position of the fulcrum with reference to these forces gives rise to three classes of levers, which can be studied in the following three diagrams:

Let C be the fulcrum about which rotation can take place; and let F and F' be two forces in equilibrium. The condition of equilibrium is now to be determined.

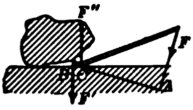


FIG. 13.

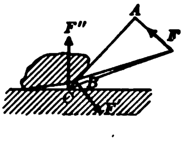


FIG. 14.

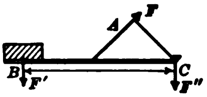


FIG. 15.

The same analysis applies to all three cases. From C draw lines CA and CB perpendicular to the lines of action of the forces. Then since the lever is in equilibrium, the sum of the moments of the forces about the fulcrum must equal zero. That is,

$$F(CA)-F'(CB)+F''0=0.$$

The mechanical advantage of a lever

$$\text{Mech. Adv.}\left[=\frac{F'}{F}\right]=\frac{CA}{CB}.$$

In hauling wagons, horses can exert all day without discomfort a pull of about one-tenth of their weight. In connecting to a wagon two or more horses abreast, a bar called an evener is employed. If the evener pin C, Fig. 16, is in the line connecting the whiffletree pins A and B, then, for any position of the evener, each horse must exert the same pull in order to keep the

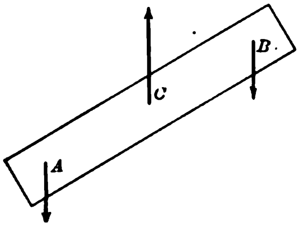


FIG. 16.

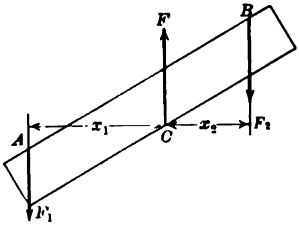


FIG. 17.

evener in equilibrium. But if the evener pin and the whiffletree pins are on opposite edges of the evener, as in Fig. 17, then, when the lighter horse is pulled back by the heavier horse, the lever arm x_2 of the force F_2 exerted by the lighter horse becomes less than the lever arm x_1 of the force F_1 exerted by the heavier horse. Then the sum of the moments of the force acting upon the evener with respect to C when in equilibrium, is

$$-F_1x_1+F0+F_2x_2=0,$$

whence,

$$F_2=F_1\frac{x_1}{x_2}.$$

That is, in order to keep this evener in equilibrium, the horse nearer the wagon must exert a greater force than the leading horse.

QUESTIONS

1. Draw a sketch of a carpenter's claw hammer in the act of drawing a nail from a board. Show the center of moments, the lever arm and all the forces acting on the hammer.

2. Show by diagrams the forces and their respective lever arms in the following cases: (*a*) a loaded wheelbarrow pushed by a man; (*b*) a nut cracker; (*c*) a pair of shears.

3. In order to assist a horse to haul a wagon out of a rut, where would a teamster apply a force most effectively?

4. When cutting a piece of sheet metal with the tinner's shears, is it easier to cut the metal when it is close to the pivot of the shears or when it is near the ends of the jaws? Why? In which case is more work done? Why?

5. A man supports a pole on his shoulder, holding one end in his hand and carrying a load attached to the other end. As he slides the pole backward, decreasing the distance between the hand and shoulder, does the force on the shoulder vary? Does the force of the man's feet on the ground vary? Diagram and explain.

6. Under the action of a horizontal force applied to the axle, a cart is on the point of going over an obstacle. Make a diagram of the forces acting on the cart. Give the moment of each relative to the point of contact of the cart and the obstacle. Show why it is easier to draw over rough ground a cart having large wheels then one having small wheels.

SOLVED PROBLEMS

PROBLEM.—A uniform plank 10 ft. long having a body weighing 25 lb. fastened to one end is balanced at a point 3 ft. from the weighted end. Find the weight of the plank.

Solution.—The plank is acted upon by three forces, viz., 25 lb. wt. downward at one end, the weight W of the plank downward at the middle point, and the reaction F' of the support upward at the point 3 ft. from the weighted end. These forces are represented in Fig. 18.

Since the plank is in equilibrium, the sum of the moments of all the forces acting upon it, about any axis normal to their plane, equals zero (Art. 22). Since the axis of moments may pass through any selected point, we will choose an axis that will make the moment of the unknown force F' equal zero. Such an axis must be at zero distance from the line of action of F'. Consequently we will take as axis of moments the line which passes through the point of support C and normal to the plane of the diagram. Therefore, with reference to this axis, we have

$$-25(3)+F'(0)+W(2)=0,$$

where the positive and negative signs are used according to the convention given in Art. 22.

Solving the above equation for W, we find the weight of the beam to be

$$W=37.5 \text{ lb. wt.}$$

Problem.—Find the least horizontal force applied at the center of a wheel 1 meter in diameter and weighing 25 kg. necessary to drag it over an obstacle 10 cm. high. (The weight of a wheel may be considered to act at its center.)

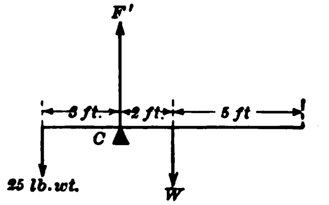


Fig. 18.

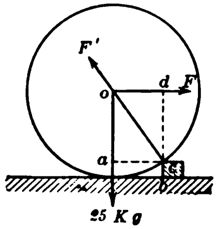


Fig. 19.

Solution.—The forces acting on the wheel are its weight, 25 kg., the reaction F' at the obstacle, and the unknown horizontal force F, applied as shown in Fig. 19. When the wheel is just on the point of moving over the obstacle, these forces are in equilibrium. In this case, the sum of the moments of all the forces acting on the wheel about any axis normal to their plane equals zero (Art. 22). By selecting as axis of moments a line passing through some point in the line of F', the moment of this unknown force is made equal to zero. Let the axis of moments be the line which passes through the point c and normal to the plane of the diagram.

From the data given in the problem, $oc=50$ cm., $bc=10$ cm., and consequently $cd=40$ cm., and $ca=30$ cm.

Consequently, when just on the point of rising over the obstacle, the sum of the moments with respect to an axis through c of all the forces acting on the wheel is

$$-25(ca)+F'(0)+F(cd)=0,$$

or

$$25(30)=F(40),$$

whence

$$F=18.75 \text{ kg. wt.}$$

§ 2. *Composition and Resolution of Forces*

24. The Resultant of Two Concurrent Forces.—Forces are said to be *coplanar* when their lines of action are in the same plane. Forces are said to be *concurrent* when their lines of action intersect. Forces are said to be *colinear* when their lines of action are coincident. Two forces or systems of forces having identical effects on the motion of a body with respect to both translation and rotation are said to be *equivalent*. A single force equivalent to two or more forces is called the *resultant* of the set. A single force which, if added to a set of forces, will produce equilibrium, is termed the *equilibrant* of the system of forces. The resultant is equal in magnitude, in the same straight line, and opposite in direction to the equilibrant. The operation of finding the resultant of a set of forces is termed *composition* of forces.

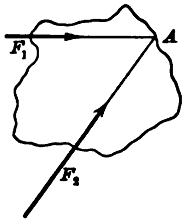


FIG. 20.

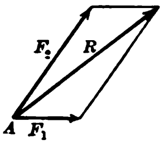


FIG. 21.

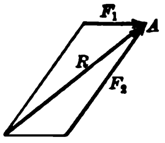


FIG. 22.

When two concurrent forces act upon a body, Fig. 20, their resultant may be found by the following method. Lay off either from a given point, A, Fig. 21, or toward a given point, A, Fig. 22, two lines representing in magnitude and in direction the two forces. On these two lines construct a parallelogram. The diagonal of this parallelogram represents in magnitude and in direction the resultant of the two forces. If the two lines representing the given forces are drawn from their point of intersection, then the resultant is directed from their point of intersection. Whereas if the two lines are drawn toward their point of intersection, then the resultant is directed toward their point of intersection. The basis of this method is the proposition called the Law of Parallelogram of forces, which may be stated as follows: *If two adjacent sides of a parallelogram represent in magnitude and direction two concurrent forces, both acting either toward or away from their point of intersection, then the diagonal of the parallelogram passing through their intersection represents the resultant of the forces both in magnitude and in direction.*

25. Computation of the Resultant of Two Concurrent Forces.—Let AB and AC represent in direction and in magnitude two forces F_1 and F_2 whose lines of action are inclined to one another at an angle ϕ. Then the resultant of the forces will be represented by the diagonal AD of the parallelogram erected on the lines AB and AC. The magnitude and direction of the resultant R can be determined by the usual trigonometric methods of solving oblique triangles. Thus, from the law of cosines,

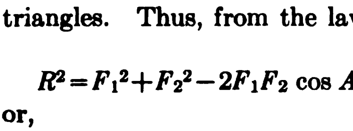


FIG. 23.

$$R^2=F_1{}^2+F_2{}^2-2F_1F_2\cos ABD=F_1{}^2+F_2{}^2+2F_1F_2\cos EBD,$$

or,

$$R^2=F_1{}^2+F_2{}^2+2F_1F_2\cos\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation gives the magnitude of the resultant of two concurrent forces in terms of the magnitudes of the two given forces and the angle between them.

26. Resultant of Any Number of Concurrent Forces.—Let F_1, F_2, F_3, etc., be any number of concurrent forces. Let the resultant of F_1 and F_2 be R_1. R_1 can now be compounded with F_3, giving a resultant R_2. Manifestly R_2 is the resultant of F_1,

F_2, and F_3. Proceeding in like manner with the other forces, the resultant of the whole system can be obtained. The following two paragraphs show how this method can be considerably abbreviated.

27. The Triangle of Forces.—From the law of the parallelogram of forces, the resultant of two forces F_1 and F_2 applied at any point A (Fig. 24), is given by the diagonal AD of the parallelogram constructed on these two sides. In the figure, since the line BD is in the same direction and has the same length as the line AC, the forces F_1, F_2, and R are represented in magnitude and direction by the sides of the triangle ABD. Whence, if the lines A' B' and B' D' (Fig. 25) are drawn in the directions of the concurrent forces F_1 and F_2 and of lengths proportional to their respective magnitudes, then the line $A'D'$ closing the triangle will represent in magnitude and direction the resultant of the two forces.

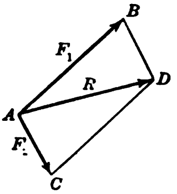


Fig. 24.

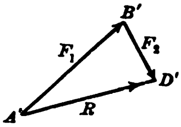


Fig. 25.

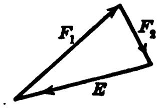


Fig. 26.

Since the equilibrant of a system of forces is equal in magnitude and is in the opposite direction to the resultant, the forces F_1, F_2, and their equilibrant E, would be represented as in Fig. 26. Consequently, *if three concurrent forces are represented in magnitude and direction by the three sides of a triangle described in succession, the forces are in equilibrium.* This proposition is called the Triangle of Forces.

28. The Polygon of Forces.—By an extension of the method used in the preceding paragraph it can be shown that if any number of concurrent forces are represented in direction and in magnitude by lines forming sides of a continuously described polygon, the line drawn from the starting point to close the polygon represents the resultant in magnitude and direction. Consequently, *if a number of concurrent forces be represented in direction and magnitude by the sides of a closed polygon taken in order, the forces are in equilibrium.* This proposition is called the Polygon of Forces.

29. Haulage by Horses.—Let a body, Fig. 27, be in equilibrium under the action of the weight W, the reaction F', and the force F. With reference to an axis through c perpendicular to the plane of the diagram, the sum of the moments of all the forces acting upon the body is

$$F'0+W(ca)-F(cb)=0.$$

Whence, the magnitude of the force F necessary to hold the body in equilibrium is

$$F=W\frac{ca}{cb}.$$

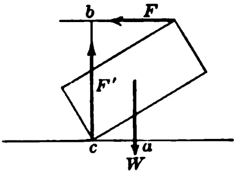


FIG. 27.

In walking, a man or a horse starts to fall, rotating about the rear foot as an axis, intercepts the fall by means of another foot, advances the foot formerly used as the axis of rotation, starts to fall by rotating about the other foot, as an axis, and so on. A cart or other body attached to the man or horse is acted upon by a force equal and opposite to F in the above equation.

In the case of a horse and cart, we will now consider the effect on the direction of the line of draft produced by changes in the distribution of the load in the cart and variations in the grade of the road.

With the load to the rear of the cart axle, there will be an upward thrust at the bellyband represented by xy, Fig. 28. Due to the grade and the friction there will act upon the horse in the direction of the traces a force xz. The resultant of these two forces is R. The moment of the resultant with respect to an axis through the point of contact of the rear foot and the ground is $-R(cb)$. At the instant when the other three feet are off the ground, this moment is counterbalanced by the moment of the weight of the horse, $w(ca)$. Thus,

$$-R(cb)+w(ca)=0,$$

or

$$R=w\frac{ca}{cb}.$$

The resultant R due to the load and the grade is in such a direction as to produce a tendency to lift the rear foot off the ground, thereby causing slipping.

The actual direction of the thrust on the rear foot is obtained by compounding R and the effective weight on the foot.

With the load forward of the cart axle, there will be a downward thrust xy', Fig. 29. The resultant of this force and the force xz due to the friction and grade is R'. The moment of this resultant with respect to an axis through c is $-R'(cb')$. This moment is counterbalanced by $w(ca)$.

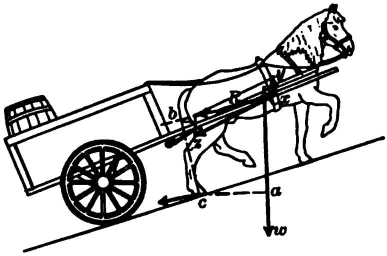


FIG. 28.

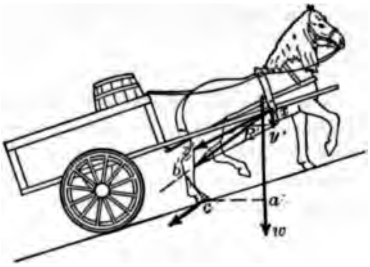


FIG. 29.

$$R' = w\frac{ca}{cb'}.$$

The direction of R' is such as to cause the rear foot to dig into the ground and not slip.

The total work done by a horse hauling a load is made up of two parts. The useful work equals the product of the distance traveled and the component parallel to the roadbed of the resultant force due to the load. The useless work equals that required to overcome friction, together with that required to lift the horse at each step. The latter equals the product of the effective weight of the horse, the vertical distance the horse lifts himself at each step and the number of steps. With the load in the rear of the cart the effective weight of the horse and the useless work are diminished. On hard level roads where there is little tendency to slip this is an advantage. But in going up hill over obstacles, the tendency of the rear foot to slip more than counterbalances the advantage of diminished useless work. In hauling up hill a cart loaded in the rear, the horse will be actually helped by an extra load on his back—a rider for example.

30. The Force Couple.—A system composed of two equal, parallel, and oppositely directed forces having different lines of action is termed a *force couple.* A couple tends to produce a change in the angular motion of the body to which it is applied, about an axis normal to the plane of the forces constituting the couple.

Consider a body (Fig. 30) acted upon by a couple consisting of two forces F_1 and F_2 whose lines of action are separated by a distance AB. It is required to find the moment of the couple, that is, the importance of the effect of the couple on the rotation of the body. The moment of a force couple about an axis normal to the plane of the forces and passing through any point C equals the sum of the moments of the two forces F_1 and F_2 about the axis. That is, the moment of a couple about this axis is,

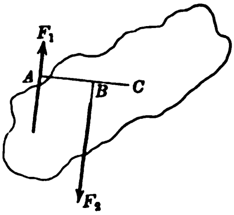


Fig. 30.

$$F_1\,(AC) - F_2\,(BC).$$

And since $F_1 = F_2$, and $AC = AB + BC$, the moment of the couple about the assigned axis is

$$F_1(AB) + F_1(BC) - F_2(BC) = F_1(AB).$$

Consequently, *the moment of a force couple equals the product of one of the forces and the perpendicular distance between the lines of action of the two forces constituting the couple, and is independent of the position of the axis of rotation.*

A couple that tends to produce rotation in the clockwise direction is said to be positive, while one that tends to produce rotation in the counterclockwise direction is said to be negative.

A couple can be held in equilibrium only by the application of another couple having an equal moment of the opposite sign. An unbalanced force produces a change in the linear motion of the body acted upon; an unbalanced couple produces a change in the angular motion of the body acted upon. The turning of a screw with a screw driver, and the turning of a bolt nut with a wrench, are examples of the action of force couples.

31. The Centroid of a System of Parallel Forces.—Let A and B (Fig. 31) be the fixed points of application of the forces F_1 and

F_2, and let P be the point of intersection of the line AB by the line of action of the resultant. So long as the two given forces remain parallel and continue to act in lines passing through A and B, then, however the body is displaced or however the direction of the forces may be changed, the line of action of the resultant will continue to pass through the same point P. Consequently, the point P is a definite point fixed in the body. This point has so many interesting properties that it has been given a name. The point of application of the resultant of a system of parallel forces is called the *centroid,* or the center of the system.

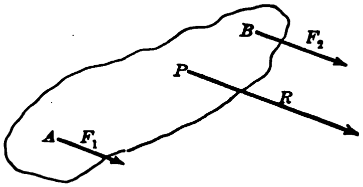


Fig. 31.

32. The Center of Gravity of a Body.—"If the action of terrestrial or other gravity on a rigid body is reducible to a single force in a line passing always through one point fixed relatively to the body, whatever be its position relative to the earth or other attracting mass, that point is called its center of gravity."* The only bodies which have true centers of gravity are uniform spherical shells, uniform spheres, and spheres whose density changes from the center to the circumference according to some definite law. In other cases the line of action of the weight of the body does not pass through the same point when the position of the body is changed.

Although when a body is acted upon by a system of nonparallel forces there will be no invariable point of application of the resultant as the position of the body is changed, the departure from parallelism of the gravitational forces acting on the different parts of a body is so small that it is customary, especially in engineering problems, to assume that there is a definite point fixed in the body at which the weight of the body is applied. Thus, it is assumed that every body has a center of gravity coincident with that point at which the resultant of the gravitational forces acting on the body would be applied if they were parallel. Custom

* Thompson and Tait, "Natural Philosophy," II, p. 78.

having sanctioned this loose use of the term, it will occasionally be employed in the succeeding pages.

If it be assumed that the gravitational forces acting on all parts of a body act in parallel lines, it can be shown that the point of application of the weight of uniform spheres, cylinders, and prisms is at their geometrical centers.

QUESTIONS

1. A telephone pole line turns a 90° corner. By means of a diagram indicate the method of determining the direction of the guy wire that should be attached to the corner pole to prevent bending.

2. A boy in a swing inclined to the vertical is acted upon by his weight and by the tension of the rope. Indicate the method of finding the resultant force.

3. A picture is hung by means of a wire attached to the frame on opposite sides, and passing over a hook. What effect will an increase or a decrease in the length of the wire have upon the tension in the wire? Explain with diagrams. If in doubt as to the ability of the wire to sustain the weight of the picture, will it be safer to use a long or short wire? Explain.

4. Suppose that the ends of the traces attached to the horse are further from the ground than the ends attached to the wagon. Show under what conditions the wagon will be drawn with greater ease when, (*a*) the traces are long; (*b*), the traces are short. If the traces were parallel to the ground, would there be any choice between a " long hitch " and a " short hitch "?

5. What are the forces that constitute the couple in each of the following cases? (*a*) the couple that acts on one of the wheels of a clock; (*b*) the couple that prevents a picture from rotating forward about the line where it touches the wall; (*c*) the couple that acts on a rotating lawn sprinkler; (*d*) the couple that acts on a door while being closed.

33. Components of a Force.—Two forces which, acting together, are equivalent to a single force are called the *components* of the given force. The operation of finding two components of a given force is called resolution of the force.

Since a straight line can be the diagonal of an indefinite number of parallelograms, it follows that any force can be the resultant of an indefinite number of pairs of components. For instance, in Fig. 32, R is the resultant of F_1, and F_2; in Fig. 35, R is the resultant of two different components F_3 and F_4; while in Fig. 34, R is resolved into still another pair of components, F_5 and F_6.

If, however, the directions of the two components be assigned, then there is but one pair of forces equivalent to the given force. The most useful resolution is into components at right angles to one another. In this case, the required components are called the rectangular components of the given force.

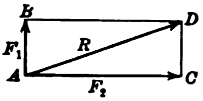


FIG. 32.

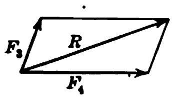


FIG. 33.

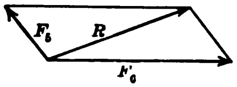


FIG. 34.

34. Rectangular Components of a Force.—To fix the ideas, let it be required to find the vertical and horizontal components of a force represented in direction and magnitude by the line AD (Fig. 32). With this line as a diagonal, construct a parallelogram having its opposite sides respectively vertical and horizontal. From the law of the parallelogram of forces, it follows that if the lines AB and AC represent in direction and in magnitude two forces F_1 and F_2, then the diagonal AD represents the direction and magnitude of their resultant. Conversely, AB and AC represent the two components of R in the required directions.

Let the angle between R and F_2 be called ϕ. Then the vertical component of R is

$$F_1 = R \sin \phi,$$

and the horizontal component of R is

$$F_2 = R \cos \phi.$$

As forces are seldom resolved in any directions except those perpendicular to one another, the terms "component of the force" and " resolved part of the force " are commonly used to denote the rectangular component of the force in the assigned direction.

SOLVED PROBLEM

PROBLEM.—In a certain steam engine the piston rod exerts a thrust of 22,500 lb. wt. Find the thrust on the guides when the connecting rod makes an angle of 15° with the line of action of the piston rod.

Solution.—At the junction of the piston rod and the connecting rod, the thrust along the piston rod is resolved into two components—one which makes an angle of 15° with the piston rod and another which makes an angle of 90° with the piston rod. Thus the present problem resolves itself into finding the magnitude of two forces having their lines of action in the directions *AB* and *AC* (Fig. 35), and which have as their resultant a force in the direction *AD* equal to 22,500 lb. wt.

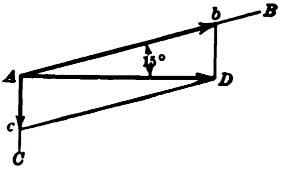


Fig. 35.

From *D* draw lines parallel to *AB* and *AC*, respectively. In the parallelogram *AbDc* thus formed, the diagonal *AD* represents the resultant of two forces which are represented by *Ab* and *Ac*. Therefore the line *Ac* represents the required thrust against the guides when the connecting rod makes an angle of 15° with the line of action of the piston rod.

From the diagram,

$$Ac = Ad \tan 15° = (22500)(0.2679) \text{ lb. wt.}$$
$$= 6027.75 \text{ lb. wt.}$$

35. The Sailboat.—Consider the case of a sailboat sailing into the wind. Let *CA* be the force of the wind on the sail. One part of this force, that is, the component *BA* parallel to the sail, is ineffective in moving the boat. The component *CB* of the force *CA*, acting normal to the sail, produces a pressure on the sail. Resolve *CB* into components parallel and perpendicular to the keel of the boat. The component *DB* is the only part of the force of the wind acting in the direction of the keel of the boat; that is, it is the only part of the force of the wind effective in propelling the boat forward. The component *CD* tends to displace the boat sideways. This tendency is partly prevented by the pressure of the water against the keel and side of the boat. This pressure acts at some point *O*, and the boat is so designed that the center of pressure of the wind is slightly aft from *O*. The wind and water together, then, tend to turn the boat so that it heads more nearly into the wind. This tendency is overcome by the rudder.

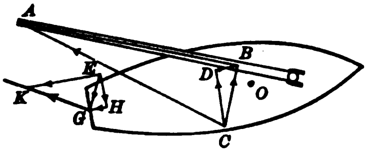


Fig. 36.

The force acting on the rudder will be represented by *EK*. Of this force the component *GK* parallel to the rudder exerts no action on the rudder. The normal component *EG* produces a pressure on the rudder. This pressure *EG* may be resolved into two components *HG* and *EH*, parallel and perpendicular respectively to the keel of the boat. The component *HG* acts as a resistance to the boat's motion. The component EH tends to push around the stern of the boat. When CD is the equilibrant of EH and the pressure of the water at O, the boat will not turn. When, in addition, $HG=DB$, the boat is in equilibrium and moves with uniform velocity.

36. The Kite.—The kite represented edgewise by AB, Fig. 37, is acted upon by the weight W and the force F due to the wind. Of the force F, however, the component parallel to the plane of the kite produces no effect. The wind pressure on the kite surface is due to the component F' perpendicular to the plane of the kite. The resultant of the weight of the kite and the part of the force of the wind that produces pressure on the kite surface is obtained by compounding W and F'. To equilibrate this resultant R, the string must act with equal force in the opposite direction.

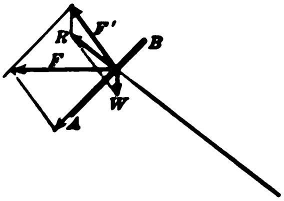


Fig. 37.

Questions

1. In what sense is it easier to load a barrel into a wagon by rolling it up a plank than by lifting it vertically from the ground to the wagon? What is the amount of work done in the two cases? Explain fully.

2. Assume a sailboat going east. Let the sail be set at some small angle with the keel and let the wind strike the sail in a direction other than at right angles. Show clearly with lines and arrows the double resolution necessary to find the effective easterly component of the wind's force. Show the position of the rudder, and resolve the force of water against it into proper components. Show the effect of each of these.

3. It is desired to sail a ship from A to B, against a wind in the direction of BA. Explain how it is possible to do this.

4. By composition and resolution of forces, (a) show how a kite is supported in the air by a horizontal wind; (b) show how an aeroplane is pushed through the air by the action of a propeller; (c) show that if the plane slants upward there will be a tendency for the aeroplane to rise.

§ 3. *Conditions of Equilibrium*

37. Equilibrium of a System of Coplanar Forces.—A body is in equilibrium when its motion does not change. In order that its linear motion may not change, the resultant force acting on the body must equal zero. In order that its angular motion may not change, the resultant torque about any chosen axis must equal zero. If all the forces are in one plane, the first requirement is satisfied if the sum of the components of all the forces is zero for each of any two directions of resolution. Therefore, *if a body is in equilibrium under the action of a system of coplanar forces,*

(a), the sum of the components of the forces is zero for each of any two directions of resolution;

(b), the sum of the moments of the forces about any axis normal to their plane equals zero.

The conditions of equilibrium of a system of coplanar forces can be expressed in the form of three equations. Two of these equations are obtained by resolving all the forces in any two convenient directions, and equating separately the sum of the components in each direction to zero. It is usually most convenient to make the resolutions in directions perpendicular to one another. The third equation is obtained by taking the moments of all the forces about an axis which is normal to their plane, and which passes through any convenient point, and equating the sum to zero. For example, if vertical and horizontal components of all the forces be found, and moments of all the forces be taken with respect to an axis through any chosen point "c," the conditions of equilibrium may be expressed by symbols in the abbreviated form,

$$\left.\begin{aligned} \Sigma f_v &= 0 \\ \Sigma f_h &= 0 \\ \Sigma L_c &= 0 \end{aligned}\right\}, \qquad \qquad (4)$$

when ΣL_c represents the sum of the moments, with respect to an axis through c, of all the forces acting upon the body.

In order to avoid, as much as possible, unknown forces appearing in the equation, it is advisable to make one of the resolutions perpendicular to an unknown force, and to take moments about an axis passing through a point in its line of action. The value of these suggestions will appear in the solution of the following problems.

38. Stability of Equilibrium.—If a body is at rest with respect to any selected point of reference, it is said to be in *static equilib-*

rium. A brick in a chimney is in static equilibrium with reference to the earth. A lump of mortar adhering to a falling brick is in static equilibrium with reference to every point of the brick, but is not in equilibrium with reference to any point of the earth. A body moving with either constant linear or constant angular motion is said to be in kinetic equilibrium.

Depending upon the behavior of a body in equilibrium after it has suffered a slight rotation, three classes of the state of equilibrium are distinguished. If after suffering a slight rotation the body recovers its former position, the equilibrium of the body is said to be *stable.* If, however, after receiving a slight angular displacement, the body departs further and further from its former position, the equilibrium is termed *unstable.* If, after suffering a slight angular displacement, there is no tendency of the body either to recover its former condition or to depart further from it, the body is said to be in *neutral* or *indifferent* equilibrium. The degree of stability of a body is measured by the amount of work necessary to effect a permanent change in the position or condition of the body.

A right cone resting with its base on a horizontal table is in stable static equilibrium; so, also, are a suspended plumb-bob and a body suspended by a vertical spiral spring. An egg balanced on an end, or a lead pencil on its point, are examples of unstable static equilibrium. A right cone lying on its side on a smooth horizontal table, or a wooden sphere floating in a basin of water, or a wheel on a smooth axle, are examples of neutral static equilibrium.

A boat being poled up a stream will be in stable equilibrium while the man is in the bow, but will be in unstable equilibrium if he poles from the stern. In order that a flying skyrocket shall not wobble, the point of application of the force acting upon it—that is, the orifice from which the gases escape—must be far forward. This is accomplished by having a long stick attached to the body of the rocket and extending backward from the fuse hole.

SOLVED PROBLEMS

PROBLEM.—The upper ends of two beams of equal length l and weight w are pinned together and the lower ends rest on the tops of two walls of equal height. There is a load W at the pin connecting the two beams. Find the thrust tending to overturn each wall and the vertical force on top of each wall.

Solution.—The forces developed when the beams are at various angles to the horizontal can be actually observed with the model illustrated in Fig. 38. The general equation now to be determined is, however, more instructive.

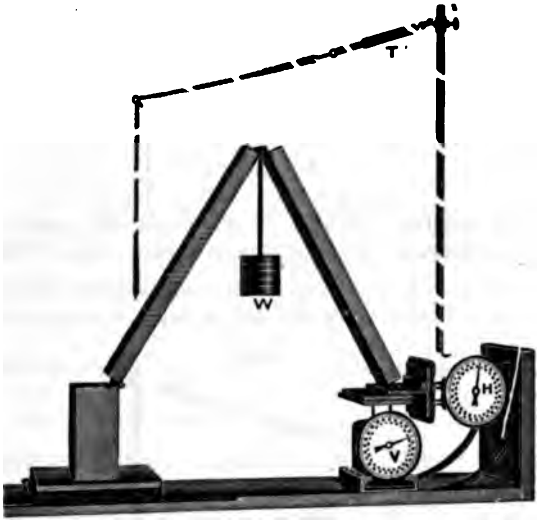


Fig. 38.

In problems on a body in equilibrium, it is essential that all of the forces acting upon the body shall be represented, and no others. In the present problem, each beam is acted upon by five forces as follows: the load $W/2$ acting vertically downward at the upper end; the horizontal thrust F, Fig. 39, due to the other beam; the weight w acting at the middle point; the vertical and horizontal reactions F_v and F_h due to the wall.

Since the beam is in equilibrium, the sum of the components of the forces is zero for any two directions of resolution. Thus,

Horizontal components $F-F_h=0$; . . (5)

Vertical components $F_v-\frac{1}{2}W-w=0$, . (6)

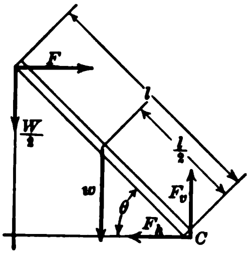


Fig. 39.

Also, the sum of the moments of the forces about any axis normal to their plane equals zero. Thus, taking moments about an axis through C,

$$-\tfrac{1}{2}Wl\cos\theta+Fl\sin\theta-w(\tfrac{1}{2}l\cos\theta)+F_h0+F_v0=0,$$

or

$$F=\tfrac{1}{2}(W+w)\cot\theta. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

From (5) and (7), the horizontal force which must be exerted at the lower ends of the beams in order to keep the beams from spreading, and consequently the horizontal thrust tending to overturn each wall, has the value

$$F_h[=F]=\tfrac{1}{2}(W+w)\cot\theta. \quad \text{. . . . .} \quad (8)$$

From (6), the vertical reaction at the lower end of each beam, and consequently the downward force on top of each wall, has the value

$$F_v=\tfrac{1}{2}W+w.$$

Note.—The relation expressed in (8) shows why semicircular stone arches require more massive abutments than pointed arches of the same span.

Problem.—A derrick has a boom 20 ft. long, weighing 150 lbs., with one end hinged to a vertical mast 20 ft. from the top. A rope extends from the other end of the boom to the top of the mast. The " center of gravity " of the boom is 8 ft. from the lower end. When the boom makes an angle of 60° with the mast, and supports at its end a load of 1200 lb. wt., find the tension in the rope, and the vertical and horizontal thrusts on the pin of the hinge connecting the boom to the mast.

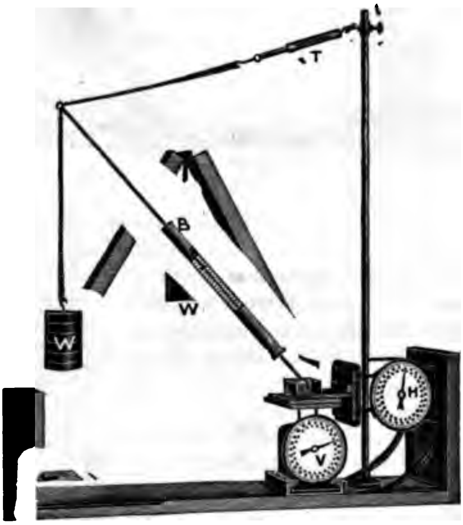


Fig. 40.

Solution.—Since the boom is in equilibrium under the action of the forces represented in Fig. 41, we have

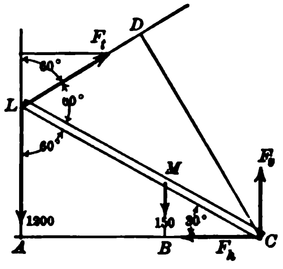


Fig. 41.

$$\text{Horizontal components, } F_h - F_t \sin 60° = 0, \quad \ldots\ldots \quad (9)$$

$$\text{Vertical components, } F_v - 150 - 1200 + F_t \cos 60° = 0. \quad . \quad (10)$$

Moments about an axis through C,

$$-1200\ (CA) + F_t(CD) - 150\ (CB) + F_h0 + F_v0 = 0$$

$$-1200\ (CL \sin 60°) + F_t(CL \sin 60°) - 150\ (CM) \sin 60° = 0$$

$$-1200\ (20) + F_t(20) - 150(8) = 0.$$

$$F_t = 1260 \text{ lb. wt.}$$

Substituting this value of F_t in (9)

$$F_h[= F_t \sin 60°] = 1260 \times 0.866 = 1091 \text{ lb. wt.}$$

Substituting the value of F_t in (10)

$$F_v[= 150 + 1200 - F_t \cos 60°] = 1350 - 1260\ (0.5) = 720 \text{ lb. wt.}$$

If desired, the magnitude and direction of the force acting on the lower end of the boom can be obtained by finding the resultant of F_h and F_v.

Questions

1. Which of the following bodies are in equilibrium? For those which are in equilibrium, the equilibrium is of which of the three types? (*a*) the bob of the pendulum at the middle of its path; (*b*) a broom balanced vertically on your finger; (*c*) an egg lying on its side; (*d*) the hour hand of a watch.

2. Three equal forces act simultaneously upon a body. Represent graphically three forces so chosen that the body is in equilibrium. Explain. Imagine two of these forces to be doubled. How must the third be changed in order to preserve equilibrium? Show graphically and explain. Show how three forces, not meeting in a point, and not all equal, may maintain a body in equilibrium.

3. Why does a balance beam return to a horizontal position when displaced therefrom? How could the beam be made to serve as an illustration of (*a*) stable, (*b*) neutral, (*c*) unstable equilibrium?

4. Show clearly why a ladder leaning against a smooth wall is more likely to slip when a man is near the top then whan he is near the bottom.

5. A painter's ladder rests with one end against a smooth vertical wall and the other on the top of a trestle. Make a diagram of the forces acting and state the condition limiting the distance the man can go up the ladder without overturning the trestle.

CHAPTER III

FRICTION BETWEEN SOLIDS

39. Force at a Smooth Surface.—A surface which offers no resistance to the sliding of a body along it is said to be *smooth.* Smooth surfaces are ideal, but some surfaces are so nearly smooth that for many purposes they may be considered to be quite so.

The force which a smooth surface exerts upon any body in contact with it is always normal to the surface: for, if it were not normal, there would be a component of the force that would oppose motion in some direction—that is, the surface would not be smooth.

40. Static and Kinetic Friction.—Consider a body resting on a horizontal plane (Fig. 42). The body is in equilibrium under the action of the force F_n normal to the plane (in this case the weight of the body), and the counteraction R of this force. If a very small force f_a be applied to the body parallel to the interface, the body will not move unless the surface of contact is smooth. The reason is that when the body is pulled forward along the supporting plane the latter reacts, pushing back against the body, parallel to the slipping surface, just as hard as the body pushes forward. This tangential reaction f which keeps the body from moving is called the *static frictional resistance* or the *static friction* between the two surfaces. If the applied force f_a be increased, the static friction f will increase at the same time; and if the applied force be decreased, the static friction will decrease at the same time—the static friction being always, so long as the body does not slip, equal to the force that tends to move the body along the surface.

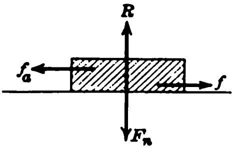


FIG. 42.

When the force parallel to the slipping surface reaches a certain definite value, the body slips. That is, the static friction cannot increase further and is no longer equal to the force that pushes the body forward. If the force f_a that is being applied at the instant when the body slips continues to be applied, the body moves along the surface with a continually increasing speed. That is, if it is desired to have the body move with uniform speed, the applied force f_a must be decreased. It is found, however, that a certain definite force does need to be applied to keep the body moving uniformly along the surface. This is because the surface reacts against the push that urges the body forward. The tangential reaction which a surface exerts upon a body moving along it is called the *kinetic frictional resistance* or the *kinetic friction* between the two surfaces.

When a body is moving with uniform velocity, it is in equilibrium, and the force that is needed to keep it moving equals the kinetic friction. It is found that both kinetic and static friction are independent of the area of the surface of contact. It is also found that if the velocity is not exceedingly small nor exceedingly great, the kinetic friction is almost independent of the velocity with which the body moves so long as the slipping surfaces remain unchanged.

The way in which friction changes when the force that tends to move a body along a surface changes may conveniently be represented by the curve shown in Fig. 43. This curve shows that as the force which urges the body along the surface increases, the frictional resistance to motion is equal to this force until the point A is reached. At that point the frictional resistance drops rather abruptly, and beyond that point the frictional resistance to motion is practically independent of the force that urges the body forward. Up to the point A the friction is static friction, beyond that point it is kinetic friction. The kinetic friction, then, has a certain rather definite value, whereas the static friction may have any

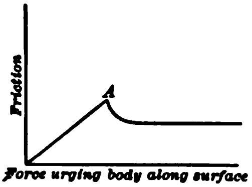


FIG. 43.

value from zero up to a value greater than that of the kinetic friction.

In the above paragraphs it has been assumed that the surface over which the body moves is horizontal. This, of course, may not be the case. But the friction is in every case a reaction that is parallel to the surface along which the body moves, and the force urging the body along the surface is in every case a force parallel to the surface.

The friction between two surfaces of the same material is usually greater than between surfaces of different materials. For example, the friction between two clean plane glass surfaces is very great. Clean surfaces of like material tend to hold together. Leather-covered pulleys are used for leather belts when great friction is required. Brass bearings are used for steel journals.

41. **Illustrations Distinguishing Static from Kinetic Friction.**—When a train is in motion and the wheels are not slipping on the rails, there is static friction between the wheel and the rail, and kinetic friction between the axle and bearings. When all the couplers of a long train of cars are tight, a locomotive may be unable to start a heavy train. But by first backing the locomotive until a number of couplers are loose, and then going ahead, the locomotive may be able to start in motion one car at a time until the whole train is moving. This is a case of overcoming piecemeal the static friction on the bearings of the train. When once in motion, the large static friction is replaced by the smaller kinetic friction so that the train can easily be kept in motion.

Consider the case of stopping a train. By increasing the pressure on the brakes the kinetic friction between the wheel and the brakeshoe is increased as long as the wheel turns in the brakeshoe. After this point the brakeshoe "seizes" the wheel and the latter skids. Just before the brakeshoe seizes the wheel, the wheel is on the verge of slipping on the rail. At this instant the kinetic friction between the wheel and brakeshoe equals the maximum static friction between the wheel and the rail—that is, is greater than the kinetic friction between the wheel and the rail would be. This means that at this instant the force that opposes the turning of the wheels is greater than the force that would oppose the sliding of the wheels along the rails if the wheels were prevented from turning. Thus, the maximum effect of the brakes is produced by tightening them until they are just on the point of seizing the wheels but still allowing them to turn.

42. **Coefficients of Friction.**—In order to keep a body A moving with uniform speed over a body B, a certain force f_p must be applied

to *A* parallel to the surface of *B*. If the force that presses *A* against *B* be doubled it is found that in order to keep *A* moving uniformly the force f_p must also be doubled. In general, the force f_p that must be applied parallel to the surface of a body in order to keep another body moving with uniform speed along it is proportional to the force F_n that presses the two bodies together. That is,

$$f_p = bF_n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where b is a quantity known as the coefficient of kinetic friction between the two given surfaces. Since the force needed to keep a body moving with uniform speed equals the force that opposes the motion, the f_p in (11) equals the kinetic friction. It follows, then, that the *coefficient of kinetic friction* may be defined as the ratio of kinetic friction to the force that presses the two bodies together.

In order to set the body *A* into motion over the body *B* a certain force F_p must be applied to *A* parallel to the surface of *B*. If the force that presses *A* against *B* be doubled, it is found that in order to start *A* the force F_p must also be doubled. In general, the force F_p that must be applied parallel to the surface of a body in order to start another to moving along it is proportional to the force F_n that presses the two bodies together. That is,

$$F_p = \mu F_n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where μ is a quantity known as the coefficient of static friction between the two given surfaces. Since, up to the point when the body begins to slip, the force that is applied parallel to the surface equals the frictional resistance to motion, the F_p in (12) equals the friction when the body is just on the point of slipping. It follows, then, that the *coefficient of static friction* may be defined as the ratio of the friction when one body is just on the point of slipping to the force that presses the two bodies together.

Since the static friction when the body is just on the point of slipping is greater than the kinetic friction, it follows that the coefficient of static friction is always greater than the coefficient of kinetic friction.

43. The Laws of Friction.—In the preceding article it has been stated that experiments show that both static and kinetic friction, (*a*) vary directly with the force normal to the slipping surfaces, and (*b*) are independent of the area of the surface of contact; and, (*c*) that kinetic friction is nearly independent of the relative speed of the two bodies so long as the slipping surface remains constant.

Numerous cases occur which have been imagined to be in opposition to the last two laws. One case under each will now be considered. By increasing the angle of contact between a belt and a pulley, the static friction between the two is increased. As the surface of contact is increased when the angle of contact is increased, some have argued that the increase of friction is due to the increase of the area of contact. In Fig. 44, let *EGHJ* represent the portion of the belt in contact with the pulley whose center is *C*. On account of the friction between the two surfaces, the tension of the belt will vary all along the length in contact with the pulley. When the belt is just on the point of slipping, let the tensions at the ends of the arc *GH* subtending the indefinitely small angle $\Delta\theta$ be denoted by f and f'. Let F and F' represent the tensions of the belt where it leaves the pulley.

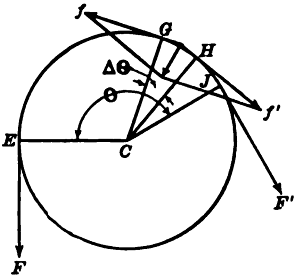


Fig. 44.

By compounding the forces f and f'—which are approximately equal because $\Delta\theta$ is very small—it is found that the normal force R against the pulley, due to the element of the belt *GH*, is given by the equation

$$R^2 = f^2 + f'^2 + 2ff' \cos (180 - \Delta\theta)$$

$$= 2f^2[1 + \cos (180 - \Delta\theta)] = 2f^2[2 \cos^2 \tfrac{1}{2}(180 - \Delta\theta)].$$

$$\therefore \quad R = 2f \cos \tfrac{1}{2}(180 - \Delta\theta) = 2f \sin \tfrac{1}{2}(\Delta\theta) \doteqdot f\Delta\theta.^*$$

From this equation it follows that the force normal to the surface of contact varies almost directly with the angle of contact between a belt and a pulley. It is for this reason that the static force of friction increases with the angle of contact of a belt.

On pressing a cast-iron brakeshoe against a car wheel with sufficient force to quickly stop the car, it is found that when the car is running 60 miles per hour the relation between the force of friction and the force normal to the rubbing surfaces is from one-third to one-half as great as when the car is running 20 miles per hour. This does not mean that the coefficient of kinetic friction of cast iron per hour when the speed is 60 miles is from one-third to one-half as great as when the speed is 20 miles per hour. The force of friction is less because the surfaces have changed. Under the high speed much heat is developed and the abraded material is sufficiently soft to be rolled into rounded forms. It is the presence of these rounded bodies between the wheel and the brakeshoe that diminishes the apparent coefficient of kinetic friction at high speed.

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* The sign "≑" means "is nearly equal to."

44. The Limiting Angle of Static Friction.—Consider a body resting upon a rough plane to be acted upon by a force inclined to the plane. In Fig. 45 the plane is inclined and the force is vertical. In Fig. 46 the plane is horizontal and the force is inclined. The force *R* acting upon the body may be resolved into two components, one *CB* parallel to the plane and another *CD* perpendicular to it. If *CB* is greater than the static friction can be, the body will slide. If *CB* is not greater than the static friction can be, the body will remain at rest and the static friction will equal *CB*. In Fig. 45 the value of *CB* can be changed by changing the inclination of the plane, and in Fig. 46 by changing the inclination of *R*.

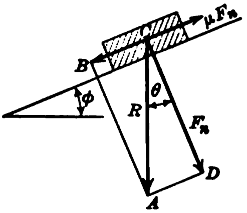

FIG. 45.

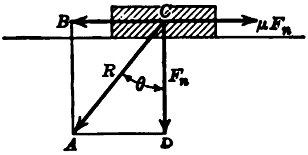

FIG. 46.

Suppose that the angle between the normal to the slipping surface, *CD*, and the line of action of the resultant force, *CA*, be such that the body is on the verge of slipping—that is, that *CB* equals the greatest possible value of the static friction. If now *R* be doubled, both components will be doubled. But when the component F_n that presses the body against the plane is doubled, (12) shows that the greatest possible value of the static friction is also doubled. It follows that the new value of CB just equals the new greatest possible value of the static friction. The same holds true however large R may be made. If the angle between F_n and R be less than that which makes the component CB equal to the force of friction μF_n, then whatever the value of R, the body will not slip.

Representing the angle between the normal to the slipping surface and the line of action of the resultant force acting upon the body by the symbol θ, it follows that there is a certain value of θ such that the body will always be just on the verge of slipping, whatever the value of the resultant force. The angle between the normal to the slipping surface and the resultant force acting upon the body when the body is on the verge of slipping is called the *limiting angle of static friction* or the *angle of repose*.

The value of the limiting angle of friction is easily determined. If the body is on the verge of slipping, then $\frac{DA}{CD} = \tan\theta$.

But when the body is on the verge of slipping, $DA\,(=CB) = \mu F_n$

Hence,
$$\frac{DA}{CD}\left[=\tan\theta\right] = \frac{\mu F_n}{F_n} = \mu,$$
$$\theta = \tan^{-1}\mu. \qquad (13)$$

That is, the limiting angle of friction equals the angle whose tangent is the coefficient of static friction.

If one pinches a wet orange seed between the thumb and finger, the seed will be projected with considerable speed. But the enormous pinching force exerted on a thin iron wedge driven into a log will not eject the wedge. In the former case, the limiting angle of friction is smaller than the angle between the normal to the slant faces and the thrusts against them; while in the latter case it is much larger. Consequently, in the latter case, however great the thrust may be, the wedge is not dislodged.

The brasses BB of a connecting rod, Fig. 47, are drawn firmly against the crank pin A by means of a strap S, which is made tight by means of a thin wedge K, called a key. The force with which the brasses press K against C makes with the normal to the inclined face of K an angle less than the limiting angle of friction for iron on iron. Consequently, however great this thrust may be, the key is not dislodged.

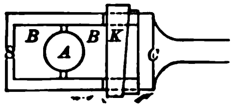

FIG. 47.

The threads of a jackscrew supporting a heavy load are acted upon by a force tending to turn the screw downward into the nut. But the pitch of the threads is so small that this force makes with the normal to the threads an angle less than the limiting angle of friction of iron on iron. Consequently, the screw does not turn.

45. Rolling Resistance.—When a sphere or cylinder rolls on a plane surface there is developed at the place of contact an opposition to the motion which is called *rolling resistance.* This opposition is also called rolling friction. But as the opposition is not a matter of rubbing, the former term is to be preferred. Rolling resistance is due to the fact that the rolling body is slightly flattened, and the plane surface is slightly depressed at the place of contact, thereby requiring the rolling body to constantly mount a slight elevation.

When moving to the right, the body shown in Fig. 48 is acted upon by a force F_n normal to the surface on which it is rolling, a force F'_p, which is required to keep the body rolling uniformly parallel to this surface, and the reaction R applied at some point c. The point c is somewhere in the interface between the two bodies.

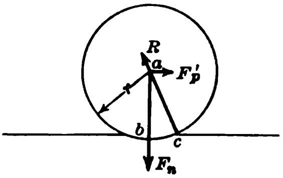

FIG. 48.

Taking moments with respect to an axis perpendicular to the plane of the diagram through c,

$$F'_p(ab) - F_n(bc) + R(0) = 0.$$

Therefore, the rolling resistance

$$F'_p = \frac{bc}{ab}F_n \doteq \frac{bc}{r}F_n. \qquad (14)$$

Therefore, when a sphere or cylinder rolls on a yielding surface that takes a permanent deformation, the rolling resistance is almost inversely proportional to the radius of the rolling body. It is found by experiment that the coefficient of F_n in this equation is not a constant quantity. In this manner rolling resistance differs from kinetic and static friction.

Experiment shows that within wide variations of load, (F_n), and radius, (r), the distance bc is nearly constant for given materials. For this reason, the distance bc is called the coefficient of rolling resistance. For a cast-iron roller on a steel surface, the coefficient of rolling resistance is about 0.015 inch.

The advantage of hauling a heavy load on a wagon instead of dragging the same load along the ground is partly due to the fact that rolling resistance is less than sliding friction. For the same reason " friction bearings," using either rolling cylinders or balls, are extensively used in many classes of machinery.

A pneumatic tire, by flattening out where it is in contact with the ground, gives the advantage of a wheel of larger diameter. On the other hand, the bulge in front of the flat place gives the disadvantage of an obstruction which the wheel must continually climb. On a soft rough road a pneumatic tire is advantageous. On a hard smooth plane road, a hard tire is superior to the pneumatic tire.

The rolling resistance of a four-wheeled wagon, on various sorts of roadways, expressed in pounds weight per ton, is about as follows:

Cubical stone block pavement	35±8
Macadam road	60±7
Gravel road	105±35
Common dirt road	150±75

46. Lubricants.—A *lubricant* is any substance which, coating with a thin film two bearing surfaces, will reduce their frictional resistance. When oil is placed between a shaft and a bearing, the slipping takes place between two films of oil instead of between two solid surfaces. The advantage in the use of the oil is due to the fact that the friction between two fluid surfaces is less than that between two solid surfaces. The laws of fluid friction are different from the laws of the friction between solids. The friction between two fluid surfaces, (*a*), is independent of the pressure between the fluid and the solid; (*b*), varies approximately with the square of the relative velocity; (*c*), varies directly with the area of rubbing surface; (*d*), varies directly with the density of the fluid.

By filling the rugosities of rough surfaces with plumbago or graphite, the friction is reduced.

On a dry day, the dust covering railway rails acts as a lubricant to such an extent that there is a considerable tendency of the wheels of a locomotive to tion to the motion which is called *rolling resis* tion is also called rolling friction. But as th matter of rubbing, the former term is to be resistance is due to the fact that the rolling tened, and the plane surface is slightly depre contact, thereby requiring the rolling body to slight elevation.

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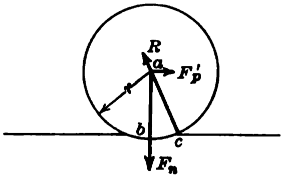

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By filling the rugosities of rough surfaces with plumbago or graphite, the friction is reduced.

On a dry day, the dust covering railway rails acts as a lubricant to such an extent that there is a considerable tendency of the wheels of a locomotive to slip. After a rain has washed off the dust, the same locomotive can draw a heavier load.

Solved Problems

Problem.—A uniform beam weighing 50 kg., inclined at an angle of 30° to the vertical, rests between a rough pavement and a smooth vertical wall and is just on the verge of slipping. Find the thrusts exerted against the pavement and the wall, and also find the coefficient of static friction between the beam and the pavement.

Solution.—Before reading the solution of this problem the student should read again the solution of the problem on p. 34.

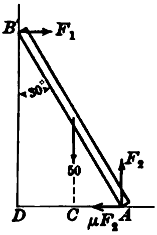

Fig. 49.

Limit the attention to the force acting upon the beam. Since the wall is smooth, there is at B no frictional resistance to the slipping of the beam down the wall. There is, however, the force F_1 with which the wall pushes against the beam to prevent it from falling over. At A the ground presses up against the beam with some force F_2, and the friction between the beam and the ground is the force that keeps the bottom of the beam from slipping to the right. When the beam is just on the point of slipping, this friction is, by (12), μF_2, where μ denotes the coefficient of static friction between the beam and the ground. Since the beam is uniform, its weight may be regarded as a single force acting at the middle point.

Since the beam is in equilibrium, we may apply the two conditions of equilibrium. The first of these conditions gives two equations: For the vertical components of the forces,

$$F_2 - 50 = 0, \qquad (15)$$

and for the horizontal components of the forces,

$$F_1 - \mu F_2 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

Any point may be chosen as the center of moments. If the point A is chosen the second condition of equilibrium gives the equation

$$F_1(BD) - 50(AC) = 0,$$

or, if the length of the beam be called l,

$$F_1(l \cos 30°) - 50(\tfrac{1}{2}\, l \sin 30°) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

The l in (17) cancels out. And when the three independent simultaneous equations (15), (16), and (17) are solved for the three unknown quantities F_1, F_2

and μ, their values are found to be respectively 14.4 kg. wt., 50 kg. wt., and 0.29.

Problem.—If, due to air resistance, friction of journals, etc., the motion of a train is opposed by a force of 5 lb. wt. per 1000 lb., find the total weight of a train that can be hauled by a 100,000-lb. locomotive on a level track such that the coefficient of static friction between the wheels and rails is 0.15.

Solution.—The greatest pull that can be exerted by the train on the locomotive—and therefore by the locomotive on the train—equals the force of static friction between the wheels of the locomotive and the rails when the wheels are on the verge of slipping on the rails. From (12), this force pushing on the locomotive is

$$F_p = 0.15(100{,}000) = 15{,}000 \text{ lb. wt.}$$

When the train is moving with uniform velocity the force pulling back on the locomotive equals the above force pushing the locomotive forward. The force pulling back on the locomotive is the kinetic friction of the journals, etc. Since the resistance to motion is 5 lb. wt. for every 1000 lb. wt., the coefficient of kinetic friction is 0.005. If the weight of locomotive and train be represented by W, then from (11), the kinetic friction

$$f_p = 0.005\ W.$$

And since, when the train is moving with constant velocity, $f_p = F_p$,

$$0.005\ W = 15{,}000 \text{ lb. wt.},$$

$$W = 3{,}000{,}000 \text{ lb. wt.}$$

Problem.—A locomotive weighs 35 tons. The coefficient of static friction between wheels and rails is 0.18. Find the total weight of itself and train which it can draw up a 1 per cent grade* if the resistance to motion on the level is 10 lb. wt. per ton.

Solution.—The advance of a locomotive is due to the wheels pushing backward on the track. If the wheels slip, the locomotive cannot pull. The greatest pull possible is that exerted when the wheels are just on the verge of slipping. When this condition occurs, the tractive force F_p is given by (12),

$$F_p = \mu F_n,$$

---

* The grade of a road is usually measured by the ratio of the vertical height to horizontal distance. Thus, a road is said to have an x per cent grade when the tangent of its inclination to the horizontal equals $x/100$.

where F_n is the force normal to the track and μ is the coefficient of static friction between the wheels and rails.

From Fig. 50, $F_n = W_1 \cos \Phi$. And since the grade is 1 per cent, $\Phi = \tan^{-1} 0.01 = 35'$. Consequently the greatest tractive force the locomotive can exert is

$$F_p = \mu W_1 \cos 35' = (0.18)(35)(0.9999) = 6.3 \text{ tons wt.} \quad . \quad . \quad . \quad (18)$$

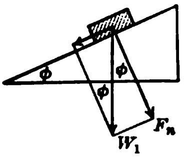

FIG. 50.

When the train is loaded to the degree that the wheels of the locomotive are just on the verge of skidding on the rails, the force F_p is exactly balanced by the sum of the component F' of the weight of the train downhill and the resistance to motion F'' due to friction of journals, etc. That is,

$$F_p = F' + F''. \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If the total weight of train, including the locomotive, is represented by W, the component of this weight in the direction of the rails is

$$F' = W \sin 35' = W(0.01). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Since the resistance to motion on the level is 10 lb. wt. per ton, the resistance on the level for the entire train is $\frac{10}{2000}$ $W[=0.005\ W]$, and the resistance to motion on the grade is

$$F'' = 0.005\ W \cos 35' \doteq (0.005)W(1). \quad . \quad . \quad . \quad . \quad (21)$$

By substituting in (19) the values given in (18), (20), and (21), we obtain

$$6.3 = W(0.01) + W(0.005).$$

Whence

$$W = 420 \text{ tons wt.}$$

QUESTIONS

1. In raising a building preparatory to moving it, jackscrews were used. In starting these with the turning bars, the workman used a number of sudden jerks instead of a steady pull. Explain why.

2. A building is lifted by means of jackscrews. Why is it that when no force is applied at the turning bars, the weight of the building does not turn the screws so as to lower itself? Draw a diagram in which the forces involved are represented.

3. Why are wagons used instead of sleighs on an earth road? Why is the reverse true on snow roads?

4. In starting a locomotive, if the wheels slip on the rails the engineer will shut off the steam and then start over again. What principle does he make use of?

5. Why is it harder for a team of horses to start a " stone boat " on a hard road than to keep it going after it is started?

6. Does an engineer gain anything by putting sand on the rails if the wheels do not slip on starting? Explain.

CHAPTER IV

THE MOTION OF A BODY UNDER THE ACTION OF ZERO FORCE

§ 1. *Uniform Linear Motion*

47. Linear Speed and Linear Velocity.—By the motion of a body is meant the change of the position of the body with reference to some other body. The velocity of a body relative to the earth is sometimes called the absolute velocity of the body. To describe a linear motion three elements must be specified. These are, the direction in which the displacement occurs, the distance traversed, and the time occupied in traveling this distance.

The ratio of the distance traveled to the time occupied in traversing this distance is called *speed.* When we are interested not only in how fast a body is moving but also in the direction in which it moves, we speak of the *velocity* of the body instead of its speed. The speed of a train may be 50 mi. per hour; its velocity may be 50 mi. per hour north.

A point which traverses equal spaces in equal times, however small the unit of time may be, is said to have *uniform speed.* When the speed is not uniform, we often speak of the instantaneous speed at a given point. The instantaneous speed at any point is the ratio of the distance the body would travel during the next interval of time to the magnitude of this interval, if from that point the speed remained uniform. Suppose two trains are running side by side in the same direction, one at a uniform speed and the other at a slower speed which is gradually increasing. A person on the train moving with increasing speed sees the other train at first moving ahead, then standing still, and finally falling behind. At the instant the train appears to be standing still, the speed of the uniformly moving train equals the instantaneous speed of the other train.

A point which preserves both its speed and the direction of its motion constant is said to be moving with *uniform velocity*. When either the direction of motion changes, or the distance passed over in equal times changes, the velocity is variable. A point on the rim of a fly wheel may traverse equal distances in equal times. Its linear speed is then constant. But as the direction of its motion is changing, its linear velocity is ununiform.

Denoting by v the constant speed of a point traversing the distance x in time t, we have, from the definition of speed,

$$v=\frac{x}{t}. \qquad \qquad (22)$$

This equation shows that the unit of speed is a speed such that a unit distance is traversed in unit time. The unit of time employed in physics is the second. It follows that a speed may be expressed in feet per second, in miles per hour, in centimeters per second, in kilometers per minute, etc., the word " per " indicating the words " divided by." A speed of one nautical mile (2024 yards) per hour is called a *knot*.

48. The Composition of Uniform Linear Velocities.—A passenger walking across a moving railway carriage furnishes an example of what are called simultaneous velocities. The passenger has a velocity relative to a point of the carriage, and at the same time the carriage has a velocity with respect to a point on the earth. These are called components of the passenger's motion with respect to the earth. His actual velocity with reference to the earth is called his resultant velocity. If the component velocities of a body's motion are in the same direction, the resultant velocity equals their algebraic sum.

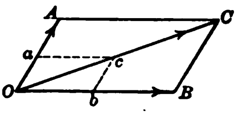

FIG. 51.

The resultant of two uniform velocities inclined to one another will now be determined. Let OB represent the velocity of the train relative to the earth and let OA represent the velocity of the man relative to the train. In other words, the line OB represents the direction and the distance traveled by the train, with reference to the earth, during some time t; while OA represents the direction and the distance traveled by the man, with reference to the train, during the same time. During the time t, the man moved across the train from O to A. During this time the line OA moved parallel to itself through the distance OB. Consequently, at the end of the time t the man has reached the point C.

It remains to be shown that throughout the time t the man was moving along the diagonal OC. Suppose that during some interval of time t', less than t, the train has traversed the distance Ob; while, with reference to the train, the man has traversed the distance Oa. Draw ac and bc parallel to OB and OA, respectively. Following the method of the preceding paragraph, it is seen that at the end of the interval t the man is at c.

But since the velocities along OA and OB are uniform,

$$\frac{t'}{t}=\frac{Oa}{OA}.$$

and

$$\frac{t'}{t}=\frac{Ob}{OB}=\frac{ac}{AC}.$$

Whence,

$$\frac{Oa}{ac}=\frac{OA}{AC}.$$

Consequently, Oc and OC are colinear. This means that at any instant during the time t the man was on the diagonal OC. Therefore, *if two simultaneous uniform linear velocities be represented by two adjacent sides of a parallelogram, the resultant velocity is represented by the diagonal which passes through their intersection.*

Since the parallelograms ab and AB, Fig. 51, are similar,

$$t' : t\,[=Oa : OA]=Oc : OC.$$

That is, the distance described along the resultant is directly proportional to the time. Therefore, *the resultant of two uniform velocities is a uniform velocity.*

49.—We have just seen that if a man moves across a train in a direction OA, with a velocity ${}_tv_m=OA$, and if at the same time, the train is moving along the earth in some direction OB with a velocity ${}_ev_t = OB$, then the velocity of the man with reference to the earth, ${}_ev_m$ is represented by OC. In general, if one body moves with reference to a second, and at the same time the second moves with reference to a third, then the velocity of the first with reference to the third is represented in direction and in magnitude by the diagonal of the parallelogram constructed upon the lines representing the two component velocities, all three lines being drawn from the same point.

If, however, we attempt to combine in this way the velocity of the first in respect to the second with the velocity of the first in respect to the third, e.g., to combine the velocity of the man in respect to the train with the velocity of the man in respect to the earth, we would get a line OD, Fig. 52, which represents nothing that we already had on our diagram, and which certainly does not represent the velocity of the man with reference to either earth or train. In fact, this combination would be meaningless. In general, if we attempt to combine the velocity of the first in respect to the second (${}_2v_1$), with any velocity except that of either the second in respect to something (${}_xv_2$), or that of something in respect to the first (${}_1v_x$), the result is meaningless. The only way in which it is permissible to compound velocities may be indicated as follows:

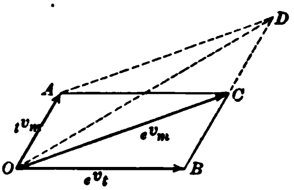

FIG. 52.

$${}_2v_1 \text{ and } {}_xv_2 \text{ give } {}_xv_1,$$

and

$${}_2v_1 \text{ and } {}_1v_x \text{ give } {}_2v_x.$$

If we adopt the notation used above, and write for any velocity a v with two subscripts, a right subscript to indicate the moving body and a left subscript to indicate the body to which the motion is referred, we see as illustrated above that two velocities can be compounded only when the right subscript of one is the same as the left subscript of the other. When we do so compound, the resultant velocity is the velocity of the body indicated by the remaining right subscript, and the body to which motion is referred is that indicated by the remaining left subscript.

50. Resolution of Uniform Linear Velocities.—If a resultant velocity be replaced by two component velocities, it is said to be resolved into these components. The remarks made in Arts. 33 and 34 with regard to the resolution of forces are also applicable to the resolution of velocities.

For example, consider the resolution of linear velocities in the sailing of an ice yacht (Fig. 53). Let the wind blow against the sail with a speed v at an angle β (Fig. 54). Denote the angle between the sail and the line of the runners by ϕ. Neglecting the slight friction of the runners on the ice, the speed V of the yacht will now be derived.

FIG. 53.

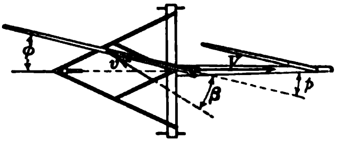

FIG. 54.

Since the frictional resistance to the yacht's motion is negligible, the speed will continue to increase so long as there is any wind pressure on the sail. The wind will press on the sail as long as the wind goes faster in a direction perpendicular to the sail than the sail is going in the same direction, i.e., until the component of the velocity of the wind in a direction normal to the sail equals the component of the velocity of the yacht in the same direction. When this occurs a particle of air will just slide along the sail without pressing on it. Under these conditions,

$$V \sin \phi = v \sin \beta.$$

Therefore, if there is no retarding force due to friction between the ice and the runners, the speed of the yacht will be

$$V = \frac{v \sin \beta}{\sin \phi}.$$

This equation shows that when β is less than ϕ, the speed of the yacht is less than that of the wind; when β equals ϕ, the speed of the yacht equals that of the wind; and when β is greater than ϕ, the speed of the yacht is greater than that of the wind which propels it.

51. The Water Turbine.—An interesting application of the composition and resolution of linear velocities is found in the design of steam and water turbines. In one form of water turbine the rotating member or "runner" consists of a series of curved blades set in a ring attached to a vertical shaft, and receiving water at all points in its periphery from the mouths of a concentric circle of passages. In Fig. 55 are represented in horizontal section an inner runner wheel and the outer stationary guides *GG* which direct the water against the blades of the runner. The whole is enclosed in a case kept filled with water under pressure. Water passing between the guides is directed against the blades *BB* of the rotating runner and, escaping into the axial space of the runner, falls into the "tail race."

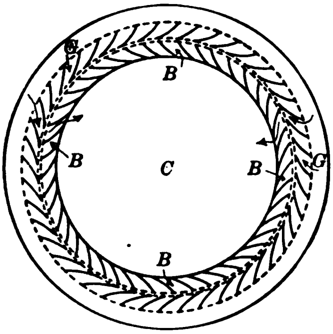

FIG. 55.

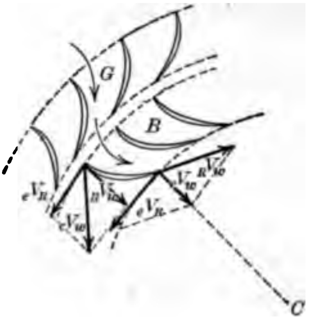

FIG. 56.

If, on striking a blade, the direction of a stream of water be suddenly changed, a turbulent eddy will be set up and a waste of energy will be produced. In order that there may be no waste of energy due to a sudden change in the direction of the stream flowing from the guides to the runner blades, the direction of the velocity of the water entering the runner, relative to the outer edge of the runner, should be tangent to the runner blades. Consequently, if the absolute linear velocity of the outer edge of the runner be $_eV_R$, and the absolute velocity of the water entering the runner be $_eV_w$, the direction of the runner blade where the water enters must be tangent to $_RV_w$ as given by the parallelogram in Fig. 56.

Again, on leaving the runner, the velocity of the water relative to the inner edge of the runner should be tangent to the blade. And in order that the water may readily escape from the axial space, the direction of the absolute velocity of the water leaving the runner should be radial. Consequently, if the absolute linear velocity of the inner edge of the runner be ${}_eV_R$ and the absolute velocity of the escaping water be ${}_eV_w$, the direction of the runner blade where the water leaves must be tangent to ${}_RV_w$, as given by the parallelogram of velocities in the figure. From these considerations the proper shape to be given to the runner blades can be determined.

SOLVED PROBLEM

PROBLEM.—An aviator wishes to travel in a line 30° south of east with an aeroplane of speed 70 miles per hour in a wind blowing from the northeast with a speed of 20 miles per hour. Find the direction in which he must point the machine.

SOLUTION.—In Fig. 57, the line OX represents the direction of the resultant velocity of the aeroplane relative to the earth. The line OB represents in direction and in magnitude the component velocity, due to the wind, of the aeroplane relative to the earth. The problem gives the magnitude of the component velocity of the aeroplane relative to the earth due to the motor, but not the direction.

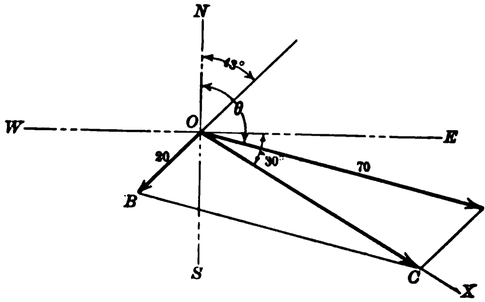

FIG. 57.

With the points of a pair of dividers separated by a distance representing 70 miles per hour, place one point at B and cut the line OX with the other point at C. Complete the parallelogram having the sides OB and BC. Then OA gives the direction and the magnitude of the velocity of the aeroplane relative to the earth due to the motor, which, when compounded with the velocity OB of the aeroplane relative to the earth due to the wind, will give a resultant in the direction OX. The angle NOA gives the required direction. The value of this angle can be obtained from the figure by ordinary trigonometric methods.

QUESTIONS

1. A battleship is moving with uniform velocity parallel to the shore. By means of a diagram show how this velocity affects the aiming of a gun on ship at a target on shore.

2. In the case of the driving wheel of a locomotive, show that for an instant one point is moving twice as fast as the locomotive and in the same direction. What point is this? To what is the motion referred?

3. A man in a boat is being rowed across a river at a uniform rate, and at the same time is carried down the stream by the current. Show by a diagram how he would direct a ball to be thrown to a man on the bank directly in front of the boat.

4. A man riding due north feels a wind from the northeast. If he rides due south at the same speed the wind appears to come from the southeast, what is the true direction of the wind? Explain with diagrams.

5. A gunner on a moving ship desires to fire a shell at a distant fixed point. What data should be known in order to avoid missing the mark?

6. By means of diagrams, explain how a gun must be aimed when the gun is (*a*), at rest, and the target moving; (*b*), moving, and the target at rest. (Motion along line of sight is excluded.)

7. Explain by means of a diagram why a person walking rapidly in rain that is descending vertically holds his umbrella somewhat in front.

§ 2. *Uniform Angular Motion*

52. Measurement of Angles.—In ordinary life the unit of angular measurement is arbitrarily taken as one ninetieth part of a right angle. This unit is called the degree. In scientific work a unit of angular measurement called the radian is frequently employed. The *radian* is the plane angle subtended at the center of a circle by an arc equal to the radius of the circle.

Thus, if AB (Fig. 58), is half as long as the radius, ϕ is one-half of a radian, and whatever the length of AB,

$$\phi = \frac{AB}{OA} \text{ radians.}$$

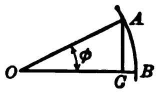

FIG. 58.

Again, from the definition,

$$360^\circ \left[= \frac{2\pi r}{r} \text{ radians} \right] = 2\pi \text{ radians.} \quad . \quad . \quad (23)$$

$$1 \text{ radian} = \frac{360^\circ}{2\pi} = 57^\circ.29578 \doteqdot 57^\circ 17' 44.8''$$

One thousandth of a radian is called a milradian. One milradian equals $\frac{1}{6283}$ of a circle, very nearly. As the unit angle, artillerists use the *mil*, which equals $\frac{1}{6400}$ of a circle.

The method of measuring plane angles which employs the radian as the unit is called circular measure.

When the angle ϕ is small,

$$\frac{AC}{OA} \doteqdot \frac{AB}{OA}.$$

Consequently for small angles

$$\sin \phi \doteqdot \phi \text{ radians.}$$

For example,

$$2° = 0.0349 \text{ radian, and } \sin 2° = 0.0349;$$
$$4° = 0.0698 \text{ radian, and } \sin 4° = 0.0698;$$
$$6° = 0.1047 \text{ radian, and } \sin 6° = 0.1045;$$
$$8° = 0.1396 \text{ radian, and } \sin 8° = 0.1392.$$

Polyedral angles are measured in a similar manner. If a sphere of any radius be constructed with the apex of the polyedral angle as center, the ratio of the area of the spherical surface included between the faces of the polyedral angle to the square of the radius of the sphere is taken to be the measure of the polyedral angle. When the radius of the sphere is unity, and the area of the included spherical surface is unity, the polyedral angle is unity. The unit polyedral angle is called the *steradian* or *space radian.*

53. Angular Velocity.—A displacement of a body such that all points of the body describe coaxial circular arcs is called *rotation.* The axis of rotation may pass through the body or it may be outside of the body. In rotation all lines perpendicular to the axis of rotation sweep through equal angles in equal times. Let XX' (Fig. 59) be a line fixed in space, and let PO be a line fixed in the body perpendicular to the axis of rotation passing through O. The rate of change of the angle $X'OP$ is called the *angular velocity*

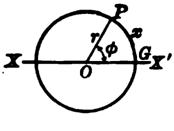

FIG. 59.

of the body about the assigned axis. The angular speed of the body is the magnitude of the rate of change of the angle $X'OP$ without reference to the axis about which rotation occurs. Thus, if w represents the angular speed of the body,

$$w=\frac{\phi}{t}\text{ radians per second.} \qquad . \quad . \quad . \quad . \qquad (24)$$

Angular speed may be measured in radians per second, degrees per second, revolutions per minute, etc.

A uniform angular velocity is one having constant angular speed in a fixed direction about an invariable axis.

54. Representation of Angular Velocity.—Three quantities are required to specify completely an angular velocity—the angular speed, the direction of the axis about which rotation occurs, and the sense of rotation, i.e., clockwise or counterclockwise. An angular velocity can be completely represented by a straight line whose magnitude is proportional to the angular speed, and whose direction is parallel to the axis of rotation.

Thus, the line AB, three units long, inclined as in the figure, represents an angular velocity of three radians per second about an axis parallel to AB. The sense of the rotation is indicated by the direction of the arrowhead. The arrowhead is so placed that on looking along the axis in the direction of the arrow, the rotation is clockwise.

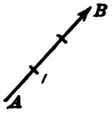

FIG. 60.

55. Instantaneous Axis of Rotation.—The fixed axis about which rotation occurs is called the axis of rotation. Sometimes, however, this axis is fixed for but a very short time; it is then called the instantaneous axis of rotation.

If a wagon axle be lifted off the ground and the wheel set in rotation, the axis of rotation with reference to both the earth and the wagon will be permanent and at the center of the axle. If, however, the wheel rolls along the ground, this axis will not be at rest with reference to the earth, but will remain at rest with respect to the wagon. If the wheel is rolling along the ground without sliding, the point in contact with the ground is instantaneously at rest with respect to the ground. Therefore the instantaneous axis of rotation with respect to the ground is at the point in contact with the ground. It must be remembered that at every succeeding instant this point is a different part of the wheel.

56. Composition of Angular Velocities.—It will now be shown that two simultaneous instantaneous angular velocities, about axes that meet in a point, can be compounded in a manner similar to that in which two linear velocities are compounded. Let OP and OQ (Fig. 61) be two axes about which a body rotates at the same time with the respective angular speeds w_1 and w_2, and in the directions indicated in the figure. From any point C erect lines CD and CE perpendicular respectively to the two axes OP and OQ. Denote CD and CE respectively by r_1 and r_2.

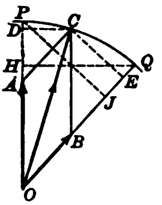

Fig. 61.

Due to the single angular velocity w_1, the line CD would rotate, about OP as an axis, in such a direction that the point C would be depressed below the plane of the paper. Denoting the angle swept through by the line CD in time t by the symbol ϕ_1, we have

$$w_1 = \frac{\phi_1}{t}.$$

Denoting by x the distance traveled by the point C while CD sweeps through the angle ϕ_1, we have

$$\phi_1 = \frac{x_1}{CD} = \frac{x_1}{r_1}.$$

Consequently, $$x_1 = \phi_1 r_1 = w_1 r_1 t.$$

Again, due to the single angular velocity w_2, the line CE would rotate about OQ as an axis, in such a direction that the point C would be raised above the plane of the paper. Denoting the angle swept through by CE in time t by the symbol ϕ_2, and the corresponding distance traveled by the point C by x_2, we find in the same manner as above

$$x_2 = w_2 r_2 t.$$

The point C will be at rest throughout the time t if it is raised by one rotation just as much as it is depressed by the other, i.e., if $x_1 = x_2$. But if the point C is at rest, the entire line OC will also be at rest; that is, OC is the direction of the axis of the resultant angular velocity. Consequently, the line OC will be the direction of the axis of the resultant angular velocity of w_1 and w_2 if

$$w_1 r_1 t = w_2 r_2 t,$$

that is, if

$$\frac{w_1}{w_2} = \frac{r_2}{r_1}. \qquad (25)$$

The condition required by this equation is easily determined. From C draw lines CB and CA parallel to the axes of w_1 and w_2, respectively. The area of the parallelogram $OACB$, thus constructed, is

$$(OA)r_1 = (OB)r_2.$$

Whence

$$\frac{OA}{OB} = \frac{r_2}{r_1}. \qquad (26)$$

From (25) and (26) it is seen that the condition that the line OC shall be the direction of the axis of the resultant angular velocity of w_1 and w_2 is that

$$\frac{OA}{OB} = \frac{w_1}{w_2}.$$

It has now been proved that if the lines OA and OB are proportional to the angular velocities about these axes, then the diagonal of the parallelogram of which these are contiguous sides is the axis of the resultant velocity. It remains to show that the length of this diagonal is proportional to the magnitude of the resultant angular velocity.

Draw QH perpendicular to OP. Since Q is on the axis of w_2, the distance traveled by Q in time t is

$$w_1 t(QH).$$

If the resultant angular velocity of the body about OC be denoted by w, we have also for the distance traveled by Q in time t, the value

$$wt(CE).$$

Since these two distances are the same,

$$w_1t(QH)=wt(CE). \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (27)$$

With O as a center and OC as a radius describe the arc PQ. Since $OC=OQ$ and $COE=ACO$, and $QOH=180°-OAC$,

$$\frac{CE}{QH}=\frac{(OC)\sin COE}{(OQ)\sin QOH}=\frac{\sin ACO}{\sin OAC}=\frac{OA}{OC}. \qquad . \quad . \qquad (28)$$

From (27) and (28),

$$\frac{w_1}{w}=\frac{OA}{OC}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (29)$$

Similarly, by drawing PJ and CD, it is found that

$$\frac{w_2}{w}=\frac{OB}{OC}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (30)$$

Taken together, (29) and (30) show that if lengths proportional to the respective angular velocities about them be measured off on the component and resultant axes, the lines so determined will be the sides and diagonal of a parallelogram.

It has now been proved that *if a parallelogram be constructed on two contiguous lines representing two simultaneous instantaneous angular velocities about concurrent axes, the diagonal of the parallelogram will represent completely the resultant angular velocity.*

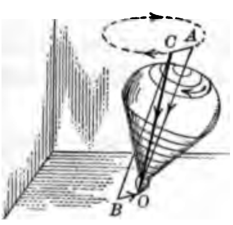

FIG. 62.

A familiar illustration of the composition of angular velocities is the case of the ordinary top spinning about its geometric axis. Usually this axis is not vertical. Let the line AO (Fig. 62) represent the angular velocity of the top about its geometric axis at any given instant. Due to the weight of the top there is another component angular velocity BO, about a horizontal axis passing through the peg. The resultant angular velocity is represented by the line CO. Thus the geometric axis of the top has moved from the position AO to the position CO. As the two component angular velocities continue to move, their resultant will rotate in the direction of the curved arrow.

Note.—It may not be amiss to here give a warning to the student in the use of vectors A vector is a mathematical conception which is completely specified by magnitude and direction. Any quantity which is completely specified by magnitude and direction can be represented by a vector. But compounding two vectors and compounding the two quantities which the vectors represent are two entirely distinct operations. The compounding of two vectors is a matter of definition. By definition, the sum of two vectors is the diagonal of the parallelogram found on the two vectors as sides. But whether this new vector, which by definition is the sum of the two given vectors, also represents the physical quantity which is the resultant of the two given physical quantities is something which has to be proved for each class of physical quantity.

Two forces acting either simultaneously or in succession can be compounded by the parallelogram law with the same result. The same is true of linear velocities occurring either simultaneously or in succession.

Two simultaneous angular velocities can be compounded by the parallelogram law. But the resultant of two angular velocities occurring in succession may not be in the same plane as the component velocities, and the resultant may be different if the order of succession be changed. This is a case in which the parallelogram law does not apply. The parallelogram law applies to the case of two successive angular velocities only if the same point of the body remains at rest whichever be the order of sequence.

57. The Relation between Angular and Linear Speed.—In Fig. 59 imagine the body to rotate about O with an angular speed w. Then any point P, fixed in the body, will move in the circumference of a circle of radius r with a linear speed v. Denoting the distance GP passed over in time t by x,

$$v=\frac{x}{t}.$$

If the angle passed over by PO in time t be called ϕ,

$$w=\frac{\phi}{t}=\frac{\frac{x}{r}}{t}=\frac{\frac{x}{t}}{r}=\frac{v}{r}. \qquad (31)$$

Whence, the angular speed of a body is numerically equal to the linear speed of any rotating point of the body divided by the distance of this point from the axis of rotation.

Questions

1. Is the angular speed of a point at the top of a moving wagon wheel greater with reference to the wagon bed or to the earth? With reference to which is its linear speed the greater? Answer the same questions concerning a point half-way from the axle to the ground, and give proof for each of the four answers.

2. A clock stands on a shelf on the north wall of a room. Explain clearly how the angular velocity of its hands can be represented by straight lines. Find the numbers which specify the lengths and state the directions of these lines.

3. Lay your watch down on the chair arm with the face up and state accurately how the angular velocities of the three hands will be represented. Give numerical values of the angular speed in each case.

4. Does the arm of a rocking chair in use rotate about a fixed axis?

5. An automobile is rounding a curve of constant radius at 10 miles per hour. Which of the following quantities are constant? Linear speed, linear velocity, angular speed, angular velocity.

6. Is angular speed or angular velocity constant in the following cases? (*a*) The wheel of a car rounding a curve of uniform radius; (*b*) the flywheel of a stationary engine after the steam has been cut off; (*c*) the hands of a clock.

CHAPTER V

THE MOTION OF A BODY UNDER THE ACTION OF A CONSTANT FORCE

§ 1. *Uniformly Accelerated Linear Motion*

58. Acceleration Produced by a Uniform Force Acting in the Direction of Motion.—When a force acts upon a body, either the direction or the magnitude of the body's motion will tend to change during the time the force acts. If the velocity changes, either in direction or in magnitude, it is said to be accelerated. When the velocity of a body is accelerated, the instantaneous velocity at any point of the path has the direction in which the body is moving at the chosen instant, and is given in magnitude by the distance the body would move in one second if from the chosen point onward the velocity were to remain uniform. If the velocity of a body changes, the ratio of the change in the linear velocity to the time occupied in producing the change is called the *linear acceleration* of the body's motion in the direction of the change. Or, more briefly, linear acceleration is the time rate of change of linear velocity. Thus, if the velocity changes uniformly from v_0 to v_t during the time t, the magnitude of the acceleration a is

$$a=\frac{v_t-v_0}{t}. \qquad \ldots\ldots\ldots \qquad (32)$$

The direction of the acceleration is the direction in which the velocity changes. The direction may be different from that in which the body moves.

This equation shows that the unit of linear acceleration is unit change of linear speed in unit time. The magnitude of the unit of linear acceleration used in science is the change in speed of one centimeter per second in one second; in engineering, the magnitude usually employed in English-speaking countries is the change in speed of one foot per second in one second, or a change in speed of one mile per hour in one second.

If the speed diminishes with time, the acceleration is said to be negative. A negative acceleration is often termed a deceleration or retardation.

Non-uniform or accelerated velocity in the direction of motion is well illustrated by a heavy ball rolling down (or up) an inclined plane. The apparatus shown in Fig. 63 consists of a grooved plank about 20 ft. long having at its upper end a device for starting the ball at any desired instant, and having along the length of the groove a series of incandescent lamps that flash up, one after the other, at one second intervals. Simultaneously with the release of the ball at the top of the incline, the lamp *O* flashes up, one second later the lamp 1 flashes up, two seconds after the ball starts the lamp 2 flashes up, and so on.

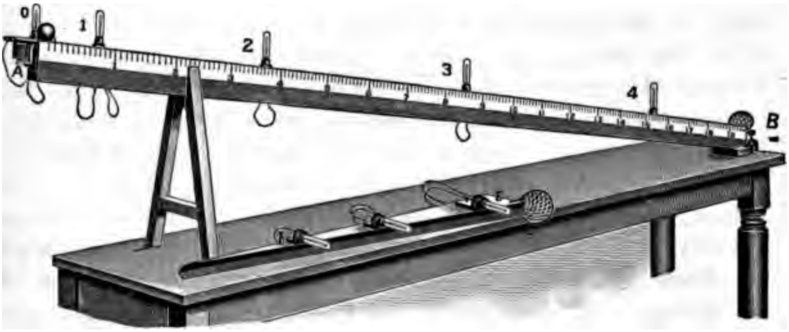

FIG. 63.

The positions of the lamps along the grooved plank can be adjusted so that the rolling ball will be exactly in front of each lamp at the instant it flashes. The distance between any two consecutive lamps will then be the distance traveled by the ball during the corresponding second of time.

In a particular trial, the ball traveled 1 ft. during the first second, 3 ft. during the second equal interval of time, 5 ft. during the third second, 7 ft. during the fourth second. Since unequal distances were traversed during equal intervals of time, the velocity was not constant, but accelerated. Since the speed increased with time, the acceleration was positive.

From (32), if the speed of a body changes from v_0 to v_t during the time t, the mean acceleration during this interval is

$$a=\frac{v_t-v_0}{t}.$$

In order to determine, by means of this equation, the magnitude of the linear acceleration of the ball's motion during this interval, it will be necessary to know the instantaneous speed of the ball at the beginning and end of the interval. The instantaneous speed of a body at any instant is numerically equal to the distance that would be traversed in one second, if from that instant the speed were to be uniform. Since there is no horizontal component of the weight of a body, the law of inertia shows that a body moving along a smooth horizontal surface, and unacted upon by any force except its weight, will move with a velocity constant both in direction and magnitude. Consequently, if the ball be intercepted at any point of its path down the inclined plank by a horizontal track, the instantaneous speed of the ball at the given point can easily be determined. In the present apparatus this is accomplished by means of a grooved wedge-shaped plank. In Fig. 64 this is shown in position for finding the instantaneous speed of the ball at the end of the first second after starting from rest.

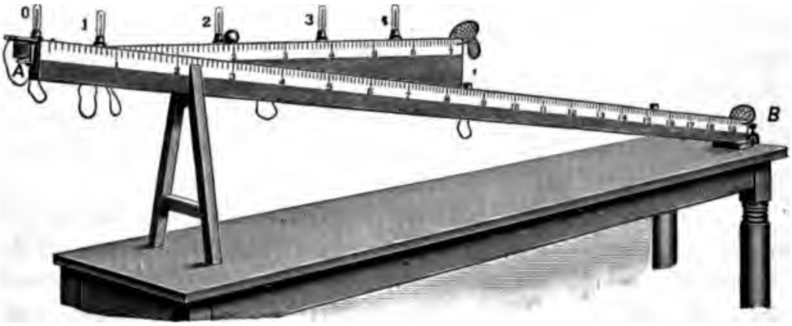

FIG. 64.

When the wedge was in this position, the ball traveled along the horizontal grooved plank 2 ft. in one second. When the acute angle of the wedge was placed at the positions reached by the ball in its motion down the incline at the end of the second, third, and fourth seconds, the intercepted ball was found to travel on the horizontal track in one second, 4, 6, and 8 ft. respectively. Whence, the instantaneous speeds of the ball down the inclined plank at the end of the first, second, third and fourth seconds were respectively 2, 4, 6 and 8 ft. per second. It thus appears that during any one-second interval of time the speed changed 2 ft. per second, or there was a constant acceleration down the plank of 2 ft. per second in one second. These results may be arranged as in the table on the following page.

If instead of being straight, the upper edge of the inclined plank had been of a curved form, the acceleration of the ball's motion would not have been uniform. Depending upon the inclination of the groove, the acceleration might be positive, zero, or negative at different instants of time.

Time, Seconds.	Total Distance, Feet.	Velocity at End of Each Second, Ft. per sec. Downward.	Acceleration, Ft. per sec. per sec. Downward.
0	0	0	0
1	1	2	2
2	4	4	2
3	9	6	2
4	16	8	2

It is one of the fundamental principles of dynamics (Art. 6), derived from experience and experiment, that when a force acts upon a body, the direction of the acceleration is that of the force, and the magnitude of the acceleration is proportional to that of the force. From this principle it follows that if a constant force be applied to a body a uniform acceleration will be produced: if the force constantly increases, the acceleration will constantly increase: if the force is very small, the acceleration will be very small, i.e. the motion of the body will be nearly uniform.

A uniform or an instantaneous linear acceleration is completely described when both its direction and its magnitude are specified. Consequently, a linear acceleration can be completely represented by a straight line. Linear accelerations can be compounded and resolved by the same methods used for forces and linear velocities.

59. Acceleration Due to Gravity.—Newton, Galileo, and others have proven experimentally that at any point of the earth terrestrial gravitation imparts to all bodies equal accelerations, but that at different places it imparts to the same body different accelerations. The acceleration due to gravity depends upon the distance of the place from the center of the earth and from the axis of the earth, together with such local conditions as the presence of mountains, large deposits of metals, etc. At sea level, at the equator, the acceleration due to gravity is 978 cm. per second per second (32.09 ft. per second per second), while at the pole it is 983 cm. per second per second (32.26 ft. per second per second). The acceleration due to gravity is usually represented by the symbol g.

A body on the inclined plane AB is under the influence of the force of gravity vertically downward. If the acceleration in the direction of this force be called g, the component parallel to a plane inclined at the angle ϕ with the horizon is directed down the plane and has the value $g \sin \phi$.

If a body starting at A with a certain initial velocity were to ascend the smooth plane AB, its velocity would experience an acceleration in the direction of the motion equal to $g \sin \phi$.

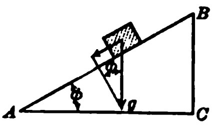

FIG. 65.

As ϕ diminishes, the magnitude of this retardation decreases, until in the limit when $\phi = 0$, the retardation is zero. Consequently, if there were no friction or other resistance, a body would move along a level surface with a uniform speed. This is in agreement with the first law of motion.

60. Distance Traveled by a Body Moving with Uniform Linear Acceleration.—If a uniform force acts upon a body in the direction of its motion, the speed of the body will change at a uniform rate, but the direction of motion will remain unaltered.

Howsoever the velocity of a body may change during a given time, there must be a certain equivalent velocity with which a uniformly moving body would traverse the same distance in the same time. In the case of a uniformly accelerated motion, the equivalent uniform velocity equals the arithmetic mean of the instantaneous velocities at the beginning and end of the time considered. Or, if during a time t the velocity changes at a uniform rate from v_o to v_t, then the equivalent velocity during this interval is

$$v_e = \tfrac{1}{2}(v_t + v_0).$$

The distance x traveled during the time t by a body whose velocity during that time changes uniformly from v_0 to v_t is

$$x[= v_e t] = \tfrac{1}{2}(v_t + v_0)t. \quad \ldots\ldots \quad (33)$$

By the use of (32) and (33) a large class of problems in uniformly accelerated linear motion can be solved.

61. Acceleration produced by a Uniform Force acting Perpendicularly to the Direction of Motion.—Since the acceleration produced is always in the direction of the applied force, if a force acts upon a body in a line perpendicular to the direction of its motion, the speed of the body will remain unaltered. An acceleration perpendicular to the direction of the motion signifies that the path of the body is changed by the force. In the following article it is proved that when a body is acted upon by a force of constant magnitude whose line of action is always in the same plane and always perpendicular to the direction of the motion, (1) *the speed of the body does not change,* (2) *the path of the body is a circle,* (3) *the body is moving with a linear acceleration which is always directed toward the center of the circle, and* (4) *the magnitude of this radial acceleration is constant and equal to the square of the linear speed of the body divided by the radius of its path.*

The converse of this proposition may be stated as follows: *Whenever a body moves with constant speed in the circumference of a circle, there is acting upon the body at every point of its path a force directed toward the center. This force produces an acceleration directed toward the center of the circular path, equal to the square of the linear speed of the body divided by the radius of its path.*

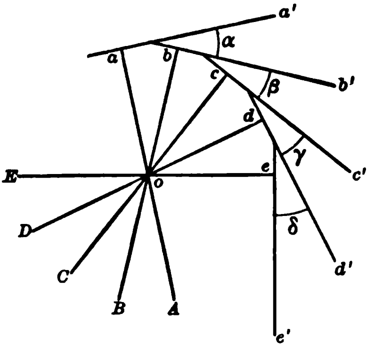

FIG. 66.

62.—The above facts may be deduced as follows. Consider a body which at some given instant is at a (Fig. 66), moving in the direction aa', and acted upon by a force which has the direction aA, perpendicular to aa' and in the plane of the paper. This force will cause the body to change the direction of its motion so that after a short time interval t the body will be at some point b and moving in a direction bb'. At b the force acts in the direction bB perpendicular to bb' and in the plane of the paper. After successive equal intervals t, the body will be sucessively at c, d, e, etc., moving in directions cc', dd', ee', etc., and the force will be acting successively along cC, dD, eE, etc. Since the body is moving with uniform speed, the lengths of the curved path in which it moves during the successive equal intervals t are equal. That is

$$\text{arc }(ab) = \text{arc }(bc) = \text{arc }(cd) = \text{etc.} \quad . \quad . \quad (34)$$

Moreover, since the force acting is constant in magnitude, the changes in direction which it produces during equal time intervals are equal. That is

$$\angle\alpha = \angle\beta = \angle\gamma = \text{etc.} \quad . \quad . \quad . \quad . \quad (35)$$

Equations (34) and (35), taken together, show that the points a, b, c, etc., are on a line of constant curvature.

Now it is shown in pure mathematics that the only plane curve of constant curvature is the circle. Consequently, the points a, b, c, etc., are all in the circumference of a circle. In the limit when t is indefinitely small, a, b, c, etc., are consecutive points in the circumference of a circle. Therefore, a body acted upon by a force always perpendicular to the direction of motion and always in the same plane, describes the circumference of a circle. The acceleration of the body's motion is directed toward the center of the circular path.

The magnitude of this acceleration will now be determined. Let the body move with uniform speed v in a circle of radius r and center O, Fig. 67. Assume that in the short interval of time t the body has traversed the distance bc. Then, (22),

$$\text{arc }(bc) = vt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

From a point P, Fig. 68, draw PB and PC representing in direction and magnitude the velocity of the body when in the positions b and c respectively. From the principle of the composition of velocities it follows that the line BC represents, in direction and in magnitude, the change in the velocity of the body during the time t.

Whence, if the acceleration perpendicular to the motion be denoted by a', we shall have (32),

$$(BC) = a't. \qquad (37)$$

Dividing each member of (37) by the corresponding member of (36).

$$\frac{(BC)}{\text{arc }(bc)} = \frac{a'}{v}. \qquad (38)$$

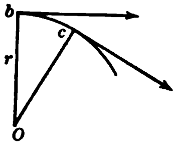

FIG. 67.

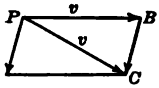

FIG. 68.

But in the limit, when t is indefinitely small, bOc and BPC are similar triangles. Hence,

$$\frac{(BC)}{\text{arc }(bc)} = \frac{(PB)}{(Ob)} = \frac{v}{r}. \qquad (39)$$

From (38) and (39) it is seen that the acceleration of a body moving with constant speed and acted upon by a force always perpendicular to the direction of its motion is

$$a' = \frac{v^2}{r}. \qquad (40)$$

SOLVED PROBLEMS

PROBLEM.—When running at 45 miles per hour the brakes are applied to an electric car with such force as to produce in the motion of the car a uniform acceleration of -4.36 ft. per second in a second. How far will the car go before coming to rest? How long a time will it take to bring the car to rest?

SOLUTION.—Since the acceleration of the car is uniform, we may apply (32) and (33). A speed of 45 miles per hour is a speed of $\frac{45 \times 5280}{3600} = 66$ ft. per second. This is the initial speed of the car. When it has come to rest its speed is zero. On substituting in (32) and (33) these values and the given value of the acceleration, we obtain

$$-4.36 = \frac{0-66}{t},$$

and

$$x = \tfrac{1}{2}(0+66)t.$$

These are two simultaneous equations which involve the two unknown quantities x and t. On solving for these quantities, we obtain

$$t = 15 \text{ sec.},$$

and

$$x = 500 \text{ ft.}$$

PROBLEM.—A cannon is fired horizontally and the ball strikes the ground at a point 11 meters below the level from which it is fired and 450 meters distant. If the only force acting upon the ball is its weight, what must have been the muzzle velocity?

SOLUTION.—From the principle of the independence of forces (Art. 11), the downward acceleration produced by the weight of the ball is the same as if the ball were not at the same time moving forward. Since the weight of the ball acts directly downward, the horizontal component of the velocity of the ball does not change. The problem resolves itself then into two: First, how long does it take a body, starting with no downward nor upward motion, to fall 11 meters? Second, if in this time a body which moves with uniform horizontal speed covers 450 meters, what is this horizontal speed?

To solve the first problem substitute in (32) and (33) the given values. Thus

$$980 = \frac{v_t - 0}{t},$$

and

$$1100 = \tfrac{1}{2}(v_t + 0)t.$$

On eliminating v_t from these equations, and solving for t, we obtain

$$t = 1.5 \text{ sec.}$$

Since the ball covers 450 meters in 1.5 seconds, its horizontal speed is

$$v = \frac{450}{1.5} = 300 \text{ meters per second.}$$

§ 2. *The Motion of Projectiles*

63. The Velocity of a Projectile in Vacuum.—Consider a body fired with initial velocity v_0 in a direction inclined to the horizontal at an angle ϕ to be acted upon by no force except its weight. The horizontal component of the initial velocity has the value,

$$v_x = v_0 \cos \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

As the force has zero component in the horizontal direction, this velocity is constant.

The vertical component of the initial velocity is $v_0 \sin \phi$, upward. But there is a constant force in the vertical direction which produces a constant acceleration g, downward. In time t, the change of velocity will be gt, downward. Consequently, at time t, the vertical component of the velocity upward has the value,

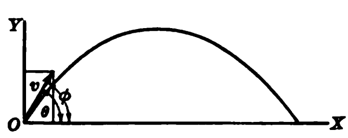

FIG. 69.

$$v_y = v_0 \sin \phi - gt. \quad \ldots\ldots \quad (42)$$

The actual velocity, at any time t after projection, is obtained by compounding these two rectangular components. Thus, the magnitude, v_t, of the actual velocity is

$$v_t = \sqrt{(v_0 \cos \phi)^2 + (v_0 \sin \phi - gt)^2}. \quad \ldots \quad (43)$$

If the direction of the actual velocity at time t be inclined to the horizontal at an angle θ,

$$\tan \theta = \frac{v_0 \sin \phi - gt}{v_0 \sin \phi}. \quad \ldots\ldots \quad (44)$$

64. The Range of a Projectile in Vacuum.—The distance between the point of projection and the point of impact is called the *range*.

If no force acts upon the projectile except its weight, there will be no change of velocity in the horizontal direction. Since in uniform motion, the distance traveled equals the product of the speed and the time, it follows that at time t, the horizontal distance of the projectile from the point of projection is, (41),

$$x[=v_x t] = v_0 \cos \phi \cdot t. \quad \ldots\ldots \quad (45)$$

In the vertical direction there is a constant force which produces a constant acceleration. In the preceding article it is shown that in time t the vertical component of the velocity changes from $v_0 \sin \phi$ to $(v_0 \sin \phi - gt)$. And since in uniformly accelerated motion, the displacement equals the product of the average velocity and the time, it follows that at time t the vertical distance of the projectile from the point of projection is

$$y = \tfrac{1}{2}(v_0 \sin \phi + v_0 \sin \phi - gt)t = v_0 \sin \phi \cdot t - \tfrac{1}{2}gt^2. \quad . \quad (46)$$

The range on the horizontal plane which passes through the point of projection is given by the value of x' in the equation, [(22) and (41)],

$$x'[= v_x t'] = v_0 \cos \phi \cdot t', \quad . \quad . \quad . \quad . \quad . \quad (47)$$

where t' is the time from the moment the projectile left the gun till it strikes the ground. This time of flight will now be determined and then substituted in the above equation.

When the projectile reaches the horizontal plane through the point of projection, $y = 0$. Hence, the time of flight is the value of t given by (46) when $y = 0$. That is,

$$0 = v_0 \sin \phi . t' - \tfrac{1}{2} g t'^2.$$

Whence, the time of flight is

$$t' = \frac{2 v_0 \sin \phi}{g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

On substituting this value in (47), the horizontal range is found to be

$$x' = \frac{2 v_0^2 \sin \phi \cos \phi}{g} = \frac{v_0^2 \sin 2\phi}{g}. \quad . \quad . \quad . \quad . \quad (49)$$

An inspection of this equation shows that since v_0 and g are constant quantities, the range is greatest when $\sin 2\phi$ is greatest; that is, when $\sin 2\phi = 1$, or $2\phi = 90°$. Consequently, if the air introduced no opposing force, the horizontal range would be greatest when the elevation of the gun is 45°.

65. The Maximum Height reached by a Projectile in Vacuum.—At the highest point in the path of a projectile, the vertical component of the velocity equals zero. At this point, (42) becomes

$$0 = v_0 \sin \phi - gt.$$

Hence the time required to reach the maximum height is

$$t = \frac{v_0 \sin \phi}{g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

On substituting this value of t in (46), we have for the maximum height reached by a projectile unopposed by air resistance,

$$y' = v_0 \sin \phi \frac{v_0 \sin \phi}{g} - \frac{1}{2} g \frac{v_0^2 \sin^2 \phi}{g^2},$$

$$y' = \frac{v_0^2 \sin^2 \phi}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

66. The Shape of the Trajectory in Vacuum.—The path described by a projectile is called the *trajectory*. If the velocity of a projectile be great, the air will offer considerable resistance to its motion. If the projectile rise to a great height, the weight will not be constant. Under these conditions the exact shape of the trajectory cannot be determined. But if a body projected in any direction be acted upon by no force except its weight, and if the weight be constant, the shape of the trajectory can readily be determined.

In Art. 63 it is shown that if a projectile be fired in air-free space with initial velocity v_0 at an angle ϕ to the horizontal, the horizontal component of the velocity will have the constant value, (41), $v_x = v_0 \cos \phi$. Whence, the horizontal displacement from the starting point at time t, is

$$x = v_0 \cos \phi \cdot t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Under the same conditions, the vertical displacement from the starting point at time t is, (46),

$$y = v_0 \sin \phi \cdot t - \tfrac{1}{2} g t^2. \quad . \quad . \quad . \quad . \quad (53)$$

From these two equations can be found the coordinates of a series of points reached by the projectile at various instants of time after starting. The curve drawn through these points is the trajectory of the projectile.

Combining (52) and (53) by eliminating t, an equation is obtained which coordinates simultaneous vertical and horizontal displacements. Thus, the equation of the trajectory is

$$y=\left(-\frac{g}{2v_0^2\cos^2\phi}\right)x^2+(\tan\phi)x. \quad . \quad . \quad . \quad (54)$$

Since the quantities within the parentheses are constants, the equation may be written in the form

$$y=-ax^2+bx.$$

This is the equation of a parabola with axis vertical and the directrix through the vertex. The fact that the coefficient of x^2 is negative means that the parabola is concave downward.

The trajectories, in vacuum, of a projectile leaving a gun with the same muzzle velocity and at various angles of elevation are shown in Fig. 70.

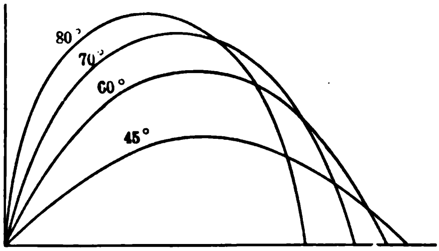

FIG. 70.

The plane of the trajectory, called also the *plane of fire*, is the vertical plane passing through the axis of the bore of the gun.

67. The Effect of Air Resistance on the Motion of a Projectile.—In the preceding articles the effect of air resistance has been neglected. On account of air resistance the trajectory is not a parabola, and the range and height attained are not so great as they would be in vacuum. In Fig. 71 the lower curve represents the trajectory in air of a projectile fired from a 3-inch gun at range

5000 yards, and the upper curve represents the trajectory in vacuum for the same muzzle velocity and angle of elevation.

Air resistance depends upon the shape, diameter, mass and velocity of the projectile, and upon the density of the air. The density of air depends upon the temperature, barometric pressure and humidity. The effects of these various factors can be determined only by long series of experiments for each model of projectile. By means of tables embodying the results of such experiments, artillery officers are able to point a gun so as to correct for the effect of air resistance.

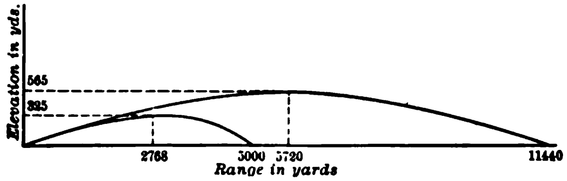

FIG. 71.

68. Axle of the Gun not Horizontal.—To give a gun such an angle of elevation that the trajectory of the projectile will intersect the target, a " sight " is used. The simplest type of sight consists of a fixed metal point attached near the muzzle, together with another point situated near the breech and capable of being moved toward and away from the axis of the barrel. In modern artillery, however, the sight in general use consists of a special form of telescope.

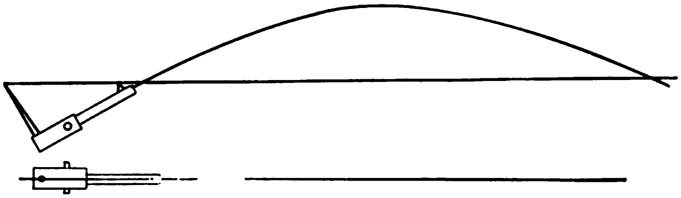

FIG. 72.

The sight is so designed that when it is set for the known range of the target, and the gun is rotated about a horizontal axis till a straight line through the sight passes through the target, the trajectory of the projectile will intersect the target. Fig. 72 represents a vertical and a horizontal view of the trajectory relative to the line of sight when the axle of the gun is horizontal.

If, however, the axle about which the gun is rotated be not horizontal, the line of sight will not be in the plane of the trajectory, as illustrated in Fig. 73. For example, if one wheel of a gun be higher than the other, the projectile will strike to the side of the target toward the lower wheel.

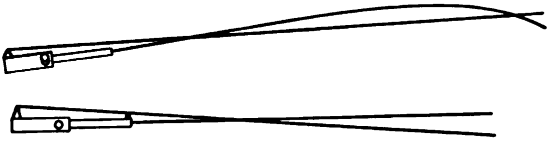

Fig. 73.

Sights are made which can be given such a lateral displacement that the departure due to the inclination of the axle is corrected.

§ 3. *Systems of Units*

69. Mass.—Matter has been already defined as anything which possesses inertia. Matter has another distinguishing characteristic. If an iron rod be moved from the equator to the pole of the earth, or be carried from the bottom of a mine to the top of a mountain, it will be found that its weight and some other of its physical qualities will be altered, but during these changes of position there is one quality that does not change. If the iron rod be distorted, electrified, heated, or magnetized, its energy, its volume, and certain other qualities will be modified, but during all of these changes there is one quality that remains unchanged. If it be dissolved in an acid, it will disappear as metallic iron and be transformed into an entirely different substance. Still, during this transformation there is one thing that does not change. That which remains invariable, however the position of a body may be changed, however its mechanical, electrical, thermal, or magnetic condition may be altered, or however its chemical constitution may be transformed is the *amount of matter* in the body.

Matter is characterized by three properties. (*a*) Matter can be neither created nor destroyed. This is called the Principle of the Conservation of Matter. (*b*) Between two portions of matter there is always a force urging them toward one another. (*c*) Matter possesses inertia.

The amount of matter in a body is called the *mass* of the body. Masses may be compared in terms of their inertiæ. The inertiæ of two bodies may be compared in terms of the accelerations produced when equal forces are applied to the bodies. If the action of a given force upon each of two bodies B_1 and B_2 gives to them the respective accelerations a and na, then the inertia of B_1 is said to be n times as great as that of B_2. That is, the inertiæ of two bodies are taken to be inversely proportional to the linear accelerations produced by the application of equal forces. Hence, the ratio of the masses of two bodies equals the inverse ratio of the linear accelerations produced by the application of equal forces.

The unit of mass is arbitrarily taken to be either the mass of a certain piece of metal deposited in the Office of the Exchequer in London, or the mass of another piece of metal preserved in the Archives of Paris. The mass of the first is called one pound, and the mass of the second is called one kilogram.

In scientific work, the thousandth part of the kilogram is the unit of mass usually employed. This is called the gram. It was originally intended that the gram should be the mass of one cubic centimeter of water at its temperature of maximum density, i.e., at 4° C. or 39.2° F. The material standard in Paris does not exactly realize this intention, but the departure is so slight that it is quite negligible except in the most refined work.

70. Comparison of Masses.—Masses can be compared in a variety of ways. For instance, the masses of two bodies can be compared by determining the ratio of the linear accelerations produced in their motion by the application of equal forces. The magnitude of this force need not be known.

Consider two bodies A and B (Fig. 74) through which passes a smooth rod capable of rotation in a horizontal plane by means of the vertical spindle *C*. Let the two bodies be connected by a light cord. When the apparatus is rotated, *A* and *B* will tend to move away from the axis of rotation. A position can be found, however, at which they will remain in equilibrium,—that is, at which the forces acting on the two bodies due to the rotation are equal.

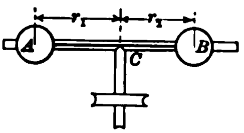

FIG. 74.

Since the two bodies are acted upon by equal forces, it follows, from the previous article, that the accelerations of the two bodies are inversely proportional to their masses. That is, if the masses of *A* and *B* be denoted by m_1 and m_2, and their linear accelerations by a'_1, and a'_2, respectively,

$$\frac{m_1}{m_2}=\frac{a'_2}{a'_1}.$$

If the apparatus makes n revolutions per second, the body *A* at a distance r_1 from the axis of rotation will have some uniform linear speed v_1, so that the acceleration of *A* toward the axis of rotation is, (40),

$$a'_1=\frac{v_1^2}{r_1}=\frac{(2\pi r_1 n)^2}{r_1}=4\pi^2 r_1 n^2.$$

In the same manner, the value of the linear acceleration of the body *B* due to the rotation is

$$a'_2=\frac{v_2^2}{r_2}=\frac{(2\pi r_2 n)^2}{r_2}=4\pi^2 r_2 n^2.$$

Consequently,

$$\frac{m_1}{m_2}\left[=\frac{a'_2}{a'_1}\right]=\frac{r_2}{r_1}. \qquad \text{(55)}$$

Whence, the masses of two bodies can be compared in terms of two accelerations, or in terms of two more easily measured distances.

There are other methods of comparing masses that offer less experimental difficulty than the one just described, but this one has been given in order to show that the essential nature of mass is quite distinct from that of force, and that masses can be compared without any knowledge regarding the magnitude of any force.

71. The Absolute Units of Force.—The units of force employed in this course thus far have been the weights of certain arbitrarily selected bodies when at definite places on the earth's surface (Art. 2). Since the weight of a body does not change greatly when it is moved from one place on the earth's surface to another, these gravitational units of force, such as the pound weight and the kilogram weight, are generally used roughly as the weight of a mass of one pound or of one kilogram at any place. This is convenient and sufficiently accurate for the requirements of engineering and ordinary life, but for scientific work it is necessary to have a unit that is absolutely constant.

The absolute unit of force is based on the relation between force and the acceleration that it produces in a body's motion. From Newton's second law of motion, when a force acts upon a body, an acceleration is produced in the line of action of the force, of a magnitude directly proportional to the applied force. That is,

$$a \propto F.$$

In Art. 69 we have seen that if equal forces act upon two bodies, the accelerations produced are inversely proportional to the masses of the bodies. That is,

$$a \propto \frac{1}{m}.$$

Since no factor except force and mass affects the acceleration of a body's motion, it follows that*

$$a = k_1 \frac{F}{m},$$

or

$$F = k(ma), \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

---

[*] "If A varies jointly as B and C, then $A = kBC$."—Hall and Knight, "Higher Algebra," p. 33.

where k is a constant of proportionality the value of which depends only upon the units adopted for F, m, and a.

It is customary to choose such units for F, m, and a that the value of k shall be unity. This can be done by selecting arbitrarily a new unit for some one of the three quantities F, m, and a, and so selecting this new unit that

$$F = ma. \qquad (57)$$

One way of doing this is to use the gram as the unit for mass, the centimeter per second in a second as the unit for acceleration, and to choose as a new unit of force, that force which if it were to act upon a mass of one gram would impart to it an acceleration of one centimeter per second in a second. This unit of force is called a *dyne*. We consequently have the relation

dynes = (grams) (cm. per sec. in a sec.).

Similarly, if we give the name *poundal* to that force which if it were to act upon a mass of one pound would produce in it an acceleration of one foot per second in a second, we have

poundals = (pounds) (ft. per sec. in a sec.).

It is to be noted that a dyne may act upon some other mass quite as well as upon a gram. If it acts upon a larger mass, it produces an acceleration that is less than one centimeter per second in a second; if it acts upon a smaller mass the acceleration is greater. A similar statement applies to the poundal.

The relations between the absolute and the gravitational units of force are easily obtained. One gram is pulled toward the earth with a force called one gram weight. At a place where the acceleration due to gravity is 980 cm. per sec. per sec., one gram weight would impart to one gram an acceleration of 980 cm. per sec. per sec. A dyne is the force which would impart to one gram an acceleration of one cm. per sec. per sec. Therefore, at the given place, one gram weight equals 980 dynes.*

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* A one-cent piece weighs about three grams. If a postage stamp be cut into 45 equal parts, each part will weigh very nearly one dyne.

One pound is pulled toward the earth with a force called one pound weight. At a place where the acceleration due to gravity is 32.1 feet per sec. per sec., one pound weight would impart to one pound an acceleration of 32.1 feet per sec. per sec. A poundal is the force which would impart to one pound an acceleration of one foot per sec. per sec. Therefore, at the given place, one pound weight equals 32.1 poundals.

In all of our problem work, unless otherwise directed, we shall take

$$\begin{aligned}1 \text{ gram weight} &= 980 \text{ dynes}\\ 1 \text{ pound weight} &= 32.1 \text{ poundals.}\end{aligned}$$

It should be kept in mind that these multiplying factors are not accelerations. They are pure numbers. They are the ratios of the magnitudes of the gram weight to the dyne, and of the pound weight to the poundal, respectively.

The gram weight and the pound weight are called gravitational units of force. The dyne and the poundal are called absolute or kinetic units of force. The unit of force employed by English speaking engineers is the pound weight. That employed by Continental engineers is the kilogram weight. In science the dyne is usually employed, although the result is sometimes expressed in grams weight. The poundal is seldom used.

SOLVED PROBLEM

PROBLEM.—What is the force equal to the weight of a kilogram at a place where a body, starting from rest, falls freely through 44.19 meters in three seconds?

SOLUTION.—The required force is given by the equation, $F = ma$. From (32) and (33) we find that if a body starting from rest moves through a distance x in time t with constant acceleration, the value of this acceleration is $a = \frac{2x}{t^2}$. Consequently, the force of gravity acting upon a kilogram mass at the given place is

$$F[=ma] = \frac{2mx}{t^2} = \frac{2 \times 1000 \times 4419}{9} = 982000 \text{ dynes.}$$

72. The Gravitational or Engineering Units of Mass.—Another way of making k in (56) unity is to choose as the unit of force the pound weight, as the unit of acceleration the foot per sec. in a sec., and as the unit of mass that mass to which a force of one pound weight would impart an acceleration of one foot per sec. in a sec. This unit of mass is the one commonly used in engineering in English speaking countries and is usually called the *British engineering unit of mass.* It is also called the *slug.*

The relation between the pound and the British engineering unit of mass is readily obtained as follows. At a place where the acceleration due to gravity is 32.1 feet per sec. per sec., one pound weight would impart to a mass of one pound an acceleration 32.1 times as great as it would impart to one British engineering unit of mass. Consequently, at this particular place, the B.e.u. of mass is 32.1 pounds.

The engineers of Continental Europe employ as the unit of mass that mass to which will be given an acceleration of one meter per sec. per sec. by the application of a force of one kilogram weight. Now at a place where the acceleration due to gravity is 9.8 meters per sec. per sec., one kilogram weight would impart to a mass of one kilogram an acceleration 9.8 times as great as it would impart to one of these Continental engineering units of mass. Therefore, at this place, this Continental unit of mass is 9.8 kilograms.

In problem work, unless otherwise directed, we shall take

1 British engineering unit of mass = 32.1 pounds
1 Continental " " " " = 9.8 kilograms.

It should be noted that the engineering unit of mass is variable, depending upon the value of the acceleration due to gravity at the place where the body is situated. But the mass itself is constant whatever the position of the body. Engineers define mass as the ratio of the weight to the acceleration due to gravity. However the acceleration due to gravity may change, the weight will change proportionally, the ratio thereby remaining constant.

SOLVED PROBLEMS

PROBLEM.—A force of 4 lb. wt. causes a certain mass to move from rest through 18 ft. in three seconds. Find the mass.

SOLUTION.—From the equation $F = ma$ it follows that a body which receives an acceleration of a ft. per sec. per sec., when acted upon by a force of F lb. wt., has a mass, expressed in British engineering units, given by the equation

$$m = \frac{F}{a}.$$

It is now necessary to find the acceleration from the data of the problem. From (32)

$$a\left[= \frac{v_t - v_0}{t}\right] = \frac{v_t}{3}.$$

From (33), $x = \frac{1}{2}(v_t + v_0)\,t$, it follows that

$$18 = (\tfrac{1}{2}v_t)3 \quad \text{or} \quad v_t = 12 \text{ ft. per sec.}$$

Whence $$a = \frac{12}{3} = 4 \text{ ft. per sec. per sec. and}$$

$$m\left[= \frac{F}{a}\right] = \frac{4}{4} = 1 \text{ B.e.u. of mass.}$$

PROBLEM.—What constant horizontal force is required to stop in one minute a train weighing 700 tons and running at 40 miles per hour.

SOLUTION.—A body of 700 tons weight has a mass of 700×2000 lbs. $= \frac{700 \times 2000}{32.1}$ British engineering units of mass. To change the velocity of this mass in t seconds from v_0 ft. per second to v_t ft. per second, requires a force

$$F[= ma] = m\left(\frac{v_t - v_0}{t}\right)$$

$$= \frac{700 \times 2000}{32.1} \cdot \frac{\frac{40 \times 5280}{3600}}{60} \text{ lb. wt.}$$

73. Weight Proportional to Mass.—The weight of a body is the force exerted upon it by the attraction of the earth. Denoting by f_1 and f_2 the weights of two bodies of masses m_1 and m_2, we have at a place where the acceleration due to gravitation is g,

$$f_1 = m_1g \text{ and } f_2 = m_2g.$$

Whence, $$f_1 : f_2 = m_1 : m_2.$$

Consequently, *at any assigned point on the earth, the masses of two bodies are proportional to their weights at that point.* This law is the basis of the ordinary method of comparing masses called weighing.

The ordinary beam balance, Fig. 75, consists of two scale pans hung from the ends of a beam supported at its middle point by a knife edge. If the scale pans are of equal mass, and the centers of mass of the two halves of the beam are equally distant from the supporting knife edge, the unloaded balance will be in equilibrium. When masses m_1 and m_2 are placed on the two pans, the balance will be in equilibrium if the moments of the weights of the two masses

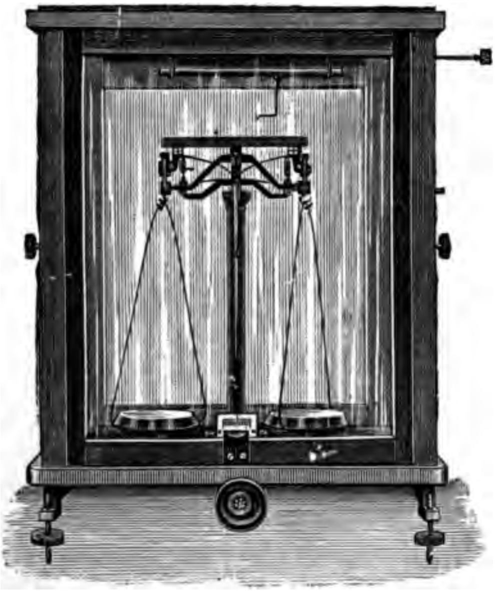

FIG. 75.

are equal. Thus, representing by l the distance from the middle knife edge to the point of support of either scale pan,

$$m_1gl = m_2gl,$$

$$m_1 = m_2$$

Consequently, if the condition of equilibrium of an equal arm beam balance is unchanged by the addition of two masses to the pans, the two masses are equal. This result is independent of the value of the acceleration due to gravity at the place of observation.

The spring balance consists of a spiral spring which is distorted by the application of a force. The spring balance can be calibrated by noting the elongations of the spring produced by a series of different standard masses. But since the elongation of the spring is due to the weight of the applied mass, and since the weight of a body depends upon the acceleration due to gravity, it follows that a spring balance calibrated for one place on the earth will give erroneous indications at any place where the acceleration due to gravity is different.

74. Change of Apparent Weight Due to Acceleration.—When a man is in an elevator which is either ascending or descending with uniform speed, the counteraction of the floor on the man equals the man's weight. But if the elevator is ascending with an accelerated motion, it has not only to support his weight, but must also impart an acceleration to his motion. In this case, the counteraction of the floor equals the man's weight just as before; but in addition to this, the floor has to support the reaction of the force that is giving the man the upward acceleration of the elevator. If the mass of the man be m, and the acceleration due to gravity be g, his weight is mg. If the upward acceleration of the elevator be A_u, then the total force exerted on the man by the floor, or his apparent weight, is

$$R = mg + mA_u.$$

If the man were suspended by a spring balance, the balance would indicate the weight given above.

If, on the other hand, the elevator were descending with an acceleration A_d, then the man's thrust on the floor would be less than his weight by the amount necessary to impart to him the downward acceleration of the elevator. Thus, his apparent weight is now

$$R' = mg - mA_d.$$

If $A_d = g$, that is, if the elevator were falling freely, the man would exert zero force on the floor of the elevator.

75. Fundamental and Derived Units.—Since all physical phenomena are intimately connected, and the quantitative relations between them can be definitely formulated, it has been found possible to select a small number of elementary units from which all the other units required for measurement can be obtained. The units selected to form the basis for a system of units are called *fundamental* units. A unit whose magnitude is determined by a relation existing between the physical quantity to be measured and quantities which are to be compared in terms of the fundamental units adopted, is called a *derived* unit. It is possible to select various sets of fundamental units. Thus, from the three units of length, mass, and time, all other desired units can be derived.* Also, the units required for all physical measurement can be derived from the units of length, force and time.

76. The Absolute Systems of Units.—A system of units employing as fundamental units, those of length, mass and time, being independent of gravitational force, is called an *absolute system of units.* If the fundamental units be the centimeter, gram and second, the system is called the *C. G. S. absolute system.* For scientific work this system is employed almost universally. In this system the unit of force is the dyne, the unit of energy is the dyne-centimeter or erg, etc.

If the fundamental units be the foot, pound and second, the system is called the *F. P. S. absolute system.* The derived unit of force is called the poundal, the unit of energy is called the foot-poundal, etc. This system of units is seldom employed.

77. The Engineering Systems of Units.—In engineering, where forces are more often considered than masses, it is convenient to select as fundamental units those of length, force and time. In a system based on these fundamentals, the unit of mass is a derived quantity and is defined as that mass which will be given unit acceleration when acted upon by the gravitational unit of force. Hence, the length-force-time systems are called gravitational or engineering systems of units.

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* Although this statement is really subject to two exceptions, it is accurate so far as dynamics is concerned.

If the fundamental units be the foot, pound weight and the second, the system is called the *F. P. S. gravitational* or the *British engineering system of units.* The derived unit of mass is about 32.1 pounds, that of energy is the foot-pound, etc.

If the fundamental units be the meter, the kilogram weight and the second, the system is called the *M. K. S. gravitational* or the *Continental engineering system of units.* The derived unit of mass is about 9.8 kilograms, that of energy is the kilogram-meter, etc.

SOLVED PROBLEMS

PROBLEM.—Find the magnitude of the constant horizontal force required to stop, in 500 ft., a train of 300 tons mass running at 45 miles per hour.

SOLUTION.—By using (32) and (33) it will be found that if a train running 45 miles per hour is brought with constant acceleration to a stop in 500 ft., the acceleration is

$$a = -4.36 \text{ ft. per sec in a sec.}$$

Therefore, from (57), the force required to impart to the train this acceleration is given by the equation

$$F[=ma] = \frac{300 \times 2000}{32.1}(-4.36) = -81{,}400 \text{ lb. wt.}$$

The negative sign signifies that the direction of the force is opposite to the direction of motion.

PROBLEM.—A mass of 40 kg. is placed on a plane inclined 30° to the horizontal. The coefficient of kinetic friction is 0.3. What force will be required to give the body an acceleration of 300 cm. per sec. per sec. up the plane?

SOLUTION.—There are three forces acting on the body in the direction of the plane. They are, (1) the force F up the plane required to give the body the assigned acceleration, (2) the component of the weight of the body down the plane, (3) the friction down the plane.

Representing the inclination of the plane by ϕ, and the weight of the body by W, as in Fig. 76, we find the component of the weight of the body down the plane to be $W \sin \phi$.

Representing by b the coefficient of kinetic friction, the force of friction down the plane is given by the expression bF_n, (11). Or, since, from Fig. 76, the normal force $F_n = W \cos \phi$, we find the force of friction down the plane to be $bW \cos \phi$.

Whence, calling forces in the direction of the acceleration positive, we have the sum of the forces acting on the body

$$F - W \sin \phi - bW \cos \phi.$$

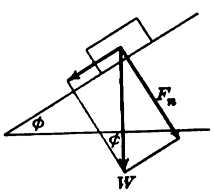

Fig. 76.

But the resultant force acting on a body in any direction equals the product of its mass and its acceleration in that direction.

Consequently, if m represents the mass of the body, and a its acceleration.

$$F - W \sin \phi - bW \cos \phi = ma.$$

$$F = W \sin \phi + bW \cos \phi + ma.$$

Using Continental engineer's units, we have,

$$F = 40(0.5) + 0.3(40)(0.866) + \frac{40 \times 3}{9.8}$$
$$= 42.62 \text{ kg. wt.}$$

78. Density and Specific Gravity.—The *density* of a substance is the ratio of its mass to its volume. The number which expresses the density of any substance depends upon the units in terms of which mass and volume are measured. For instance, the density of copper is 8.92 grams per cubic centimeter, or 557 pounds per cubic foot.

The relative density, or *specific gravity*, of a substance is the ratio of its density to the density of some standard substance. In other words, the specific gravity of a body is the ratio of its mass to the mass of an equal volume of a standard substance. Specific gravity is an abstract number which is independent of the units employed. In the case of solids and liquids, water at the temperature of its maximum density (4° C. or 39.2° F.) is arbitrarily taken as the substance with which the densities of other substances are compared.

Since the gram is the mass of one cubic centimeter of water at the temperature of its maximum density, it follows that the number which expresses the density of a substance in grams per cubic centimeter is the same number which expresses its specific gravity.

79. The Center of Mass.—If a body or system of bodies be conceived to be divided into particles of equal mass, then that point whose distance from any given plane is equal to the average distance from that plane of all the constituent particles, is termed the *center of mass*, or *center of inertia*, of the body or system of bodies. In case the bodies composing the system have incommensurable masses, then they may be conceived to be divided into particles of as nearly equal mass as is desired by making the particles sufficiently small. This point has several important properties, a few of which will here be enumerated:

(1) If the particles of a rigid body moving with pure translation are acted upon by systems of forces whose resultants are all in the same direction, the centroid of this system of forces coincides with the center of mass of the body.

(2) If the resultant of all the forces acting on a rigid body is a single force whose line of action passes through the center of mass of the body, the motion of the body is without angular acceleration.

(3) The weight of a body acts approximately at its center of mass.

(4) The center of mass of a body is so situated that the linear motion of the body would not be changed if the total mass were concentrated at this point and all the forces acting on the body were transferred to this point without change of magnitude or direction.

(5) The motion of the center of mass of any material system is not affected by the internal forces between the parts of the system, but only by external forces.

(6) No material system can of itself, without the action of external forces, change the motion of its center of mass.

(7) An unconstrained body, when acted upon by a system of forces equivalent to a couple, will rotate about its center of mass with constant angular acceleration.

Questions

1. Would it require a different force to impart a certain speed to a cannon ball on the surface of the moon than it would if the ball were on the surface of the earth—(*a*) in a horizontal direction; (*b*) in a vertical direction? Note—The acceleration due to lunar gravitation is about one-sixth the acceleration due to terrestial gravitation.

2. Explain why a heavy body and a light one of the same size will fall equal distances in the same time.

3. In each of the following cases state whether a man standing on an elevator floor will exert a force downward different from his weight. Explain each case. (*a*) Elevator at rest; (*b*) elevator at uniform velocity upward; (*d*) rising elevator going more slowly; (*e*) elevator starting to descend; (*f*) elevator descending at uniform speed; (*g*) descending elevator going more slowly.

4. A body is lifted by an endless chain and dropped automatically. The instant it reaches the bottom it is caught by the chain and lifted again. Can the number of strokes per minute be increased by adding mass to the body? Explain.

5. A man who wears a pair of very heavy boots dives from a bridge into a river. While he is still in the air will the boots feel heavier or lighter than if he were sitting on the railing of the bridge? Why?

6. In "hefting" a body by lifting it up and down, does one attempt to estimate its weight or its mass? Give reasons for answer.

7. Two packages of similar size and appearance are lying on the ground. Without lifting or weighing them, state how one might determine which has the greater mass. Give the physical principles underlying the method.

8. Why can the blow of a hammer exert a greater force on a nail than the weight of a considerably heavier object that simply rests on top of the nail?

9. Why is a watch more likely to be damaged when falling to the ground from a table 2 ft. high than by falling from a point an inch above the ground?

10. Aristotle considered that if two bricks, one upon the other, be allowed to fall freely, the upper brick by pressing upon the lower will cause the latter to fall with greater acceleration than if it were not present. Discuss this idea.

§ 4. *Circular Motion*

80. Material Particles.—It is often desired to omit the consideration of any rotatory motion that a body may have and to limit one's attention to its motion of translation. If the size of a body were reduced to that of a mathematical point, the body could have a translatory motion, but no rotatory motion. An ideal body so small that the only motion of which it is capable is translatory motion is called a *material particle*. Although no actual body is so small as this, it is true that in so far as motion is concerned any body which has no rotation may be thought of as a material particle. In this sense the term material particle is often used in physics.

81. Forces in Circular Motion.—In Fig. 77 let $ABCD$ represent a vertical rail inclosing a horizontal circle. Imagine a particle moving initially with constant velocity of magnitude v in the direction AB, to be deflected from its rectilinear path by the smooth circular rail BCD. Since the rail is smooth, no force parallel to the rail is acting upon the particle; consequently the linear speed of the particle remains constant. It has been shown (Arts. 61, 62) that at every point in the path of a body moving with uniform speed in the circumference of a circle, the velocity of the body has directed toward the center of the path an acceleration equal to the square of the linear speed divided by the radius of the path. Consequently, the force required to deflect the particle out of the rectilinear path into a circular path of radius r is directed toward the center of its path and, (40), has a magnitude

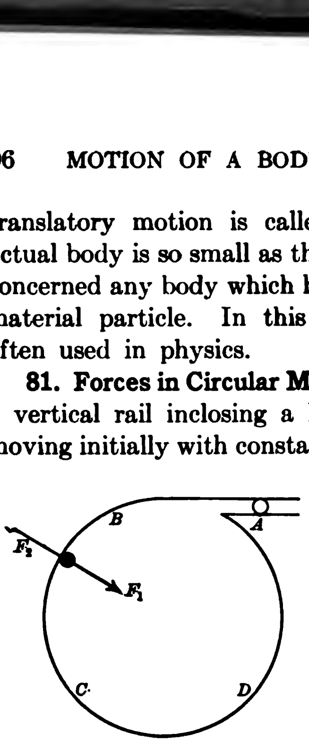

FIG. 77.

$$F_c[=ma]=\frac{mv^2}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (58)$$

The force required to overcome the inertia of a body in deflecting it from a rectilinear path into a circular path is called *centripetal* (center-seeking) force. In the above case, this force is due to the thrust of the rail against the particle.

The particle itself exerts on the agent which constrains it to move in a circular path a force ${}_rF_c$, which is equal in magnitude and opposite in direction to the centripetal force. This, frequently called *centrifugal* (center-fleeing) force, is the reaction of the centripetal force and may be defined as the resistance which the inertia of a body in motion opposes to whatever deflects it from the rectilinear path. It should be kept in mind that the centripetal force acts upon the particle, while its reaction, the centrifugal force, acts, not upon the particle, but upon the agent which constrains it to move in a circular path. Since, in the above illustration, the reaction of the particle ceases as soon as the rail ceases to push it toward the center, both the centripetal and the centrifugal forces disappear simultaneously. On the cessation of these forces the particle pursues a straight path tangential to its former circular path. At no time is there any tendency of the particle to move radially out from the center of its path. If the body move with a greater speed, (58) shows that it will require a greater force to constrain it to move in a circular path of given radius. If the linear speed be increased while the centripetal force is kept constant, (58) also shows that the radius of the circular orbit must increase.

The centrifugal drier used in laundries for drying clothes consists of a cylindrical drum, with perforated sides, capable of rapid rotation about a vertical axis. When rotated, the contents of the drier tend to continue moving in a path tangential to the sides of the drum. The clothes cannot go through the perforations, but the drops of water can. In this manner most of the water is very quickly removed from the clothes. A similar machine is used to recover the oil mixed with the metal turnings and shavings of machine shops. In sugar refineries, sugar crystals are separated from admixed molasses by means of a similar machine.

Every part of a belt tends to continue moving in its present direction. In going around a pulley, this tendency causes each portion of the belt to increase its distance from the axis of rotation. The diminution of the force pressing the belt against the pulley causes high-speed belts to slip.

It is due to this same principle that the circus performance called " looping the loop " is possible. In order that the bicycle may move in the circular path some force must press the bicycle toward the center of the circle. At the bottom and on the sides of the circle this force must be supplied by the track; at the top of the circle a part of it is suppled by the weight of the bicycle and rider. Consequently the track does not need to be so strong at the top as at the bottom—there may even be a gap at the top.

82. The Centrifugal Pump.—In many cases where electric or steam power is available, the centrifugal pump is preferable to a reciprocating pump. One form consists of a hollow drum, provided with passages extending from the axial space to the periphery, and rotating within a casing, as illustrated in

Fig. 78. If, when filled with water, the drum be rotated in the direction indicated, each particle of water within the drum passages will tend to move in a tangential path. Thus the inertia of each particle will cause it to retreat from the axis in the tangential direction till the particle is acted upon by a force given by (58). Commercial centrifugal pumps are made that will push water against a force of 25 lbs. per square inch.

83. The Centrifugal Emulser.—Although the supply of milk and cream on the market is nearly the same from one day to the next, the demand for ice cream fluctuates several hundred per cent. This difficulty can be successfully met only by making ice cream from materials that can be stored in anticipation of any demand. Instead of milk and cream, factories at the present time frequently use butter and dried skim milk. When intimately combined with the proper amount of water, the result is essentially the same as that obtained from milk and cream. After the mixture of milk powder, water and washed butter has been raised to about 150° F., it can be made into a homogeneous emulsion by forcing it through minute holes. In some plants high pressure steam pumps are employed to push the mixture through very small holes. But a simpler device is the DeLaval emulser. This consists of a column of disks as in Fig. 79, with their edges separated by almost microscopic spaces. The column of disks rotates with a speed of about 10,000 revolutions per minute. Under this enormous angular speed, the warm mixture traverses the thin spaces between the edges of the disks and emerges thoroughly emulsified.

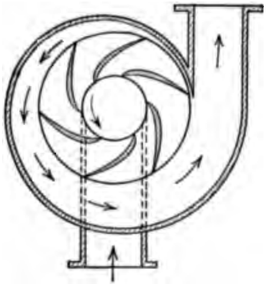

Fig. 78.

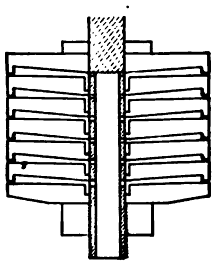

Fig. 79.

84. The Centrifugal Cream Separator.—From (58) it follows that if the rotating body consists of a mixture of two substances of different densities, then each particle of the substance of greater density, i.e., of greater mass for a given volume, will require a greater force to keep it moving in the circle than will a particle of the substance of smaller density. If the particles are free to move amongst one another, the forces exerted upon all the particles in the same neighborhood must be equal. If, in a given region, the force is just sufficient to hold the particles of the less dense substance in a circle, it will not be great enough to hold the particles of the denser substance. Consequently the latter will move farther away from the center. This is the principle employed in the centrifuge and in the centrifugal cream separator.

Milk is a mixture of fat globules, called cream particles, and a liquid called milk serum or skim milk. Let the mass and density of a particle of milk serum of volume V be represented by m and d, respectively, and the mass and density of an equal volume of cream by m' and d', respectively.

When at rest in a pan, the cream particles are pushed upward by a force equal to the weight of serum displaced by them (Archimedes' Principle, Art. 110). Thus in pan or gravity separation, the separating force is

$$F_g = mg - m'g = Vg(d - d'). \quad . \; . \; . \; . \; . \; . \; . \qquad (59)$$

But if the vessel containing the milk be rotated, the sides of the vessel must push the cream particles toward the axis of rotation with a force

$$F' = \frac{m'v^2}{r} = \frac{Vd'v^2}{r},$$

where r represents the radius of the rotating vessel and v is the linear speed of a point on the circumference. Similarly, the serum particles are pushed toward the axis of rotation with a force

$$F = \frac{mv^2}{r} = \frac{Vdv^2}{r}.$$

Hence the separating force due to rotation is

$$F_c = \frac{Vdv^2}{r} - \frac{Vd'v^2}{r} = \frac{Vv^2}{r}(d - d'). \quad . \; . \; . \; . \; . \; . \qquad (60)$$

Consequently the ratio of the force of centrifugal separation to the force of gravity separation is, (59) and (60).

$$\frac{F_c}{F_g} = \frac{v^2}{rg} = \frac{4\pi^2 rn^2}{g}, \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (61)$$

where n represents the number of revolutions made by the vessel in one second.

In Fig. 80, the rotating bowl of a centrifugal separator is represented in heavy lines, and the stationary collector is represented in light lines. After descending the axial tube, the milk flies to the periphery of the rotating bowl.

The more dense serum collects at the periphery, crowding the less dense cream toward the axis. Due to the crowding of the serum at the periphery, together with the pressure of the milk in the axial tube, the serum rises and emerges from a vent in the lower collector. The cream emerges from the upper collector, in some machines being directed upward by a series of cones as shown in the diagram.

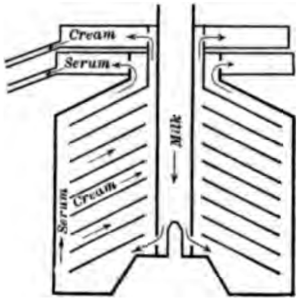

Fig. 80.

85. A Vehicle Moving on a Curved Track.—When going around a horizontal curved track, every particle of matter composing a bicycle and the rider tends to continue moving in a straight line and thereby to increase its distance from the center of curvature of the track. The friction between the road and tires prevents the lower portions of the wheels from retreating from the center of the curve, but the rider will fall outward if he does not incline his body toward the center of the curve. Again, when riding in a straight path, if he feels a tendency of the bicycle to fall over, he will right himself by turning the front wheel in such a direction that the center of the curved path thus formed is on the side toward which he feels himself falling.

When a train is going around a curve, every particle composing its entire mass tends to continue moving in a rectilinear direction. The outer rail produces on the flanges of the wheels in contact with it a horizontal thrust directed toward the inside of the curve, and thus constrains the trucks of the cars to follow the curve. As there is nothing acting in a similar manner on the superstructure of the cars, they tend to increase their distance from the center of the curvature of the track, thereby tending to lift the wheels off the inside rail. In practice the outside rail is somewhat higher than the inside rail, so that a component of the weight of the car acts toward the inside of the curve.

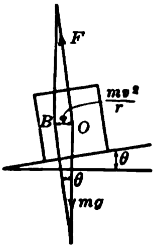

Fig. 81.

The elevation that the outer rail of a railway track on a curve of radius r must have in order that the flanges of the wheels on a car moving with a speed v shall produce no lateral thrust against the rails can easily be computed.

In Fig. 81 let the car be moving away from the observer. Acting upon the car are two forces—its weight mg vertically downward, and the force F with which the rails press against the car. By hypothesis the conditions are to be such that F is perpendicular to the track. In order that the car may be moving in the circumference of a circle, the resultant force acting upon it must be a force of value $\frac{mv^2}{r}$ acting toward the center of the circle. That is, the resultant

of F and mg must be horizontal, and must have a value $\frac{mv^2}{r}$. When the parallelogram is completed, the angle between F and mg will equal the angle θ, which is the angle of super-elevation of the outer rail. From the figure,

$$\frac{\frac{mv^2}{r}}{mg} = \tan\theta.$$

Whence $$\theta = \tan^{-1}\frac{v^2}{rg}. \quad \ldots \ldots \ldots \quad (62)$$

If the speed of the train on the curve is to be 45 miles per hour, and the radius of the curve is 2000 ft., then if the rails are 4 ft. 8½ in. apart, (62) shows that the outer rail should be 3.8 in. above the inner.

SOLVED PROBLEM

PROBLEM.—A pail of water is rotated in a vertical plane in a circle of 1 meter radius. Find the least period of revolution necessary to prevent spilling of the water.

SOLUTION.—In order that the water may not move in a tangential path, it must be acted upon by a force directed toward the center of the circle having the magnitude given by (58). This required force may be supplied by the weight of the water or by the bottom of the bucket pushing on the water. When the bucket is at the highest point of the path, the water will not spill if the speed be such that

$$\frac{mv^2}{r} = mg \quad \text{or} \quad v^2 = rg.$$

Denoting the period of revolution by t, $v = \frac{2\pi r}{t}$. Hence

$$t\left[=\frac{2\pi r}{v}\right] = \frac{2\pi r}{\sqrt{rg}} = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{100}{980}} \doteq 2 \text{ seconds}.$$

QUESTIONS

1. When an automobile is running at constant speed, what stresses are there in one of the wheel spokes? How does each of these stresses vary in magnitude during one revolution of the wheel?

2. It is found that even when there is no production of air pockets between a belt and a high-speed pulley, the friction between the belt and the pulley is less than when running at a lower speed. Explain.

3. Show how the weight of a body depends in two particulars on the latitude of the place in which the body is situated.

4. In the case of a body moving with uniform speed in the circumference of a circle, what is the value of, (*a*) the angular acceleration, (*b*) the linear acceleration in the direction of motion, (*c*) the linear acceleration perpendicular to the direction of motion?

5. In one form of governor for a steam engine a heavy mass at one end of an arm is so pivoted inside the flywheel that it can move toward or away from the circumference of the latter. Which way will an increase in the speed of the wheel cause this mass to move? Explain fully.

6. In the circus performance, "looping the loop," what is the effect of a change in (*a*) the initial speed, (*b*) the radius of the loop? Under what conditions will the rider fail to negotiate the loop? How does the pressure vary on different parts of the track?

7. Explain the tendency of a motor car to "skid" on turning corners. Why does this occur more often on wet pavements? A reckless driver of a four-wheeled vehicle is sometimes characterized by the statement, "he turned the corner on two wheels." Which two wheels will these be? Explain.

8. A man stands on a platform balance and swings a pail of water in a vertical circle. Do the indications of the balance vary? Explain.

CHAPTER VI

THE MOTION OF A BODY UNDER THE ACTION OF A CONSTANT TORQUE

§ 1. *The Axis of Torque and the Axis of Rotation Coincident*

86. Torsional Stress and Torque.—If a wooden rod be held in the two hands and one end be twisted about the length of the rod as an axis, there will be developed in the rod a stress which resists further torsion and which also tends to diminish the twist already produced. The stress which tends to twist a body in such a manner that each section of the body turns on the next adjacent section about an axis normal to the plane of section is called a *torsional stress*. While the rod is under torsional stress it tends to rotate, in opposite directions, the two hands that hold it twisted. Any cause which (like those at the two ends of the rod) either changes or tends to change the angular velocity of one part of a system with respect to another is called a *torque*.

The magnitude of a torque is the moment of the forces acting with respect to the axis of rotation. The direction is given by the axis of rotation and the sense of rotation about that axis. A torque is in the positive direction when, on looking along the axis of the torque the rotation is clockwise. A torque can be represented by a straight line in the direction of the axis of torque and having a length proportional to the magnitude of the torque. The direction of the twist about the axis may be indicated by an arrowhead on the line so pointing that in looking along the line in the direction of the arrowhead the rotation is clockwise.

Torques acting simultaneously can be compounded and resolved by the same method used for angular velocities.

87. Angular Acceleration.—If a torque acts upon a stationary body capable of rotation, the body will be set into angular motion. If the body be already rotating, the application of the torque will change the angular velocity of the body. In either case a change in the angular velocity of the body will be produced. When the angular velocity of a rigid body varies, the time rate of change of the angular velocity is called the *angular acceleration* of the body's motion. Thus, if during the time t the angular velocity about a given axis varies uniformly from w_0 to w_t, there is during the given interval a uniform angular acceleration $\mathbf{a}$, the magnitude of which is given by

$$\mathbf{a}=\frac{w_t-w_0}{t}. \qquad (63)$$

This equation shows that angular acceleration may be measured in radians per second in a second, degrees per minute in an hour, revolutions per minute in a second, etc.

Howsoever the rotation of a body about a fixed axis may change during a given time, there must be a certain equivalent angular velocity with which a uniformly rotating body would rotate through the same angle in the same time. If the rotation is uniformly accelerated, the equivalent uniform velocity may, by the method of Art. 60, be shown to be the arithmetic mean of the instantaneous velocities at the beginning and end of the time considered. Consequently, if the angular velocity of the body changes uniformly during the time t from w_0 to w_t, then the angle Φ swept through during this time by any line in the body perpendicular to the axis of rotation is

$$\Phi=\tfrac{1}{2}(w_t+w_0)t. \qquad (64)$$

By the use of (63) and (64) a large class of problems in uniformly accelerated angular motion can be solved.

Angular accelerations can be represented by right lines in the same manner as angular velocities (Art. 54). Angular accelerations can be compounded and resolved by the parallelogram law precisely as can angular velocities (Art. 56).

88. The Relation between Angular and Linear Acceleration.—From (63), (31), and (32), we have

$$\mathbf{a}\left[=\frac{w_t-w_0}{t}\right]=\frac{\frac{v_t}{r}-\frac{v_0}{r}}{t}=\frac{v_t-v_0}{tr}=\frac{a}{r}. \qquad (65)$$

Whence, the magnitude of the angular acceleration of any body is equal to the tangential linear acceleration of any point of the body divided by the distance from that point to the axis of rotation.

A body moving with uniform linear speed in the circumference of a circle will have $\mathbf{a}=0$; from (65) a will also equal zero along the tangent. There will, however, be an acceleration along the radius of the value (40), $a'=v^2/r$.

89. Three Laws of Angular Motion.—(1) Whatever is capable of producing linear acceleration is called force; whatever is capable of producing angular acceleration is called torque. If there is no torque, there is no angular acceleration. Consequently, *a body will continue to move with its present angular speed about an invariable axis of rotation until acted upon by an external torque.* This is the analogue in rotation of Newton's First Law of Motion. A torque is required to change either the angular speed or the direction of the axis of rotation.

To diminish air resistance, the diameters of modern projectiles are small. To have sufficient mass, the length must be greater than the diameter. In order that such a long projectile may strike head-on, it is given a rapid angular velocity about the long axis.

(2) *When a torque acts upon a body, there is produced in the body's motion an angular acceleration whose magnitude is directly proportional to the sum of the moments of the applied forces.* This is the analogue in rotation of Newton's Second Law of Motion.

Since that which produces angular acceleration is called torque, and the angular acceleration produced is proportional to the resultant force moment, torque is measured by the resultant force moment applied to a body.

(3) Since the reaction of any force is equal in magnitude, has the same line of action, and is in the opposite direction to the force, it follows that the reaction to any force has a moment, about any axis, equal and opposite to the moment of the force. Consequently, *for every torque there acts upon some other body another torque equal to the first and tending to produce rotation about the same axis in the opposite direction.* This is the analogue in rotation of Newton's Third Law of Motion.

§ 2. *Moment of Inertia*

90. Angular Acceleration Produced by a Uniform Torque about the Axis of Rotation.—The apparatus depicted in Fig. 82 consists of an easily turning spindle to the upper end of which various bodies can be attached. The lower end is provided with a drum *E*, to which a constant torque can be applied by means of a weight acting through a flexible cord arranged as shown in the engraving. The angular acceleration of the rotating system is proportional to the linear acceleration of the vertically falling mass *A*.

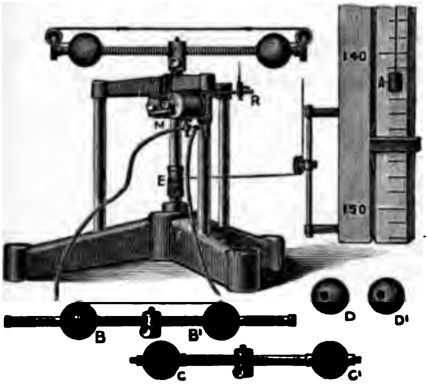

Fig. 82.

The body shown attached to the upper end of the rotating spindle consists of two similar spheres, connected by a spiral spring, and capable of sliding along a horizontal rod. These spheres can be drawn apart by means of a cord stretched over two small pulleys at the ends of the rod. If the spindle be released, it will rotate with constant angular acceleration under the influence of the constant torque produced by the weight acting at the end of the cord. If now the cord holding the spheres apart be burned, the stretched spring will pull the spheres toward the axis of rotation, and the angular acceleration of the rotating system will be increased. The body *BB'* is arranged to take the place of the one just described and consists of two similar spheres held apart by a spiral spring. These spheres can be drawn together by means of a cord. If this cord be burned while the system is rotating, the spheres will fly apart, and the angular acceleration of the rotating system will be diminished. These two experiments show that the angular acceleration of a given body, under the action of a constant torque, depends upon the distribution of the mass with respect to the axis of rotation.

The body *CC′* consists of a horizontal rod with two solid spheres attached to the ends. These solid spheres can be removed and two hollow spheres, *DD′* of the same diameter, substituted for them. When this body is attached to the rotating spindle, and the solid spheres are in place, the angular acceleration of the moving system is found to be less than when the hollow spheres are substituted for them. This shows that the angular acceleration of a body of definite shape, under the action of a constant torque, depends upon the mass of the body.

The relation between the magnitude and distribution of the mass of a body, the torque applied, and the angular acceleration developed, will now be considered. It has been shown (Art. 71) that if a particle of mass m be acted upon by a resultant force F, there will be produced a linear acceleration.

$$a=\frac{F}{m}.$$

It has also been shown (Art. 89) that if a torque be applied to a body, there will be produced an angular acceleration of its motion proportional to the torque applied. That is,

$$\mathbf{a}=\frac{L}{K}, \quad \ldots\ldots\ldots \quad (66)$$

where L is the torque, i.e., the moment of the applied force, about the axis of rotation, and K is some function of the inertia of the body yet to be determined. From the experiment considered at the beginning of this article, it appears that this quantity K depends upon both the mass of the body and upon its distance from the axis of rotation. Since the numerical measure of the importance of any physical agency on the rotation of a body is frequently called the " moment " of the particular agency, it is customary to call the quantity K the " moment of inertia " of the body. The *moment of inertia* of a rigid body with respect to a given axis is that property of the body which requires a torque to change the angular velocity of the body.

[illegible] **Moment of Inertia of a Particle.**—The value of the moment of inertia K of a particle will now be determined.

Consider a particle of mass m attached to one end of a massless rod of length r (Fig. 83), capable of rotation about c. If a force F be applied at B in such a way as to be always perpendicular to the rod and to the axis of rotation, the particle will move in the circumference of a circle of radius r with constant linear acceleration. In Art. 89 we have seen that the same angular acceleration would be produced if, instead of the force F applied at B, another force F', of such a magnitude that its moment

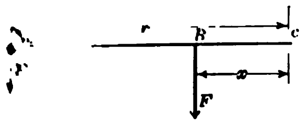

FIG. 83.

$$F'r = Fx,$$

were applied to the particle.

Denoting the linear and angular accelerations of the particle by a and $\mathbf{a}$, respectively, we have, from (65),

$$F'[=ma] = m\mathbf{a}r.$$

Substituting this value of F' in the preceding equation, we obtain

$$\mathbf{a} = \frac{Fx}{mr^2}$$

Comparing this with (66), we obtain

$$\mathbf{a}\left[=\frac{L}{K}\right] = \frac{Fx}{mr^2}. \qquad (67)$$

Since torque is measured by the force moment acting on the body, (Art. 89), $L = Fx$. Consequently the above equation shows that the moment of inertia of a particle equals the product of the mass of the particle and the square of its distance from the axis of rotation. Moments of inertia may be expressed in the various units of mass and distance. There are no names for the different units.

It can be shown, although the proof will not be given here, that the moment of inertia of any body equals the sum of the moments of inertia with respect to the given axis of the separate component particles of the body. That is, the moment of inertia of any rigid body is

$$K=\Sigma(mr^2). \qquad (68)$$

Therefore, the angular acceleration of a rigid body under the action of a resultant torque L is

$$\mathbf{a}\left[=\frac{L}{K}\right]=\frac{L}{\Sigma(mr^2)}. \qquad (69)$$

For any rigid body revolving about any axis fixed in space, the moment of inertia is a constant quantity quite independent of both the speed of rotation and the force acting. Consequently, the angular acceleration of the motion of any body is numerically equal to the ratio of the moment of the force applied, to the moment of inertia of the body about the axis of rotation.

Rotating machinery is acted upon by frictional torques which tend to produce negative angular accelerations. In order that the angular speed may remain constant there must be applied a torque of the same magnitude in the opposite direction. If the machinery be operated by a reciprocating engine, the resulting angular acceleration will not be zero. However, by adding a flywheel of large moment of inertia, the angular acceleration due to the variable torque produced by the reciprocating piston can be made as nearly constant as is desired.

For a given mass, the moment of inertia of a flywheel will be larger when the diameter of the spokes is small and the radius of the rim is large.

92. The Reason that a Falling Cat Alights on its Feet.—It is due to the application of (69) that a freely falling cat always alights on its feet. Imagine a body consisting (Figs. 84 and 85) of a rod CD, to the ends of which are hinged four rods carrying the masses A, A', B, and B'. Assume that by means of some internal mechanism, a torsional stress can be given to the body about the axis CD. This torsional stress will cause the two ends of the body to rotate in opposite directions.

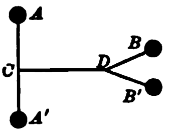

FIG. 84.

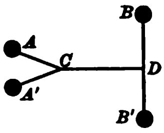

FIG. 85.

Equation (69) shows that, for a constant torque, the angular acceleration produced is inversely proportional to the moment of inertia of the body, and also that the moment of inertia, about any assigned axis, of a body of given mass, can be considerably altered by small changes in the distances of its parts from the axis of rotation.

With the parts arranged as in Fig. 84, the moment of inertia of the left end of the body, about the axis CD, is greater than that of the right end of the body about the same axis. With the parts arranged as in Fig. 85, the reverse is true. Consequently, if, with the configuration of Fig. 84, a torsional stress be developed about the axis CD, the left end of the body will experience a smaller angular acceleration than the right end; that is, in a given time the left end of the body will rotate through a smaller angle than the right end, and in the opposite direction. With the parts arranged as in Fig. 85, the left end will rotate through a greater angle than the right end. On this principle depends the ability of the cat to rotate its body while falling freely through the air.

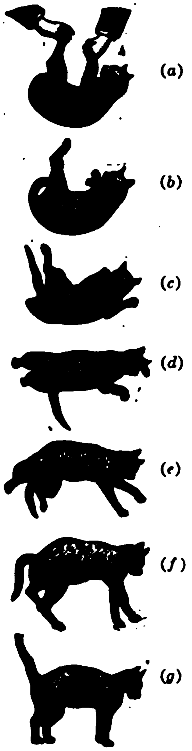

Fig. 86.

Fig. 86 is an engraving made from a series of kinetoscope photographs of a freely falling cat. An inspection of these figures shows that the first operation was simultaneously to extend the hind legs and tail perpendicular to the median axis of the body, and draw the fore legs close in to the body. A torsional stress now applied about the median axis has resulted (Fig. 86c) in a rotation of the fore quarters nearly 90° in advance of the hind quarters. By drawing in the hind legs and tail, extending the fore legs, and exerting another torsional stress in the direction opposite to the previous one, the hind quarters have been rotated (Fig. 86e) in advance of the fore quarters. By this series of operations, an agile animal can produce a sufficient rotation of the entire body to enable it always to alight on its feet, even when falling from a comparatively small height.

93. Values of the Moment of Inertia of Certain Bodies.—The moments of inertia of bodies having regular geometrical shapes can be computed. But since, in general, their computation involves mathematical methods that would be inappropriate in a course having the limitations of the present one, such calculations will be omitted. For use in solving problems, values of the moments of inertia of three regular bodies about different axes are given below:

The moment of inertia of a uniform right solid cylinder of mass M and radius r, about its geometric axis, is

$$K_c = \tfrac{1}{2}Mr^2; \qquad \ldots\ldots \qquad (70)$$

while about an element of its surface parallel to its geometric axis, its moment of inertia is

$$K'_c = \tfrac{3}{2}Mr^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (71)$$

The moment of inertia of a cylindrical ring of mass M, external radius r_1, and internal radius r_2, about its geometric axis is

$$K_r = \tfrac{1}{2}M(r_1^2 + r_2^2); \quad . \quad . \quad . \quad . \quad . \quad . \qquad (72)$$

while about a line parallel to the geometric axis and distant r_3 from it, its moment of inertia is

$$K'_r = \tfrac{1}{2}M(r_1^2 + r_2^2) + Mr_3^2. \quad . \quad . \quad . \quad . \quad . \qquad (73)$$

The moment of inertia of a uniform solid sphere of mass M and radius r, with respect to an axis through the center is

$$K_s = \tfrac{2}{5}Mr^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (74)$$

while with respect to an axis tangent to the sphere, the moment of inertia is

$$K'_s = \tfrac{7}{5}Mr^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (75)$$

94. Radius of Gyration.—If material were taken from near the axis of any body and fastened to the body farther from the axis, the moment of inertia of the body would be increased. If the material were taken from the outer part of the body and moved in toward the axis, the moment of inertia would be decreased. These two processes might be carried on at the same time in such a way that the moment of inertia of the body would be unaltered. The two processes might be kept up until the entire material of the body had been brought to the same distance from the axis. The distance from the axis at which the entire mass of a body might be concentrated without altering the moment of inertia of the body is called the *radius of gyration* of the body about the given axis. Since all the material is at the same distance from the axis, it follows from (68) that the moment of inertia of a body of mass M and radius of gyration k is

$$K = Mk^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (76)$$

95. Comparison of Force with Torque, and Inertia with Moment of Inertia

Force is any cause which either changes or tends to change the linear motion of one part of a material system relative to another.	Torque is any cause which either changes or tends to change the angular motion of one part of a material system relative to another.
Inertia is the name of that characteristic of matter by virtue of which a force is required to change the linear velocity of a body either in direction or magnitude. Inertia is measured by the tendency of a body to keep its linear velocity of constant magnitude in an invariable direction. The inertia of a body is numerically equal to the sum of the masses of its component particles.	The moment of inertia of a rigid body is the name of that quality of the body by virtue of which a torque is required to change the angular velocity of a body either in direction or magnitude. Moment of inertia is measured by the tendency of a body to keep its angular velocity of constant magnitude about an invariable axis of rotation. The moment of inertia of a body about a given axis is numerically equal to the sum of the products of the masses of the particles composing the body and the squares of their respective distances from the axis of rotation.
A body will continue to move with its present linear speed in a straight line until acted upon by an external force.	A body will continue to move with its present angular speed about an invariable axis of rotation until acted upon by an external torque.
When a force acts upon a body, there is produced in the body's motion a linear acceleration whose direction is that of the force and whose magnitude is directly proportional to that of the force. The magnitude of the linear acceleration is equal to the ratio of the applied force to the inertia of the body.	When a torque acts upon a body, there is produced in the body's motion an angular acceleration about the axis of the torque whose magnitude is directly proportional to that of the torque. The magnitude of the angular acceleration is equal to the ratio of the applied torque to the moment of inertia of the body.
For every force there is an equal and oppositely directed reaction which acts in the line of action of the force and is applied to a different body.	For every torque there is an equal and oppositely directed torque which acts about the same axis and is applied to a different body.

96. Moment of Inertia of a Plane Area.—In Mechanics, in the study of strength of materials, it is often convenient to consider a body to be composed of thin plane laminæ. In the discussion of the displacements of these laminæ with respect to a given line, when the body is distorted, a quantity of the form ΣAr^2 is of frequent occurrence. In this quantity, A represents the area of one of the small elements into which the face of one of the laminæ is conceived to be divided, and r is the distance of this element from the axis of reference. As this quantity is of the same form as the expression Σmr^2 for the moment of inertia of a body with respect to a given axis, it is called the *moment of inertia of the area of the lamina* with respect to the assigned axis. The moment of inertia of a plane area with respect to an axis normal to its plane is called a *polar* moment of inertia.

SOLVED PROBLEMS

PROBLEM.—A flywheel of radius of gyration 5 ft., is making 40 revolutions per minute when thrown out of gear. In what time does it come to rest if the diameter of the axle is 6 in. and the coefficient of kinetic friction is 0.05?

SOLUTION.—At the surface of the axle the flywheel is acted upon by a torque due to friction. Representing the weight of the flywheel by mg, the radius of the shaft by r, and the coefficient of kinetic friction by b, the torque acting on the wheel is

$$L = bmgr.$$

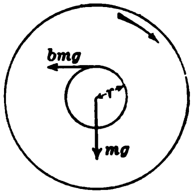

FIG. 87.

This torque will produce an angular acceleration, (66), (76):

$$\alpha\left[=\frac{L}{K}\right]=-\frac{bmgr}{mk^2}=-\frac{bgr}{k^2}$$

where K represents the moment of inertia of the wheel, k the radius of gyration, and the negative sign indicates that the angular speed is diminishing.

A body rotating with an angular acceleration α will change in angular speed from ω_0 to ω_t in the time, (63),

$$t=\frac{\omega_t-\omega_0}{\alpha}.$$

In the present problem $\omega_t=0$, and $\omega_0=40$ revolutions per minute or $\frac{40(2\pi)}{60}$ radians per second. Consequently,

$$t\left[=\frac{\omega_0 k^2}{bgr}\right]=\frac{40(2\pi)5^2}{60(0.05)(32.1)(0.25)}=261 \text{ sec.}$$

Problem.—An empty corn sheller has a speed of 60 revolutions per minute when a force of 20 lb. wt. is applied to the crank having a lever arm of 18 in. When an ear of corn is introduced, there is a resisting torque of 40 lb. ft. It is desired to add a flywheel such that the speed shall not drop below 50 revolutions per minute during the two seconds taken for the shelling of one ear. Find the moment of inertia that the flywheel must have. If the radius of gyration of the flywheel be 1.5 ft., find the mass.

Solution.—From (66) and (63),

$$K\left[=\frac{L}{a}\right]=\frac{Lt}{w_t-w_0}$$

where L represents the resultant torque. If the torque be expressed in pound-feet, K will be expressed in British engineering units.

Substituting in the equation the data of the problem,

$$K=\frac{[20(\frac{3}{2})-40]2}{2\pi\left(\frac{50}{60}\right)-2\pi\left(\frac{60}{60}\right)}=19 \text{ B.e. units.}$$

Again, from (76), we have

$$m=\frac{K}{k^2}.$$

When K is expressed in British engineering units and k in feet, m will be expressed in British engineering units of mass. Hence the mass required is

$$m\left[=\frac{K}{k^2}\right]=\frac{19}{(1.5)^2}=8.44 \text{ B.e. units of mass}$$

$$=8.44\times 32.1 \text{ lb.}$$

Problem.—A cylinder rolls down an inclined plane of height h. Find its linear speed at the bottom.

Solution.—Let r denote the radius, and m the mass of the cylinder; l the length of the plane, and θ its inclination to the horizon.

Since the rolling body is a cylinder, the line from its axis O to the point A where it touches the plane is normal to the plane. Since the weight of the cylinder acts vertically, the angle $AOB=\theta$.

A is the instantaneous axis of rotation; mg is the force that urges the cylinder to rotate about A; and $AB[=r \sin \theta]$ is the lever arm of the force. The torque that tends to make the cylinder rotate about A is, therefore,

$$L=mgr \sin \theta.$$

From (71) the moment of inertia of the cylinder about A is

$$K=\tfrac{3}{2}mr^2.$$

From (66) it follows that if $\mathbf{a}$ denotes the angular acceleration with which the cylinder rotates about A,

$$\mathbf{a}\left[=\frac{L}{K}\right]=\frac{mgr\sin\theta}{\frac{3}{2}mr^2}=\frac{2g\sin\theta}{3r}.$$

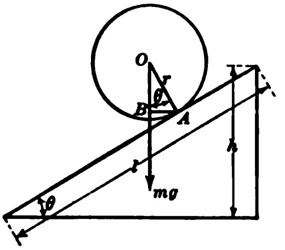

FIG. 88.

From (65), the linear acceleration of the axis of the cylinder is

$$a[=\mathbf{a}r]=\frac{2g\sin\theta}{3}.$$

From Fig. 88

$$\sin\theta=\frac{h}{l},$$

so that

$$a=\frac{2gh}{3l}.$$

From this equation it is seen that the linear acceleration of the cylinder down the inclined plane is constant. Therefore the formulæ for uniformly accelerated linear motion may be used. Substituting in (32) and (33) the values found above, we have then

$$\frac{2gh}{3l}=\frac{v_t-0}{t},$$

and

$$l=\tfrac{1}{2}(v_t+0)t.$$

On eliminating t from these two equations and then solving for v_t, we find

$$v_t=\sqrt{\frac{4gh}{3}}.$$

Since this equation does not involve the mass or radius of the cylinder, nor the length of the plane, it follows that the final speed is independent of these quantities.

QUESTIONS.

1. If you are given two cylinders of the same dimensions and mass, but the one is hollow, while the other is solid, how could you determine, by allowing them to roll down an inclined plane, which is hollow?

2. If two flywheels, one of wood and one of iron, with equal radii of gyration, are given equal angular speeds on similar shafts and then thrown out of gear, how will the times required for the two to come to rest in a vacuum compare? Explain.

CHAPTER VII

ENERGY

97. Different Aspects of Energy.—The accomplishment of a change in the position of a body against a resisting force is called *work*. Work is measured by the product of the force acting on the body moved, and the resolved part of the displacement of the body in the line of action of the force. *Energy* has been defined as stored work, or as the ability to do work. The quantity of energy possessed by a system of bodies is the amount of work the system can do against external forces in passing from its present condition to some standard condition.

The energy of a system may be due to the motion of part of it with reference to other parts. This type is called *kinetic energy*. Or, the energy of the system may be due to stresses between different parts of the system. This type is called *potential energy*.

In certain cases work may be done by a body, not on account of the motion or the strained condition of the body, but on account of the heat energy stored in the body. For example, a body can do work either by expanding and thereby overcoming pressure, or by expanding and thereby liberating heat,* which in turn is converted into work. In the former case the work is due to a diminution of the potential energy of the strained body, while in the latter case the work is due to a diminution of the internal energy of the body. Again, the heat absorbed when water is converted into steam is stored up as internal energy until the steam resumes the liquid form. Steam equals water plus a quantity of internal energy. According to the kinetic theory of matter (Art. 126), the internal energy of a body is a form of potential energy due to the arrangement of the ultimate parts of which a substance is considered to be composed.

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* Many supersaturated solutions expand on crystallizing with an evolution of heat.

Work and energy are measured in the same units. In the C. G. S. absolute system of units, the unit of work is the amount of work done when a body is moved through a distance of one centimeter against a force of one dyne. This unit is called the *dyne-centimeter*, or *erg;* 10^7 ergs is called a *joule.* In the F. P. S. gravitational system, the unit of work is the amount of work done when a body is moved through a distance of one foot against a force of one pound weight. This unit is called the foot-pound.

The rate of doing work, that is, the amount of work performed per second, is called *power.* If W denotes the work done in time t, then the power P is

$$P=\frac{W}{t}. \qquad (77)$$

In the C. G. S. absolute system of units, the unit of power is the *erg per second.* One joule (10^7 ergs) per second is called a *watt.*

In the British engineering system of units, the unit of power is called the *foot-pound per second;* 550 foot-pounds per second is called a *horse power.*

Units of work often used in engineering are the watt-hour and the horse-power-hour.

98. Work Done and Power Developed by Forces and Torques.—If, in opposition to a uniform force F a body is moved a distance x along the line of action of the force, the work done is, (1),

$$W=Fx.$$

But since when a radial force constrains a particle to move with constant speed in a circular path the force and motion are at right angles to one another, no work is done by the radial force. In this case, the energy of the particle is constant although a force is acting upon it.

If a linear displacement x is effected in time t, with a constant linear velocity v, then the power developed by a force F in the direction of the displacement is

$$P\left[=\frac{W}{t}\right]=\frac{Fx}{t}=Fv. \qquad (78)$$

Also, consider a body capable of rotation about a fixed axis passing through the point C and acted upon by a force F having a constant moment Fl about the axis of rotation. When the body has turned through an angle ϕ radians, the point of application of the force will have moved a distance $l\phi$ along the line of action of the force. Consequently, the work done by the torque L is

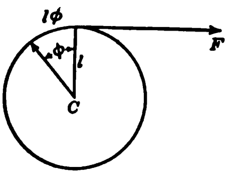

FIG. 89.

$$W[=Fx]=Fl\phi=L\phi. \quad . \quad . \quad . \quad . \quad (79)$$

If the angular displacement ϕ be effected in time t, with a constant angular velocity w, then the power developed by the torque is

$$P\left[=\frac{W}{t}\right]=\frac{L\phi}{t}=Lw. \quad . \quad . \quad . \quad . \quad . \quad (80)$$

SOLVED PROBLEMS

PROBLEM.—From the definitions of the horse-power and of the watt, together with the relation between the pound and the gram, and the relation between the foot and the centimeter, find the number of watts in 1 H.P.

SOLUTION.—

1 H.P. $=550$ ft.-lb. per sec

$=550\times30.5\times454$ gm. cm. per sec.

$=550\times30.5\times454\times980$ dyne-cm. per sec. or ergs per sec.

$=\dfrac{550\times30.5\times454\times980}{10^7}$ joules per sec. or watts

$=746$ watts.

This transformation constant is so frequently used that it should be memorized.

PROBLEM.—In determining the output of a small electric motor a strap was wrapped half way around the pulley and a spring balance attached to each vertical free end of the strap. The pulley had a diameter of three inches. When the pulley was rotating 3000 times per minute the difference in the tensions on the two ends of the strap was 4 lb. weight. Find the output at this speed, expressed in horse-power and also in watts.

SOLUTION.—

$$\text{H.P.}\left[=\frac{\text{ft.-lb. per sec.}}{550}\right]=\frac{4\left(2\pi\dfrac{1.5}{12}\cdot\dfrac{3000}{60}\right)}{550}.$$

$$\text{Watts}[=\text{H.P.}\times 746]=\frac{4\left(2\pi\frac{1.5}{12}\cdot\frac{3000}{60}\right)746}{550}.$$

Problem.—While a cut is being made on a 2-in. iron shaft in a lathe making 20 revolutions per minute, the power expended at the cutting tool is found to be 0.15 H.P. Find the force acting on the cutting tool.

Solution.—By definition (Art. 97),

$$\text{H.P.}=\frac{\text{ft.-lb. per sec.}}{550}$$

The work done per second equals the product of the force acting on the cutting tool and the distance traveled in one second by a point in the circumference of the shaft. Distance traveled in one revolution $=\frac{\pi}{6}$ ft.

$$\therefore\ \text{Speed}=\frac{20\pi\ \text{ft.}}{6\times 60\ \text{sec.}}=0.174\ \text{ft. per sec.}$$

Whence, (77),

$$0.15=\frac{0.174\ F}{550}.\quad \text{Therefore, } F=474\ \text{lb. wt.}$$

SOLVED PROBLEMS

Problem.—A fire-engine pump of 80% efficiency lifts 2400 lb. of water per min. through a vertical height of 15 ft., and discharges it through the horizontal nozzle with a speed of 20 ft. per sec. Find the horse-power of the engine required to operate the pump.

Solution.—The work done by the pump in one second equals the sum of the work required to raise 2400 lb. 15 ft., and that required to impart to it a horizontal velocity of 20 ft. per sec. Thus, the work done in one second by the pump is

$$\begin{aligned} W&=Fx+\tfrac{1}{2}Mv^2\\ &=\frac{2400\times 15}{60}+\frac{1}{2}\left(\frac{2400}{32.1}\right)(20)^2=15{,}550\ \text{ft. lb. per sec.}\end{aligned}$$

Since the efficiency of the pump is only 80%, the engine must supply power amounting to

$$\text{H.P.}\left[\frac{\text{ft.-lb. per sec.}}{550}\cdot\frac{100}{80}\right]=\frac{15550\times 100}{550\times 80}=35.3.$$

99. Kinetic Energy of a Body in Linear Motion.

—Kinetic energy has been defined as that energy which a system possesses because its parts are moving with respect to each other. The kinetic energy of any system could then be determined if all the parts were to be brought to rest with respect to each other, and the amount of work determined that the system could do against the forces that brought its parts to rest. As we have already considered the laws of uniformly accelerated motion, and since uniformly accelerated motion is produced by a constant force, a convenient way of determining the work that a moving body can do is to let a constant force act upon it until it comes to rest, and then find the amount of work that the body does during this time.

Suppose that in overcoming a constant force $-F$ the body is brought to rest in t seconds, and that during this time the body traversed a distance x with a uniform acceleration $-a$. The negative signs before F and a indicate that the directions of the force and the acceleration are opposite to that of the displacement. It follows that, when moving with the velocity v the kinetic energy of the body was

$$W[=-Fx]=-max.$$

On setting the final speed of the body v, equal to zero in (32) and (33) and then eliminating t between them, we obtain

$$-ax=\tfrac{1}{2}v.$$

On substituting this value of $-ax$ in the preceding equation we obtain

$$W[=-max]=\tfrac{1}{2}mv^2. \quad . \quad . \quad . \quad . \quad . \quad (81)$$

If m be measured in grams and v in centimeters per second, W will be expressed in ergs. If m be measured in British engineering units of mass and v in feet per second, W will be expressed in foot-pounds. But if m be measured in pounds and v in feet per second, W will not be expressed in foot-pounds, but in terms of a unit of work called the foot-poundal.

In driving a nail, the kinetic energy lost by the hammer in coming to rest is transferred to the nail, thereby setting the nail into motion. The considerable energy received by the nail is expended in doing work through a short distance, and therefore with great force. This force is sufficient to overcome the resistance to penetration offered by the wood. If the resistance to penetration is very great, the nail will advance a very short distance and hence with enormous force. If the nail is not very rigid, it will break or bend. A mallet drives nails less well then a hammer because the deformation of the head causes an absorption of energy which in the case of the hammer would be imparted to the nail.

In chipping iron or granite, a great force is required at the chisel edge for a very short time. In cutting wood and soft stone, a force is required that is of less magnitude but of longer duration. In the first case one would use a hammer of rather small mass that would rebound on striking the chisel. In the second case one would use a mallet of larger mass.

When the wheels of a moving vehicle strike an obstacle, an acceleration is given to the load and vehicle, and there is an absorption of kinetic energy. If the vehicle have springs, less acceleration is given to the load and there is less absorption of energy. This effect is more important in the case of two-wheeled vehicles than in the case of four-wheeled vehicles since when a wheel of a two-wheeled vehicle passes over an obstacle one-half of the load is lifted, whereas in the case of a four-wheeled vehicle only about one-quarter of the load is lifted.

The kinetic energy of a rapidly moving stream of water or steam is often used to eject ashes from a steamship or the ash pit of a power plant. In ejecting ashes from a steamship through an aperture in the hull under water, a stream of water at high velocity traverses the space x, Fig. 90, and passing through the throat y and the outward-opening valve z enters the sea. The impact of the stream of water on ashes dropped into x will carry them outside the ship.

In ejecting ashes from a power plant steam is used. If the ejector pipe contains bends, there is usually a steam jet at each bend as represented in Fig. 91.

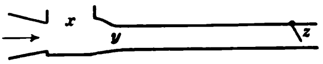

FIG. 90.

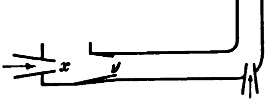

FIG. 91.

100. Kinetic Energy of a Rotating Body.—In the preceding Article an expression was obtained for the kinetic energy of a body moving with a motion of pure translation. The kinetic energy of a rigid body rotating about a fixed axis with angular velocity w will now be considered.

At any given instant any particle of mass m at a distance r

from the axis of rotation is moving with a certain linear speed v. The kinetic energy of this particle is $\frac{1}{2}mv^2$ units of work. But since $v=wr$, (31), the kinetic energy of the given particle equals

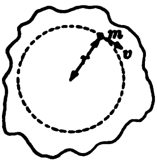

FIG. 92.

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}mw^2r^2=\tfrac{1}{2}w^2mr^2.$$

Now the energy of the whole body equals the sum of the energies of its constituent particles. Consequently the kinetic energy of the rotating body equals, (68),

$$W_{ro}[=\tfrac{1}{2}w^2\Sigma mr^2]=\tfrac{1}{2}Kw^2 \text{ units of work,} \quad . \quad . \quad (82)$$

where K represents the moment of inertia of the body with respect to the axis of rotation.

If K be measured in grams and centimeters, and w in radians per second, W_{ro} will be expressed in ergs. If K be measured in pounds and feet, and w in radians per second, W_{ro} will be expressed in foot-poundals. If K be measured in feet and British engineering units of mass, and w in radians per second, W_{ro} will be expressed in foot-pounds.

In case a rigid body has a motion both of translation and of rotation, its kinetic energy consists of two parts,—one given by (81) and another given by (82).

Projectiles from rifled guns have a motion of translation and also of rotation. Some such projectiles have hardened steel noses shaped like gimlet points. Due to the kinetic energy of translation, the shell will strike a hard blow. Due to the kinetic energy of rotation it will bore into the target with great destructive force.

101. Potential Energy.—The potential energy of a body is measured by the work it can do in going from its given position or condition to some standard position or condition. Consider the particular case where the potential energy of the body is due to the attraction between it and the earth. If a body of mass m be lifted a distance h, the work done upon it will be mgh. Consequently the work that it could do while descending a distance h is also mgh. That is, when a body of mass m is lifted a distance h, its potential energy is increased by an amount mgh. Since no work is required to move a body in a direction perpendicular to the resultant force that acts upon it, the potential energy of the body is not altered by moving the body horizontally. Since the potential energy of a body is the same at all points of any one horizontal surface, a horizontal surface is sometimes called a *gravitational equipotential surface.*

If a body be moved from any position A to some other position B, and the only force that opposes the motion is its weight, then the work done is the same no matter along what path the body passes from A to B. For, any path may be resolved into a series of steps, part of the components being vertical and the rest horizontal. Along the horizontal components no work is done, and the sum of the vertical components is the difference between the heights of the two given points. The above is a particular case of the following general proposition:

When a body is moved from one position to another along any frictionless path and against any set of forces which remain constant in direction and magnitude during the displacement, the work done is independent of the path.

In strictness, it is not correct to speak of the potential energy of a body, because the energy really belongs to whatever agent is under stress. But in order to avoid such circumlocutions as "the potential energy of the strained medium between the parts of the given system of bodies" custom has sanctioned such expressions as " the potential energy of the given body with respect to the earth." When the context shows what is taken to be the standard position from which potential energy is reckoned, it is common to speak simply of the " potential energy of the body."

If one keeps clearly in mind that potential energy always implies a system under stress, and does not refer to a single body, one will not be confused by such a paradox as the following: " When a body falls freely, gravity does work upon it, thus continually increasing its energy, yet at every point of its fall the sum of its kinetic and potential energies is the same as when it started to fall."

102. The Sum of the Kinetic and Potential Energies is Constant.—That greatest of all the generalizations of physics—the principle of the conservation of energy—has been enunciated by Maxwell in the following form: "The total energy of any material system can neither be increased nor diminished by any action between the parts of the system, though it may be transferred into any of the forms of which energy is susceptible." Since energy can be only kinetic and potential, it follows, as a direct deduction from the above principle, that, so long as no external forces act upon a system, the sum of its kinetic and potential energies must be a constant quantity.

Falling water decreases in potential energy and either gains in kinetic energy itself or imparts kinetic energy to something else. As the distance from the earth to the sun increases from midwinter to midsummer, the potential energy of the system increases and the kinetic energy decreases, causing the speed of the earth in its orbit to diminish.

SOLVED PROBLEMS

PROBLEM.—A sled with rider weighing 96.3 lb., reaches the foot of a hill 70 feet high with a speed of 40 feet per second. Find the amount of work done against friction.

SOLUTION.—The work done against friction equals the difference between the energy of the sled and rider when at the top of the hill and when at the bottom. When at rest at the top of the hill all of the energy is potential and is given by the equation $W_1 = mgh$. When at the bottom of the hill all of the energy is kinetic and is given by the equation $W_2 = \frac{1}{2}ms^2$. Consequently, the work done against friction is

$$W = W_1 - W_2 = mgh - \tfrac{1}{2}ms^2.$$

$$= \frac{96.3}{32.1}(32.1 \times 70) - \frac{1}{2}\left[\frac{96.3}{32.1}\right](1600)$$

$$= 4341 \text{ ft.-lb}$$

PROBLEM.—Find the horse power required to haul a car weighing 100,000 pounds weight at the rate of 30 mi. per hour up a 2% grade when the resistance of friction is 5 lb. wt. per 1000.

SOLUTION.—In Fig. 93 the line AB represents the weight of the car ($= 100{,}000$ lb. wt.); AE represents the component of this weight down the plane ($= AB \sin \phi$); AC represents the force normal to the roadbed

($=AB \cos \phi$); ED represents the force of friction ($=b \times AC$, where b is the coefficient of friction); and $AH(=AE+ED)$ represents the force that must be exerted up the plane in order to just overcome friction and the component of gravity down the plane.

Since a horse power equals 550 ft.-lb. per second, if the above force expressed in pounds weight be denoted by F and the velocity in feet per second be denoted by v, we have,

$$\text{Horse power} = \frac{Fv}{550}.$$

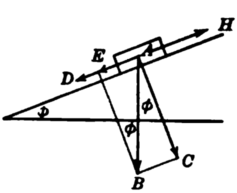

FIG. 93.

Again, since

$$AH = AB \sin \phi + b(AB \cos \phi)$$

$$F = 100000 \ (\sin 1^\circ 9' + 0.005 \cos 1^\circ 9') \text{ lb. wt.}$$

Therefore,

$$\text{Horse power} = \frac{100000 \ (\sin 1^\circ 9' + 0.005 \cos 1^\circ 9')}{550} \cdot \frac{30 \ (5280)}{3600} = 200.$$

PROBLEM.—The mass of a flywheel is 500 lb. and its radius of gyration is 2½ ft. Calculate what horse power applied for one minute will develop an angular speed of 180 revolutions per minute.

SOLUTION.—From (82) the kinetic energy of a body of moment of inertia K, rotating with an angular speed w, is

$$W_{ro} = \tfrac{1}{2} K w^2.$$

From (76) the moment of inertia of a body of mass M and radius of gyration k is

$$K = Mk^2.$$

An angular speed of 180 revolutions per minute equals 3 revolutions per second, or $3 \times 2\pi$ radians per second. Whence,

$$W_{ro}[=\tfrac{1}{2}Kw^2 = \tfrac{1}{2}Mk^2w^2] = \frac{1}{2}\left[\left(\frac{500}{32.1}\right)(2\tfrac{1}{2})^2\right](6\pi)^2 = 17280 \text{ ft. lb.}$$

By definition,

$$\text{H. P.} = \frac{\text{ft.-lb. per sec.}}{550}.$$

Therefore, in the present problem, since the power is applied for 1 min.,

$$\text{H. P.} = \frac{17280}{60 \times 550} = 0.52.$$

QUESTIONS

1. In order to haul a heavy train up a steep grade, engineers try to reach the foot of the grade with the greatest possible speed. Why?

2. A boy riding in a wagon is moving with the wagon and therefore has kinetic energy. He jumps out of the back of the wagon in such a way that he drops straight down to the ground without moving either forward or backward. What has become of his energy?

3. A pendulum swings in the arc of a circle. What kind of energy has it, (*a*) at the end of the path, (*b*) at the middle of the path, (*c*) at a point one-fourth of the distance from the end of its path? What is the measure of the amount of energy it possessed at the end of its path? At the middle point? How do these two values compare?

4. A particle of smoke when it comes out from the funnel of a steamer into still air is moving with the speed of the steamer, and therefore has kinetic energy with respect to the earth. It soon, however, takes up the motion of the air—that is, it loses its motion and consequently its kinetic energy. What has become of its energy?

5. If a bicycle rider takes his feet off the pedals at the bottom of a hill, he will ascend a certain distance against the force of gravity. Where does the energy come from?

6. By applying a constant force, a horse draws a carriage along a level road at a constant speed. (*a*) Does energy accumulate in the carriage? (*b*) What kind of energy does the carriage possess? (*c*) If the carriage were drawn up a hill, would energy accumulate in the carriage? (*d*) If the carriage is in uniform motion at the top of the hill, give the value of the energy it possesses.

7. Is there any difference in the amount of work required to bring a wagon wheel to rest, (*a*) when, after being raised off the ground it is spinning on the axle at a given angular speed, (*b*) when, after being removed from the wagon it is rolling along horizontal ground with the same angular speed?

8. A sphere rolls and a sled of equal mass slides side by side down an inclined plane. If the friction of the sled runners is negligible, show that at the bottom of the hill the linear speed of the sphere will be less than that of the sled.

THE C.G.S. ABSOLUTE UNITS AND THE F.P.S. GRAVITATIONAL UNITS COMPARED

QUANTITY	SYSTEM OF UNITS	
	C.G.S. absolute	F.P.S. gravitational or British engineering
Displacement		
Linear [x]	cm.*	ft.
Angular [Φ]	radian†	radian
Inertia		
Linear [M]	gram	32.1 pounds‡ or slug
Angular [$K=\Sigma(mr^2)$]	gram-cm.^{2}	slug-ft.^{2}
Time [t]	sec.	sec.
Speed		
Linear $\left[v=\frac{x}{t}\right]$	cm. per sec.	ft. per sec.
Angular $\left[w=\frac{\Phi}{t}\right]$	radian per sec.	radian per sec.§
Acceleration		
Linear $\left[a=\frac{v_t-v_0}{t}\right]$	cm. per sec. per sec.	ft. per sec. per sec.
Angular $\left[\alpha=\frac{w_t-w_0}{t}\right]$	radian per sec. per sec.	radian per sec. per sec.
Force [$F=ma$]	dyne ‖	pound wt.
Moment of Force [Fx]	dyne-cm.	pound-ft.
Torque [$L=\Sigma(Fx)$]	dyne-cm.	pound-ft.
Work [$W=Fx(=L\Phi)$]	erg ¶	ft.-pound
Power $\left[P=\frac{W}{t}(=Fv=Lw)\right]$	erg per sec. **	ft.-pound per sec. ††
Potential Energy [P.E. $=Fx(=L\Phi)$]	erg	ft.-pound
Kinetic Energy [K.E. $=\frac{1}{2}mv^2(=\frac{1}{2}Kw^2)$]	erg	ft.-pound

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[*] Other units of linear displacement in frequent use are the millimeter (*mm.*), the micron (μ), the millimicron ($m\mu$) or micromillimeter ($\mu\mu$), and the Ångström unit (.å). [1 mm. $=0.1$ cm.; $1\mu=0.001$ mm.; $1\ m\mu=1\ \mu\mu=0.001\mu=0.000001$ mm.; 1 å. $=10^{-10}$ meter $=0.1\mu\mu$. $=0.0001\mu$.]

[†] Other units of angular displacement are the mil, the degree, and the revolution. [1 rev. $=360°=6400$ mils $=2\pi$ radians.]

[‡] The numerical coefficient is not a constant quantity.

[§] 1 rev. per minute $=\frac{\pi}{30}$ radians per second.

[‖] 1 megadyne = 1,000,000 dynes. 1 metric ton = 1,000 Kg. wt.

[¶] 1 megalerg = 1,000,000 ergs. 1 joule = 10,000,000 ergs.

[**] 1 watt = 10,000,000 ergs per second.

[††] 550 ft.-lb. per sec. is called a horse-power.

CHAPTER VIII

FLUIDS

103. Pressure.—All bodies yield gradually to forces which tend to change their form. The property of a body by virtue of which time is required to change its form is called *viscosity*. A body possessing small viscosity is said to be *mobile*. The resistance which a body offers to a change of form is called *rigidity*. The property of changing form under continuous stress with the development of but small reaction is called *plasticity*.

A body possessing considerable rigidity is called a *solid*, while a body possessing little rigidity is called a *fluid*. A perfect solid is a body that does not remain distorted after the removal of a stress. *A perfect fluid* is a body that may be deformed by any force, however small. Fluids are divided into liquids and gases. A solid has a shape and a volume of its own. A liquid has a volume of its own, but takes up the shape of whatever contains it. A gas has neither volume nor shape of its own.

By the *pressure* P of a fluid at a given point is meant the force F exerted by the fluid per unit of area A in the specified region. That is,

$$P=\frac{F}{A}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (83)$$

Fluid pressure is often called hydrostatic pressure.

If F be measured in pounds weight and A in square inches, P will be expressed in pounds weight per square inch, the word per signifying that to get the number that expresses the pressure we divide the number of pounds weight by the number of square inches. When the pressure in a boiler is said to be 100 lb., it is really meant that the pressure in the boiler is 100 lb. wt. per square inch plus the pressure of the air outside the boiler. Other units for pressure are the pound weight per square foot, dyne per square centimeter, etc. Pressure is often expressed in centimeters of mercury, inches of water, etc., these expressions meaning the height of the column of mercury or water that the given pressure would support. Pressure is also expressed in atmospheres, one atmosphere being the pressure that the air about us usually exerts.

104. General Properties of Perfect Fluids at Rest.—A perfect fluid is a body that can be deformed by any force however small. From this definition can be deduced several general properties.

(*a*) *In case of a fluid at rest, the force it exerts on any surface in contact with it must be normal to the surface.*

Conversely, at any point, the surface of a fluid at rest must be normal to the resultant force acting at the assigned point. For example, in the case of a liquid, if its weight be the only force acting upon it, the free surface of the liquid will be horizontal.

(*b*) *At any point within a fluid at rest, the pressure is equal in all directions.*

(*c*) *If a fluid completely filling a vessel is free from the influence of gravity and other external forces, it will exert the same pressure at all points throughout the fluid and at all points against the walls of the vessel.*

The preceding law (*c*) governs the action of the hydrostatic press. This machine consists essentially of two cylinders of different diameters, closed by pistons, and connected by a tube. The cylinders and the connecting tube are filled with liquid. Let the areas of cross-section of the pistons be denoted by A_1 and A_2 respectively. If a force F_1 be applied to the piston having the area A_1, a force F_2 must be applied to the piston having the area A_2 in order to balance the force F_1. Since the pressure is transmitted equally throughout the liquid, the pressures on the two pistons are equal. That is, the force F_2 is given by the equation

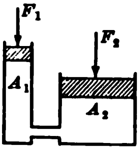

FIG. 94.

$$\frac{F_1}{A_1}=\frac{F_2}{A_2}.$$

By this device a small force applied to a small piston can produce a great force on a large piston.

In the special case of a liquid in a vessel open to the air, there is a uniform pressure on the free surface of the liquid due to the weight of the air. This produces a uniform pressure throughout the liquid and against the walls of the vessel, exactly as though the mouth of the vessel were closed with a piston acted upon by a pressure equal to that of the atmosphere.

105. Fluid Pressure Due to Weight.—In the case of a fluid at rest, the pressure at two points lying in the same horizontal plane will first be considered. Let A and B (Fig. 95), lying in the same horizontal plane, be the two points under consideration. Consider the forces acting upon a cylindrical element of the liquid extending from A to B. Suppose the ends of this element are normal to its axis. Acting on this element are, (a) the oppositely directed thrusts on the end faces; (b) the thrusts on the curved surface; (c) the weight of the fluid composing the element. The only forces acting in the direction of the axis of the element are the thrusts on the ends at A and B. Since the liquid is at rest, these forces must be equal and oppositely directed. Since the areas of the two end faces are equal, it follows that the fluid pressure at A equals that at B. Consequently, *in the case of a fluid at rest, the pressures are equal at all points in the same horizontal plane.* This is sometimes called the Law of Balancing Columns.

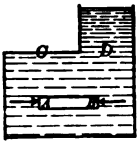

Fig. 95.

The pressure at any point in a fluid depends upon the depth of the point below the surface. The pressure of a fluid at any point will now be determined. If there is no force acting upon the fluid except that due to its own weight, the total force acting upon any horizontal surface of area A at a distance h below the upper surface of the fluid is equal to the weight of a column of the fluid of height h and area of cross-section A. If the mean density of the fluid be denoted by D, the mass of this volume is DAh, and its weight is $DgAh$. Whence, the pressure downward at the given point, due to the weight of the fluid is

$$P\left[=\frac{F}{A}\right]=\frac{DgAh}{A}=Dgh. \quad . \quad . \quad . \quad . \quad (84)$$

If, in addition to the weight of the fluid, there is acting on its surface a pressure of magnitude P_1, the pressure at any point in the fluid at a distance h below the upper surface is

$$P'=P+P_1=Dgh+P_1. \quad . \quad . \quad . \quad . \quad (85)$$

This pressure is independent of the shape of the containing vessel.

For instance, in Fig. 95 the pressure at A is as great as that at B although the height of the fluid column above A is less than that above B. The reason is that the horizontal surface of the vessel at C presses down upon the fluid immediately under it just as hard as the fluid at the same level near D presses down upon the fluid below it. If the horizontal surface of the vessel at C could not press downward with this force, the pressure of the fluid would break this part of the vessel.

A popular statement of the law that, in the case of a fluid at rest the pressures are equal at all points in the same horizontal plane, is that "fluids find their own level." This statement, however, should be used with caution. For example, consider a reservoir R, Fig. 96, with water level at A. Suppose that the water main after leaving the reservoir descends into a valley, thence passes over a hill, and finally empties at a point O at a lower level.

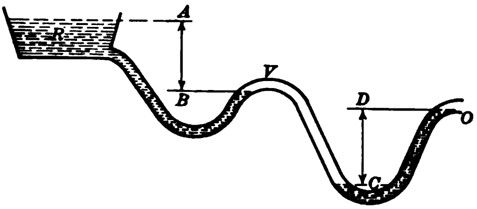

FIG. 96.

Since air is lighter than water, any air that may be carried along by the water will gradually collect on top of the hill. There may thus be formed in the bend of the pipe an "air block" which completely breaks the water column. Since the density of air is small compared with that of water, the pressure on the water surface at C will be only slightly greater than that at B. The pressure at D is less than the pressure at C by an amount which depends upon the height CD. Thus if AB is less than CD, the water may stop flowing, even though A is higher than D.

In practice a valve is placed at V which permits air to escape but prevents the escape of water. If the valve operates properly an air block is not produced.

It is sometimes observed that the gas jets in the upper part of a tall building burn more brightly than do those in the lower stories. Does this mean that the gas pressure is greater on the upper floors, and that it would be to the advantage of the consumer to have his meter placed in the attic instead of in the cellar?

Consider a tube XYZ filled with gas and open to the air at Z. The pressure, P, of the gas at the open end equals that of the atmosphere at the same level. Let the pressure of the gas in the tube at the level of X be denoted by P_1, and the pressure of the air at the same level by P_2. Denote the mean density of the gas from X to Z by D_g, and the mean density of the air between the same levels by D_a.

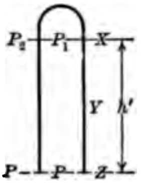

FIG. 97.

Then, from (85),

$$P_1 = P - D_g\, gh'$$

and

$$P_2 = P - D_a\, gh'.$$

Whence,

$$P_1 - P_2 = gh'(D_a - D_g).$$

Consequently, if the density of the gas is less than that of air, the pressure inside the tube is always greater than the pressure outside at the same level, and this difference of pressure will be greater for points at greater elevations than for points at lower elevations. That is, the pressure of the gas at any point in the tube diminishes as the elevation is increased, but the difference between the pressure of gas inside the tube and that of the air outside of the tube at the same level increases as the elevation is increased. Therefore, the gas escaping from an orifice at a considerable elevation has not greater pressure nor density than the gas which escapes from an orifice at a less elevation.

106. Atmospheric Pressure. The Barometer.—If a long tube AB, Fig. 98, closed at one end, be filled with liquid and then, without any air being admitted, the open end be submerged in a reservoir of the same liquid, then the column of liquid in the tube will be held in equilibrium under the action of the weight of the liquid in the tube and the pressure on the surface of the liquid in the reservoir due to the weight of the atmosphere. It was shown in the previous article that the pressure is the same at all points of any specified horizontal plane. The only pressure on the free surface of the liquid in the reservoir is that due to the weight of the atmosphere. The pressure at the same level inside the tube is Dgh, where D is the density of the liquid and h is the height of the free surface of the liquid in the tube above the free surface of the liquid in the reservoir, plus the pressure due to the vapor above the liquid column. Consequently, the pressure of the atmosphere is

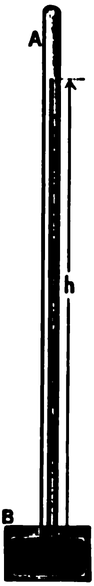

FIG. 98.

$$P = Dgh + P_1,$$

where P_1 is the very small pressure due to the vapor of the liquid above the column of liquid in the tube. This pressure, P_1, is so small that it is usually negligible compared with Dgh.

The apparatus above described was devised by Torricelli in 1644 and is called a *barometer*. The length h of the vertical column of liquid supported by the atmosphere is called the barometric height at the given place at the specified time. The barometric height is different for different places and at different times. At sea level, at latitude 45°, the mean barometric height is about 76 centimeters of mercury. That is, the atmosphere will support a column of mercury 76 cm. high, and the atmospheric pressure is said to be "76 cm. of mercury." The pressure may also be expressed by saying that it is "one atmosphere." It may also be expressed in terms of (84). Thus,

$$P = Dgh = 13.596 \times 980 \times 76 \text{ dynes per sq. cm.}$$

$$= 1.013 \times 10^6 \text{ dynes per sq. cm.}$$

$$= 14.7 \text{ pounds weight per sq. in.}$$

107. The Open Manometer.—By balancing the pressure of a gas against the pressure due to the weight of a column of liquid of known density, the pressure of the gas can be determined. Let an open glass tube AB (Fig. 99) containing some liquid (e.g., water or mercury) be bent as in the figure and joined by one end to the vessel R containing the gas whose pressure is sought. If the pressure of gas in the reservoir is different than the pressure of the air outside, there will be a difference in level of the liquid in the two arms of the tube.

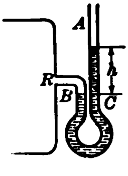

FIG. 99.

The pressure on the liquid surface at A is the atmospheric pressure P. The pressure in the liquid at C, which is a distance h below the surface at A, is by (84) greater than the pressure at A by an amount Dgh, where D denotes the density of the liquid and g the acceleration due to gravity. If the point C is chosen at the same level as the top of the liquid at B, then the pressure at B equals the pressure at C. But the pressure on the liquid surface at B is the desired pressure, P', of the gas in the reservoir. Whence,

$$P' = Dgh + P.$$

This instrument is called an open manometer.

108. The Common Pump.—The common lifting pump (Fig. 100) consists of a cylinder supplied with an upward opening valve V and a closely fitting piston supplied with one or more upward opening valves vv. While the piston is being raised, the valves vv close, the pressure of the air in the lower part of the cylinder is diminished below the atmospheric pressure, the valve V is forced open by the water below it, and the pressure of the atmosphere on the surface of the water into which dips the end of the supply pipe forces water up into the cylinder. If the piston be now lowered, the pressure of the water in A closes the valve V and then opens the valves vv, thus allowing the water in the cylinder to flow above the piston. On again raising the piston, the water above it flows through the outlet O, and more water is forced up into the low pressure space below the piston.

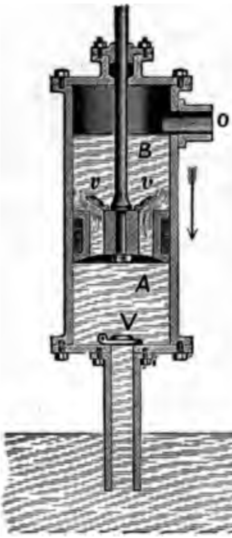

Fig. 100.

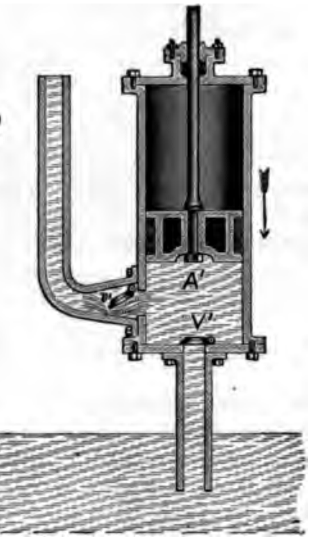

Fig. 101.

The maximum height h' to which a perfect lifting pump could raise a liquid of density D' is given by the condition that the atmospheric pressure is

$$P = D'gh'.$$

In the case of water, D' is 62.4 lb. per cubic foot. The atmospheric pressure which supports the column of water has a mean value,

$$P' = 14.7 \text{ lb. per sq. in.} = 14.7 \times 144 \text{ lb. per sq. ft.}$$

Consequently, from the equation $P = D'gh'$,

$$14.7 \times 144 = \frac{62.4}{32.1}(32.1)h'.$$

Whence, the maximum height to which a perfect lifting pump could raise water is

$$h' = \frac{14.7 \times 144}{62.4} \doteq 34 \text{ ft.}$$

The action of the common force pump will be made clear by a study of Fig. 101.

109. The Air Lift.—When the depth of water in a well, h_1, is greater than the height to which water is to be raised above the free surface, h_2, the air lift can be successfully employed.

In Fig. 102 is represented a well casing in which water would stand at the level AB. Within the casing is a pipe supplied with compressed air. Air rising from the outlet of this pipe forms bubbles having a diameter nearly that of the bore of the casing. In moving upward, the bubbles increase in size and push up the liquid above. The space above the air outlet becomes filled with alternate layers of water and air. Then there issues from the delivery pipe an intermittent flow of water.

At Sulphur, Louisiana, melted sulphur is raised about 500 ft. by means of an air lift represented diagrammatically in Fig. 103. Hot water is sent into the sulphur-bearing stratum through a pipe outside of the air lift. The sulphur melted by the hot water is then raised by air pressure as above described.

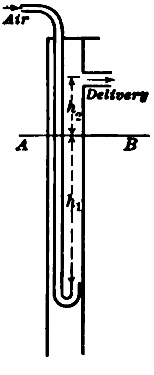

FIG. 102.

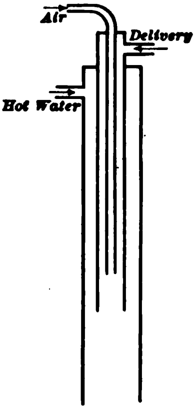

FIG. 103.

110. Archimedes' Principle.—Imagine any portion X, Fig. 104, of a fluid at rest to be separated from the remainder by an imaginary enveloping film. Since the entire mass of fluid is at rest, the portion X must be acted upon by a force whose magnitude is equal and whose direction is opposite to its weight. This force is due to the pressure of the fluid Y being greater on the lower parts of the portion X than upon the upper parts. If the fluid within the imaginary film be replaced by another body having the same shape and size, the pressure on the outside of the film will not be altered. Whence, *any body immersed in a fluid at rest is buoyed up by a force equal to the weight of the fluid displaced by the body.* This is called Archimedes' Principle.

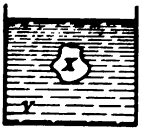

FIG. 104.

A body having a volume of 1 cubic yard is buoyed up by a force of 1684 lb. wt. when immersed in water, and about 2.2 lb. wt. when in air.

If W denotes the weight of a body in a vacuum, W' the weight of a fluid which it is displacing, and F the resultant force pulling down on the body, Archimedes' principle may be stated by the equation

$$F = W - W'. \qquad (86)$$

It follows from Archimedes' principle that if the weight of the fluid displaced is less than the weight of the body, the body is still pulled downward, but not so strongly as when not immersed in the fluid; that if the weight of the fluid displaced is equal to that of the body, the body is not pulled down at all; and that if the weight of the fluid displaced is greater than that of the body, the body is urged upward instead of downward.

For example, the envelope of a balloon distended with hydrogen, coal gas, or hot air will rise in the atmosphere. Since the sole object of the gas in the balloon is to displace a large volume of air, a gas should be selected that has the greatest possible volume per unit of mass.

If the weight of a body equals the weight of the fluid displaced, the body will be in equilibrium in the fluid. A body floating in a liquid is an example of this case. An iron ship floats, not because the materials entering into its construction have a smaller density than water, but for the reason that its hollow form causes it to displace a mass of water equal to its own mass. A ship of "5000 tons displacement" is one that displaces 5000 tons of water; that is, one that weighs 5000 tons.

111. Determination of Density by Immersion.—Whenever a solid body is of such simple geometrical form that its volume can be computed from its linear dimensions, its density can be obtained directly from the definition (Art. 78),

$$D = \frac{m}{V}.$$

Very often, however, a body is of such irregular shape that its volume cannot be computed from its linear dimensions. In such cases the principle of Archimedes furnishes a simple method of finding the volume.

The weight of the air displaced by most bodies is so slight that except in rather accurate work it may be neglected. This means that the force pulling downward on a body when it is in air is almost the same as if the body were in a vacuum. Suppose that a piece of copper is weighed and found to have a mass of 20 g. Suppose that it is then suspended by a very fine wire, immersed in a tumbler of water, and weighed while hanging in the water. On account of the buoyant effect of the water, the force with which the copper pulls downward on the wire is now less than the weight of the copper. Suppose that it now takes 17.75 g. to balance it. Then, by Archimedes' principle (86),

$$17.75=20.00-W'.$$

That is, the weight of the water displaced is $20.00-17.75=2.25$ g. Since the density of water is 1 g. per cc., 2.25 g. of water has the volume 2.25 cc. Since the copper was entirely immersed, the volume of water displaced equals the volume of the copper. That is, the volume of the copper is 2.25 cc. Its density is, therefore,

$$D\left[=\frac{m}{V}\right]=\frac{20.00}{2.25}=8.9 \text{ g. per cc.}$$

In the case of a body of such small density that it floats in the liquid, the body must be submerged by means of a sinker. The method of determining the density in this case is left as an exercise for the student.

112. The Hydrometer.—From Archimedes' principle, the mass of liquid displaced by a body floating in it is equal to the mass of the floating body. But the mass of liquid displaced equals the product of the density of the liquid and the volume of the immersed portion of the floating body. Consequently, if a body of given mass be allowed to float in a liquid, the density of the liquid can be determined from the volume of the submerged portion.

An instrument depending upon this principle is called an aerometer or hydrometer. The ordinary hydrometer (Fig. 105) consists of a graduated glass stem to which is attached a weighted glass bulb. By placing the instrument into two or more liquids of known densities, the scale on the stem can be calibrated to read specific gravities directly.

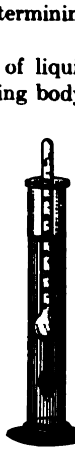

FIG. 105.

113. Pressure and Speed of a Flowing Fluid.—Consider a fluid flowing steadily through a tube of ununiform cross-section. Since the speed is greater where the cross-section is smaller, and since an acceleration implies a force in the same direction, in going toward a constriction the liquid is moving from a place of greater to a place of smaller pressure. Similarly, in going away from a constriction the liquid is moving from a place of less to a place of greater pressure. That is, other things being the same, the pressure is less where the speed is greater, and greater when the speed is less. The phenomenon that in a moving fluid the pressure is greater when the speed is small than when the speed is great is called the "Bernoulli effect."

The Pitot meter is an instrument in which the Bernoulli effect is utilized for the measurement of velocities of liquids and gases. One form used on aeroplanes for determining the velocity of the air relative to the ship is illustrated in Fig. 106. Air flowing through the tube with the constriction *A* produces a diminution of pressure on one side of the flexible diaphragm *B*, while air flowing into the tube *C* produces an increase of pressure on the other side of the same diaphragm. The displacement of this diaphragm is indicated by a pointer moving over a scale divided so as to read in miles per hour.

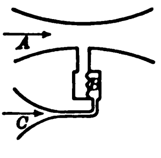

Fig. 106.

By giving a motion of rotation to a projected ball, a difference of air pressure on opposite sides will be developed, and the ball will be deflected out of the original course. With reference to a ball advancing in still air from left to right, without rotation, the air currents are as indicated in Fig. 107. The motion of the ball is opposed by a mass of air piled up on the advancing side. A ball rotating without translation will carry air around with it as indicated in Fig. 108. If the ball advances from left to right and also rotates as in Fig. 109, there will be at *A* an air current which is the resultant of two currents in the same direction, and at *B* an air current which is the resultant of two currents in opposite directions. It follows that at *A* the speed of the passing air is greater than at *B*. And since lower pressure goes with greater speed, the air pressure at *A* is less than at *B*. Thus the Bernoulli effect tends to deflect the nose *N* of the advancing ball in the direction of rotation. The amount of deflection depends upon the inertia of the ball, the magnitude of the angular speed compared with the linear speed and the position of the air

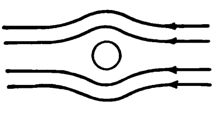

Fig. 107.

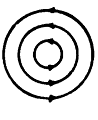

Fig. 108.

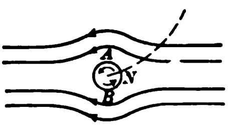

Fig. 109.

cap in front of the advancing ball. When the air cap is symmetrical with respect to the line of flight, and the angular speed is great compared with the linear speed, the deflection is small. But when the linear speed compared with the angular speed becomes lower than a certain value, the ball is in unstable kinetic equilibrium, the air cap becomes displaced toward the side of greater pressure, and the ball shoots out of its original path. A skillful baseball pitcher can so control the relation between the linear velocity to the angular velocity that the ball will not "break" till it is close to the batter, and it will then shoot upward, downward, to the right or to the left as he may desire.

There are many devices the action of which depends upon a combination of the energy of impact and the Bernoulli effect. In the case of the ordinary atomizer, air emerging from the end of the horizontal tube, Fig. 110, produces a sufficient diminution of pressure to cause liquid to rise to the top of the vertical tube. The current of air blows this liquid off in fine drops or spray. The increase in draft of a chimney when wind blows across the top is due to the same cause.

In the jet pump, Fig. 111, water at high speed passes through the nozzle *A* and escapes through the outlet *C*. At the throat *B* the flowing water produces a sufficient diminution of pressure to cause a liquid to rise in the tube *D* to a height of several feet. If it rises to the throat, it will be forced along the outlet pipe *C* by the impact of the high-speed stream. Some jet pumps have an efficiency of as much as seventy per cent.

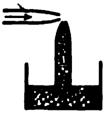

FIG. 110.

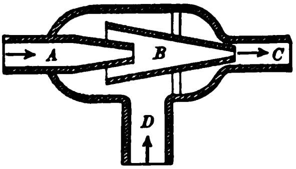

FIG. 111.

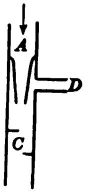

FIG. 112.

The aspirator, Fig. 112, is a jet pump for exhausting air or other gas. When pumping air, the stream in the outlet pipe consists of masses of water separated by masses of air. The impact of the high-speed water on these water pistons produces quickly an evacuation to one-half of an atmosphere pressure or less.

SOLVED PROBLEMS

PROBLEM.—A body placed on one pan of an equal arm balance is counterpoised by brass standard masses (often called "weights") amounting to 834.30 g. When immersed in water the body weighs 733.05 g. The densities of air and brass are 0.0013 and 8.5 g. per cc., respectively. What would the body weigh in vacuum?

Solution.—When the balance is in equilibrium, the total force acting on the body equals the total force acting on the standard masses. That is,

The weight of the body in vacuum − weight of air displaced by the body = weight of standard masses—weight of air displaced by the standard masses.

Consequently, the weight of the body in vacuum = weight of air displaced by body + weight of standard masses − weight of air displaced by the standard masses.

From Archimedes' principle, the volume of the body equals (834.30 − 733.05) = 101.25 cc. And since $D=\frac{m}{V}$ or $m=VD$, the weight of air displaced by the body equals

$$(101.25)(0.0013)=0.132 \text{ g.}$$

The volume of the standard masses

$$V\left[=\frac{m}{D}\right]=\frac{834.30}{8.5}=98.15 \text{ cc.}$$

Hence the weight of the air displaced by the standard masses equals (98.15)(0.0013) = 0.128 g. Therefore, the weight of the body in vacuum equals (0.132 + 834.30 − 0.128) = 834.304 g.

Problem.—Find the mass of lead of density 708 lb. per cu. ft. that is required to submerge in water a life preserver weighing 10 lb. made of cork of specific gravity 0.24. (The density of water is 62.4 lb. per cu. ft.)

Solution.—When in equilibrium, the sum of the forces acting upward on the cork and on the lead equals the sum of the forces acting downward. Or, representing the volume of cork and of lead by V_c and V_l respectively, the mass of lead by m_l, and the density of water by D_w, we may write,

$$V_cD_wg+V_lD_wg=10g+m_lg. \qquad \ldots \ldots \cdot (87)$$

Since $D=m/V$, the volume of the lead required to submerge the cork has the value

$$V_l\left[=\frac{m_l}{D_l}\right]=\frac{m_l}{708} \text{ cu. ft.}$$

Since the specific gravity of a substance equals the ratio of the density of the material to the density of water (Art. 78), the density of cork equals (0.24) (62.4) lb. per cu. ft. And since $D=m/V$, the volume of the cork

$$V_c\left[=\frac{m_c}{D}\right]=\frac{10}{(0.24)\ (62.4)} \text{ cu. ft.}$$

(87) may now be written

$$\frac{10(62.4)}{(0.24)\ (62.4)}+\frac{m_l(62.4)}{708}=10+m_l$$

Whence, the mass of lead required to submerge the cork life preserver is

$$m_l = 34.8 \text{ lb.}$$

PROBLEM.—How far would a stone of density 2.5 g. per cc., starting from rest, sink in sea water in two seconds? Take the density of sea water as 1.025 g. per cc., and neglect the retardation due to friction.

SOLUTION.—Denote the mass of the stone by m, the mass of water displaced by it by m', and the density of the stone and of sea water by D and D', respectively.

The resultant force acting downward on the stone has the value (86),

$$F = (mg - m'g) = g(VD - VD') = 980(2.5 - 1.025)V = 1446\ V \text{ dynes.}$$

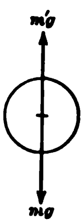

FIG. 113.

This force will impart to the stone a linear acceleration given by the equation $F = ma$.

Since $$D = \frac{m}{V}, \quad F[=ma] = DVa,$$

and since $$F = 1446V, \text{ and } F[=DVa] = 2.5Va,$$

$$1446V = 2.5Va.$$

Whence, $$a = 578 \text{ cm. per sec. in a sec.}$$

By eliminating v_t from (32) and (33), and remembering that since the stone starts from rest $v_0 = 0$, we find that the distance traveled in time t is

$$x = \tfrac{1}{2}\,at^2.$$

Whence $$x = \tfrac{1}{2}(578)2^2 = 1156 \text{ cm.}$$

QUESTIONS

1. Why does not the weight of the greater quantity of liquid in a filled coffee pot cause the liquid to rise higher in the spout than in the pot?
2. Show why the bubble in a spirit level goes to the highest part of the vial.
3. Sketch the form of a dish such that the total force due to the hydrostatic pressure on its bottom shall be (*a*) greater than, (*b*) equal to, (*c*) less than the weight of the contained liquid. Explain fully.
4. A tumbler may be filled with water, a piece of paper laid over it, and the tumbler inverted and held inverted without the paper dropping off or the water coming out. To what is this due? Is the force required to support the inverted glass containing the water greater than or equal to that required to support the same glass right side up and empty?, Why?

5. How would the height of the liquid column in a barometer be altered by each of the following operations? (*a*) doubling the length of the tube; (*b*) doubling the cross-section of the tube; (*c*) replacing the mercury by a liquid of density half as great; (*d*) carrying the barometer to a place where the pressure is the same, but the acceleration due to gravity is smaller.

6. Explain the action of the open manometer. Would the same graduation of the tube hold for different points on the earth's surface and for different altitudes?

7. It is desired to raise water a distance of 50 feet. Will the following arrangements give any difference in the force to be exerted in each stroke? (*a*) A lift pump, the piston being 25 ft. above the water; (*b*) a force pump, the piston being 25 ft. above the water; (*c*) a force pump, the piston being 1 ft. above the water.

8. Explain the action of the fountain pen filler.

9. Why will not liquid flow out at the faucet of a barrel unless there is another hole in the upper part of the barrel? Can a flexible rubber bag be emptied of water if the bag has a single small hole?

10. Two non-miscible liquids are poured, one into each branch of an upright U tube, so that their place of contact is at the bottom of the tube. What will be the relative height of each liquid? What will be the effect of doubling the cross-section of one of the tubes?

11. A common form of drinking fountain for poultry is made by inverting a jar filled with water and submerging the neck of the jar in a saucer of water. As water is removed from the saucer, water from the jar will take its place thereby maintaining nearly constant the amount of water in the saucer. Explain.

12. Which would require the stronger dam, a mill pond of large area and shallow, or one of small area and deep? Explain.

13. You are provided with a glass U tube, a rubber connecting tube, and a glass of water, and are asked to measure the pressure of the illuminating gas at a given gas burner. How would you measure it? Give an expression showing the pressure of gas. What effect will the size of the tube have on your measurement?

14. Define density, specific gravity. Under what conditions will these have numerically the same value? How can Archimedes' principle be applied to the determination of these quantities?

15. A piece of cork counterbalances a piece of lead on an equal arm balance. Have they the same mass? Explain the answer fully.

16. State the condition under which a balloon will—(*a*) rise; (*b*) fall; (*c*) remain at a given elevation.

17. Not enough gas is put into the envelope of a balloon to distend it fully when it is at the surface of the earth. When it rises to points where the air pressure is less, the inclosed gas farther distends the envelope. Assuming the pressure inside the envelope always equal to the air pressure outside, neglecting volume of the car and aeronauts, and assuming temperature at all elevations the same, determine whether the buoyant effect of air on the balloon is greater at great elevations or at smaller. Give reasons.

18. State the law governing the following cases and show what conditions are present in each: a balloon rises in air; a kite rises in air; a bullet sinks in water; a cork floats on water.

19. A vessel partly filled with water is balanced on one pan of a pair of scales. A piece of wood is placed in the water. What is the effect of the introduction of the piece of wood on the pressure on the bottom of the vessel, and also upon the weight on the scale pan? Would the pressure on the bottom of the vessel or the weight on the scale pan be altered by completely submerging the block of wood by fastening it to the bottom of the vessel? If either be different, state the cause and the factors upon which it depends.

20. Show the fallacy of the following. A hollow flexible tube is used as a belt on two pulleys, Fig. 114. On this tube are chambers which communicate with the tube. Iron cylinders slide in and out of these chambers as they are turned up or reversed. For example, in the diagram, cylinder *A* has slid out of *B*, while *C* has slid into *D*.

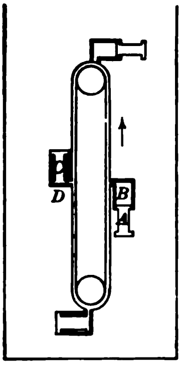

FIG. 114.

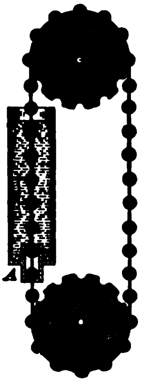

FIG. 115.

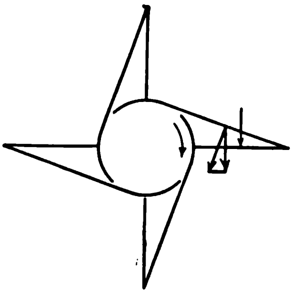

FIG. 116.

Now according to Archimedes' principle, the buoyant effect on *A* and *B* is greater than that on *C* and *D*, since more water is displaced. Consequently, *C* and *D* exert a greater resultant force downward than *A* and *B*. This force will urge the belt around the pulleys. If a sufficient number of these cylinders and chambers are present, frictional force can be overcome and work done by the machine.

21. An endless chain of light spheres is mounted on two wheels as shown in Fig. 115. At *A* is an arrangement whereby the spheres may pass from air into a liquid without any of the liquid escaping.

Since the spheres on the left side are immersed in liquid, the left side of the chain is buoyed up by a force of a magnitude depending upon the number of spheres immersed. As this buoyant force may be made sufficiently great to exceed friction why could not continuous motion be produced?

22. On a shaft is mounted a drum carrying several compartments, each having one side radial and one side tangential to the edge of the drum, as shown in Fig. 116. The device is filled with air under pressure.

The air pressures on the radial and on the tangential sides will be equal. But the components tangential to the edge of the drum will be unequal. Why will not continuous rotation result?

23. The following mechanism has been proposed as a perpetual-motion machine. A wheel with bucket-shaped pockets on its circumference is capable of rotation about a horizontal axis. Near the wheel is a tank of water somewhat taller than the wheel. Spheres of a material of less density than water are dropped into the pockets on one side of the wheel and the wheel is thereby set into rotation. On reaching the lower part of the wheel the spheres are dumped onto a chute and enter the lower part of the water tank through a suitable trap. Being lighter than water, the spheres rise to the top of the tank. They then slide onto another chute and into the buckets on the wheel and repeat the cycle.

Assuming that effective devices exist for allowing the spheres to enter the tank without spilling water, and for transferring the spheres from the top of the water to the buckets on the wheel, show why the machine would fail.

CHAPTER IX

PROPERTIES OF MATTER

114. Empirical Laws.—In the present state of our knowledge, the laws of such properties of matter as friction, elasticity, surface tension, and solution appear to be generalizations of isolated groups of phenomena which cannot be deduced from the fundamental principles of dynamics, but which can be derived only from the experimental study of matter under the special conditions imposed by the nature of the investigation. Generalizations based upon relations obtained solely from experiment or observation are called *empirical laws.* In the present chapter several such laws will be studied.

§ 1. *Elasticity*

115. The Deformation of a Stretched Rod.—Suppose that a soft steel rod is subjected to a stretching force which can be increased at a uniform rate. If two marks are placed on the rod at a and b, Fig. 117, the distance x between them will increase as the stretching force increases. Let some accurate means be provided for measuring this increase in length. It is found that for a time the increment of length is directly proportional to the increment of the force. But when a certain force has been reached, the ratio between the change of length to the change of force is much greater than before. If the applied force continues to increase, the specimen continues to stretch at a gradually increasing rate, until at a certain maximum force the cross-section at

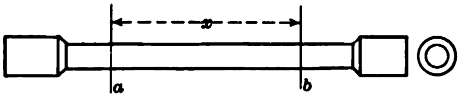

Fig. 117.

some point of the specimen suddenly begins to reduce and a so-called "neck" begins to form. Thereafter, the length of the specimen increases rapidly with the application of small force, till the specimen is broken.

When rods of the same material but of different lengths and cross-sectional areas are tested, it is found that identical results are obtained if the loads be reduced to force per unit area of cross-section, and elongations be reduced to elongation per unit length. The force per unit area of cross-section is called the *unit stress* and the elongation per unit length is called the *unit strain.*

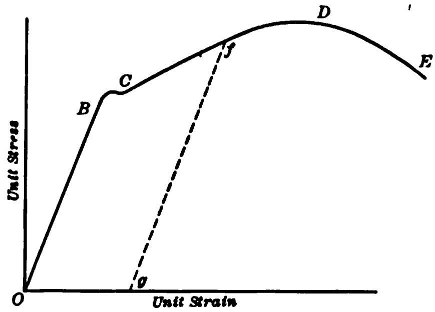

FIG. 118.

The results of tests such as above described are best shown by plotting a stress-strain diagram, Fig. 118.

As the load is gradually increased the curve coordinating unit stress and unit strain is at first a straight line OB. The unit stress at B, where the curve begins to bend, is called the *elastic limit* of the material. The unit stress at the point C at which the material begins to flow, i.e., to yield continuously without any increase in stress, is called the *yield point.* Above C the stress increases to the maximum value D. From D to E occurs the formation of the neck in ductile materials. Rupture takes place at E.

If the load be gradually diminished from a condition represented by a point below the elastic limit, the curve OB will be described in the reverse direction, and the specimen will return to the original length. If, however, the removal of the load is not begun until some point f beyond B has been reached, the relation between unit stress and unit strain will be represented by the line fg nearly parallel to BO. When the load is completely removed there remains an increase in length Og called the *permanent set.*

116. Hooke's Law.—*Elasticity* is the name given to "that property by virtue of which a body requires force to change its bulk or shape, and requires a continued application of the force to maintain the change, and springs back when the force is removed; and if left without the force, does not remain at rest except in its previous bulk and shape." (Kelvin.)

A body is highly elastic which not only offers a great resistance to distortion but which also completely recovers its size and shape on the removal of the deforming force, e.g., steel, glass. A body is slightly elastic which is either deformed by a small force or which can sustain but a small deforming force without permanent deformation, e.g., rubber, clay. When a body is *perfectly elastic,* a deforming force will develop in it an equal and opposite force of restitution which will not diminish with the lapse of time. A body which does not completely recover its original shape or size on the removal of the deforming force is said to be *plastic.* A body that can be deformed through wide limits without being permanently distorted, is said to be *tough.* A body that can be distorted to but a very small amount without breaking is said to be *brittle.* For example, rubber is very tough, though it is not highly elastic. Glass, on the other hand, is slightly tough but is very elastic; that is, glass cannot be distorted through a great range, but a distortion of glass of unit amount develops a great force of restitution.

For a perfectly elastic body, that is, one not distorted beyond the elastic limit, the stress due to a distortion tends to restore the body to the original condition and has a magnitude proportional to the strain. This is called Hooke's Law.

The ratio of unit stress to unit strain of a body not distorted beyond the elastic limit is called *a coefficient of elasticity.* As there are different types of strain, there are different coefficients of elasticity.

117. Young's Modulus.—For a rod under tension or compression the unit tensile stress equals the force of restitution divided by the area of cross-section perpendicular to the force; and the unit tensile strain equals the change of length divided by the original length. Consequently, the tensile coefficient of elasticity, stretch modulus, or Young's modulus of elasticity,

$$E=\frac{\dfrac{\text{force of restitution}}{\text{area of cross-section}}}{\dfrac{\text{elongation}}{\text{original length}}}=\frac{\dfrac{F}{A}}{\dfrac{\Delta l}{l}}=\frac{Fl}{A\Delta l}. \quad . \quad . \quad . \qquad (88)$$

If a beam be bent, the layers on the convex side will be stretched and those on the concave side will be compressed. There is one layer, called the "neutral surface" which is neither stretched nor compressed. Young's modulus of a material can also be determined from the amount of flexure produced in a beam of known dimensions when subjected to a given force perpendicular to its length.

For steel, Young's modulus is about $22(10^{11})$ dynes per sq. cm. [$30\ (10^6)$ pounds per sq. in.]. For cast iron it is about $11.5\ (10^{11})$ dynes per sq. cm. [$16.8\ (10^6)$ pounds per sq. in.]

Solved Problem

Problem.—A rod of mild steel ¾ inch in diameter was tested in tension. When the elastic limit was reached, the load was 14,550 lb. wt. The corresponding elongation in a length of 8 in. was 0.0088 in. The maximum load was 27,400 lb. wt. Compute the elastic limit; the Young's modulus; the maximum or breaking unit stress.

Solution.—

$$\text{Area of cross-section} =\frac{\pi}{4}\left(\frac{3}{4}\right)^2=0.44 \text{ sq. in.}$$

$$\text{Elastic limit} =\frac{14550}{0.44}=33000 \text{ lb. per sq. in.}$$

$$\text{Unit strain} =\frac{0.0088}{8}=0.0011 \text{ in.}$$

$$\text{Young's modulus} =\frac{33000}{0.0011}=30(10^6) \text{ lb. per sq. in.}$$

$$\text{Maximum unit stress}=\frac{27400}{0.44}=62200 \text{ lb. per sq. in.}$$

118. Bulk Modulus.—When a body is pressed on all sides without its shape being changed, the force per unit area with which it opposes its decrease in volume is called the *bulk restoring stress.* The ratio of its decrease in volume to its original volume is called the *bulk strain* in the material. As long as the same material is used and the elastic limit is not passed, the ratio of the stress to the bulk strain is constant. This ratio is called the coefficient of volume elasticity, or the *bulk modulus,* of the given material. The bulk modulus of a given material is numerically equal to the change of pressure required to reduce the volume to one-half.

The bulk modulus of water is about 155 tons wt. per square inch. Hence, to compress a cubic foot of water so that each dimension of the cube would be shortened 0.01 in. would require a pressure of about 775 lb. wt. per square inch.

The elastic limit for liquids and gases has never been reached.

119. Simple Rigidity.—If a rectangular parallelopiped of rubber, *ac*, Fig. 119, has two opposite faces glued to two boards, and if one of these boards is pushed sidewise in its own plane, there is no change in the volume of the block, but its shape is changed to *fgcd.* In this case the strain is the ratio of *af* to *ad*, and is called a *shear*, or a shearing strain. If *F* is the force applied, and *A* is the area of the face *ab*, then *F* divided by *A* is called a *shearing stress.* If the block of rubber is very thin in the direction normal to the paper, and if it is bent around until *ad* coincides with *bc*, it is seen that a shear is the kind of strain involved in the twisting of a wire about its geometric axis. As long as the elastic limit is not reached, the ratio of the shearing stress to the shearing strain that produces it is a constant quantity. This ratio is called the *simple rigidity, shearing modulus* or *slide modulus,* of the material sheared.

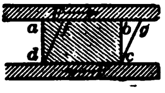

Fig. 119.

In punching a round hole of diameter d in a plate of thickness t, the area subjected to shear is πtd. If the mean force required to punch the hole be F, the mean shearing stress equals $F/\pi td$.

120. Viscosity.—In the distortion of any body there is usually a sliding of some portions of matter with respect to the other portions composing the body. This may develop an internal resistance which retards the relative motion of the parts of the body. That property of bodies by virtue of which they resist an instantaneous distortion is called *viscosity*. A body that has little viscosity is said to be *mobile*. Thus pitch, tar, and molasses are viscous fluids, while water, chloroform, and ether are mobile. The ratio of shearing stress to shearing strain produced in unit time is called the *coefficient of viscosity* of the substance. An increase in temperature causes an increase in the coefficient of vicosity of a gas and a decrease in that of a liquid.

A well-lubricated journal does not come into contact with its bearing, but is kept separated from it by a film of the lubricant. In this way the friction between two metallic rubbing surfaces is replaced by the smaller viscous resistance offered by the lubricating oil. The resistance to the motion of a journal in a lubricated bearing depends upon the coefficient of viscosity of the oil used, the thickness and area of the lubricating film, and the relative speed of the two surfaces. For a lightly loaded bearing an oil should be used whose coefficient of viscosity is small. Such an oil, however, could not be used on a heavily loaded bearing because it would be squeezed out from between the moving surfaces, thus causing the journal to run dry. For a rough bearing or journal it would also be necessary to use an oil having a high coefficient of viscosity. Otherwise the rugosities of one of the surfaces would protrude through the lubricating film and come into direct contact with the other surface. Since, in general, the coefficient of viscosity of oils greatly diminishes as the temperature is raised, oils intended for use in steam cylinders must have a high coefficient of viscosity at ordinary temperature. For any particular service, a lubricating oil should be used that has the smallest coefficient of viscosity consistent with the requirement that it shall keep an unbroken film between the journal and bearing.

SOLVED PROBLEM

PROBLEM.—The flywheel of a punch press has a moment of inertia of 1500 lb. ft., and when running freely, has a speed of 60 revolutions per minute. The punch makes a round hole ¾ inch in diameter in an iron plate ½ inch thick. The shearing strength of the iron is 40,000 lb. per square inch of area of sheared surface. Assuming that when going through the iron, the average force acting on the punch is one-half the maximum force, find (*a*) the work done; (*b*) the speed of the flywheel when the punch is emerging from the plate.

SOLUTION.—The maximum force $=\pi\frac{3}{4}\cdot\frac{1}{2}\cdot 40{,}000=15{,}000\pi$ lb. and the average force $=7500\pi$ lb.

Therefore, the work done in punching one hole $=\dfrac{7500\pi}{2\times 12}=312.5\pi$ ft. lb.

Representing the moment of inertia of the flywheel by K, the angular speed of the flywheel before the punch enters the plate by w_0, and the angular speed when the punch emerges from the plate by w, the loss of energy of the flywheel due to punching the hole has the value, (82), $\frac{1}{2}Kw_0^2 - \frac{1}{2}Kw^2$.

Since the work done in punching the hole equals the energy lost by the flywheel,

$$\tfrac{1}{2}Kw_0^2 - \tfrac{1}{2}Kw^2 = 312.5\pi \text{ ft. lb.}$$

Now $w_0 = 60$ revolutions per minute $= 2\pi$ radians per second (Art. 52), and $K = 1500$ lb. ft. Consequently,

$$\tfrac{1}{2}\, 1500(4\pi^2 - w^2) = 312.5\pi,$$

$$w^2 = 4\pi^2 - \frac{312.5\pi}{750} = 4\pi^2 - \frac{5\pi}{12},$$

$$w = \pi\sqrt{4 - \frac{5}{12\pi}} \text{ radians per second}$$

$$= \pi\left(\frac{60}{2\pi}\right)\sqrt{4 - \frac{5}{12\pi}} \text{ revolutions per minute.}$$

Questions

1. A wire of given diameter and length is to be used to support a varying weight, and it is desirous to have the wire remain as nearly as possible of the same length. The Young's modulus of steel is greater than that of brass. Would steel or brass be a better material to use? Explain fully.

2. With which coefficient of elasticity are we concerned in each of the following cases? Give reasons. (*a*) The stretching of a spiral spring; (*b*) the stretching of a rubber band; (*c*) the compression of a pillar by the load which it bears; (*d*) the change in size of a bubble as it rises in water; (*e*) the bending of a plank that rests on two supports when you walk along it.

3. Upon what physical property of matter does the measurement of force by means of a spring balance depend? State what physical change takes place in the spring when the balance is overloaded. Define all physical terms used in your answer.

4. What modulus is represented in each of the following cases? (*a*) A wire is stretched; (*b*) air is compressed by means of an air pump; (*c*) a rod is twisted.

Give in (*a*) and (*b*) an expression for the modulus in terms of the force applied and the dimensions of the object distorted.

§ 2. *Universal Gravitation*

121. Newton's Law of Gravitation.—From an analysis of the laws of motion of the planets about the sun formulated by Kepler, Newton reached the conclusion that between the sun and each of its planets there is an attractive force which is inversely proportional to the square of the distance between their centers of mass. With the insight of genius, Newton at once imagined that the same law that applies to these celestial bodies applies to all bodies, whether celestial or terrestrial. After a long series of investigations, he finally enunciated the law that, (1) *every portion of matter attracts every other portion of matter; and the gravitational force between any two bodies is proportional,* (2) *to the product of their masses,* (3) *is inversely proportional to the square of the distance between their centers of mass,* (4) *is independent of the kind of matter, and* (5) *is independent of the intervening medium.* This very far-reaching generalization is called Newton's Law of Universal Gravitation.

A force existing between two bodies that tends to move them apart is termed positive, while one that tends to bring them together is termed negative. Thus, the gravitational force existing between two bodies of masses m_1 and m_2, whose centers of mass are separated by the distance d, is

$$F = -G\frac{m_1 m_2}{d_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (89)$$

where G is a positive constant.

122. The Gravitation Constant.—The simplest method of measuring the attraction between two small bodies and of determining the gravitation constant is by means of the torsion balance. This consists of two equal stationary masses, m_1 and m'_1 (Fig. 120), together with a suspended system including a very delicate vertical fiber supporting on its lower end a thin uniform horizontal rod carrying on its two ends equal masses m_2 and m'_2. The four bodies are so arranged that their centers of mass lie in the same horizontal plane, the distance between the centers of mass of the stationary bodies equals that between the suspended bodies, and the axis of rotation of the suspended system passes through the point midway between the centers of mass of the stationary bodies. m_1 attracts m_2, and m'_1 attracts m'_2. These two forces produce

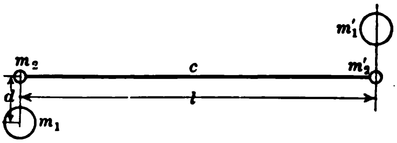

Fig. 120.

a couple that tends to rotate the suspended system. The rotation is opposed by twisting the wire that supports m_2 and m'_2. The wire is twisted until m_2 and m'_2 when supported by the twisted wire are in the same position that they take when the wire is not twisted and m_1 and m'_1 are not near them. From the angle through which the supporting wire has to be twisted, together with the known masses and dimensions of the apparatus, the gravitation constant can be calculated.

This method was first used by Cavendish. Later it was considerably refined by Boys, who found the gravitation constant to be

$$G=6.6576(10^{-8})\text{*C.G.S. units.}$$

This means that if two lead spheres each 2 ft. in diameter were placed so that they lacked 0.1 in. of touching each other, the force with which they would attract each other would be equal to the weight of about $\frac{1}{30}$ of a gram, i.e., to about $\frac{1}{150}$ part of the weight of an American nickel five-cent piece.

Knowing the value of the gravitation constant, together with the radius of the earth and the acceleration due to gravity, the mass and mean density of the earth can be computed. Thus, if the mass of the earth is represented by m_2, the magnitude of the gravitational force between the earth and a mass of one gram at its surface is

$$F=-1g=-G\frac{1m_2}{r^2}.$$

Whence,

$$m_2=\frac{gr^2}{G}.$$

At the poles the radius of the earth is about 6357 km., and the acceleration due to gravity is about 983 cm. per second in one second. Using these data and assuming that the earth is a sphere, we find the mass of the earth to be about 6 (10^{27}) g., and its mean density to be about 5.5 g. per cc.

Questions

1. As the earth travels around its elliptical orbit its velocity alternately increases and decreases. Does this mean that the total energy of the earth alternately increases and decreases? Explain.

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* This may be read either 6.6575 times ten to the negative eighth power, *or*, 6.6576 eighthets.

2. Would the following changes increase or decrease weight? (*a*) Doubling the mass of the earth; (*b*) doubling the size of the earth without changing the mass; (*c*) doubling the density without changing the mass.

3. Explain why a given body does not weigh as much in a deep mine as on the surface of the earth at the same latitude.

§ 3. *Properties of Gases*

123. Boyle's Law.—In 1661 it was announced by Robert Boyle that, to a high degree of approximation, the volume of a given mass of gas varies with the pressure, so long as its temperature remains unchanged. Thus, for a given mass of gas at constant temperature,

$$V \propto \frac{1}{P}.$$

Now, for a gas at given pressure and temperature,

$$V \propto m.$$

Therefore, for a gas at constant temperature,

$$V \propto \frac{m}{P}.$$

Whence, if the constant of proportionality be denoted by k,

$$V = \frac{mk}{P}.$$

or

$$PV = mk, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

where k is a constant for the particular kind of gas at a definite temperature. Thus, if at the pressures P_1, P_2, P_3, etc., the temperature remaining constant, a given mass of gas has the respective volumes V_1, V_2, V_3, etc., then

$$P_1 V_1 = P_2 V_2 = P_3 V_3 = \text{etc.} = \text{a constant quantity.} \quad . \quad . \quad (91)$$

If the density of a gas be denoted by D, then, from (90),

$$P = kD. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

The result expressed by (90), (91), or (92) is called Boyle's Law. These equations are approximately true for most gases throughout a considerable range of pressure. Boyle's Law is usually enunciated in the form of (91), or, in words, *the product of the pressure and the volume of a gas kept at constant temperature is a constant quantity.* *A perfect* or *ideal* gas is an hypothetical substance that would obey Boyle's Law for all pressures.

124. The Closed Manometer.—Boyle's Law furnishes a simple method of determining the pressure of a fluid whose pressure is too great to be conveniently measured by means of an open manometer (Art. 107). The simplest form of closed manometer consists of a glass tube XY (Fig. 121) closed at one end and connected at the other to the reservoir R, the pressure inside of which is desired. In the tube is a globule of mercury C which separates the air or other gas in the farther end of the tube from the fluid in the reservoir. Suppose that when the manometer is connected to the reservoir the mercury index stands in the position indicated, and that when the manometer is disconnected from the reservoir and the end X is subject to the atmospheric pressure, the index stands at C'. If the tube is of uniform cross-section A, the volume of the air beyond the index is Al in the first case and in the second Al'. If the temperature of this air is the same in both cases, it follows from (91) that

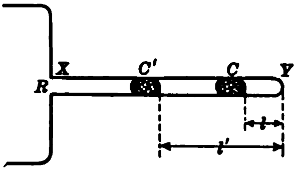

Fig. 121.

$$PAl = P'Al',$$

where P denotes the pressure to be measured and P' the pressure of the air in the room. Hence,

$$P = \frac{l'}{l} P'. \qquad (93)$$

So that if the pressure P' of the air in the room is known, P can be calculated.

Kelvin's sounding machine is an instrument in common use by ships' captains for making soundings as a vessel approaches land. It consists of a narrow, straight tube of uniform bore closed at one end, coated inside with a substance that changes color when wet with salt water. The bore of the tube is so small that water will not enter without pressure. On lowering this tube into the sea, water is forced into the tube by the pressure of the water above. By measuring the length of the tube, and the distance from the end of the coloration to the closed end of the tube, the pressure of the water at the bottom of the sea can be obtained by means of Boyle's Law. Knowing this pressure, and the density of sea water, the depth can be computed. In practice these tubes are calibrated so as to indicate depths directly without computation.

125. Dalton's Law.—If a number of different gases which do not react upon one another are placed in the same reservoir, a homogeneous mixture is quickly formed. It has been shown from measurements made by Dalton that the pressure at any point in the mixture is equal to the sum of the pressures which each of the gases would separately exert in the given space. This fact is known as Dalton's Law, and this law suggests that a gas is composed of discrete particles which are in rapid motion and whose size is small compared to their distance apart.

§ 4. *Molecular Properties of Matter*

126. The Constitution of Matter.—Many phenomena suggest that all forms of matter are built of minute bodies which are in violent motion. Since there are no known facts that are in opposition to the assumption that gases, liquids, and solids are thus constituted, and as the conception of such a mechanism serves a valuable purpose by fixing the thoughts upon an easily apprehended model which obeys the same laws as do the many abstruse phenomena of matter, it is convenient to assume that matter is composed of minute bodies which are in rapid motion. This is one of the fundamental hypotheses of the kinetic theory of matter.

The smallest particle of a body that can possess the properties of the substance composing the body is called a *molecule.* The motion of molecules is subject to the attraction of neighboring molecules. When two molecules collide, each will rebound.

In a gas the molecules are separated by distances that are so great compared with their diameters, that a molecule is but slightly influenced by the attraction of neighboring molecules, and will travel a considerable distance before striking another molecule. This nearly free wandering of the molecules explains why a gas is able to expand to any extent and fill any vessel inclosing it. Whenever the expansion is resisted by inclosing a gas, the impact of molecules against the sides of the vessel causes a pressure. It is probable that the diameter of a hydrogen molecule is about $6(10)^{-8}$ cm., its mass is about $5\ (10)^{-24}$ g., and when under atmospheric pressure, its mean free path is about 200 times its diameter.

In a liquid the molecules are much closer together than they are in a gas. The mean distance between the centers of adjacent molecules in a liquid is not greater than twice their diameter, and the mean free path is less than half the diameter of a molecule. The short distances between molecules in a liquid cause greater forces to be exerted between molecules of a liquid than between the molecules of a gas. In the interior of a liquid the molecular forces acting upon a molecule are sensibly in equilibrium. A molecule at the surface, however, is pulled inward with an enormous force. It is estimated that this internal molecular force is of the order of magnitude of 900 atmospheres pressure. This force resists the escape of molecules from the liquid.

In solids the molecules are packed even more closely than in liquids, their mean free path is shorter, and the force between adjacent molecules is greater.

127. Adhesion and Cohesion.—If a horizontal glass plate be brought into contact with the free surface of a vessel of water, a considerable force will be required to lift the plate from the liquid surface. After the plate has been withdrawn, its surface will be found to be covered with a film of water. This shows that there is an attractive force between the molecules of liquid, an attractive force between the liquid and the glass, and that the latter force is greater than the former. The force required to separate two bodies of the same material, per unit area of contact, is called the *cohesion* of the given material. The force required to separate two bodies of different material, per unit area of contact, is called the *adhesion* of one material to the other.

Two plane polished glass plates pressed firmly together will cohere so strongly that it is impossible to separate them without rupture. Glue and cements adhere so strongly to some substances that the substances will break rather than the joint between the glue and the substance. Gases adhere to solids very strongly. To free glass from the adhering air film requires heating to a high temperature.

Filtration is an example of adhesion. A filter is a porous substance which arrests solid bodies suspended in a fluid. That a filter is not simply a sieve is shown by the fact that it will arrest bodies much smaller than the interstices of the filter. A piece of unglazed porcelain will completely prevent the passage of bodies one-thirtieth of the diameter of the pores of the porcelain.

128. Surface Tension.—It is a matter of common observation that a small body of any liquid, not in contact with other bodies, assumes a globular form. This fact is explained by the mutual attraction of the molecules of which the drop is conceived to be composed. A particle of liquid inside the bounding surface is acted upon by a system of forces due to the attractions of other molecules on all sides of it. A molecule well inside the bounding surface would be attracted no more in one direction than in another, and would therefore be in equilibrium under the action of these forces. But since a molecule of liquid situated in the surface is not entirely surrounded by similar molecules, the resultant of the attractions of the neighboring molecules will urge it toward the interior of the body. If no outside force acts upon the liquid it can be shown that it will assume the shape having the least surface, viz., a spherical form. The force in a liquid surface causing it to contract is called *surface tension.* The tendency of the surface of a liquid to assume the smallest area consistent with the volume of the liquid and the external forces acting upon it is similar to the action that would be produced if the surface of the liquid were transformed into a thin contractile membrane.

One important difference between the action of surface tension and that of a thin contractile membrane inclosing the fluid is that, whereas the tension in the membrane depends on the amount of stretching and may be greater in one direction than in another, the surface tension of a liquid is constant, however much the surface is extended, and at any point the surface tension is the same in all directions. With this exception there is a close analogy between surface tension and a stretched film. This analogy will serve a useful purpose in helping one to foresee the effect of surface tension in special cases.

The magnitude of surface tension is measured by the force acting in the fluid surface perpendicular to a line of unit length situated in the surface. The magnitude of the tension in a given fluid surface depends upon the temperature and also upon the substance in contact with the surface. When the temperature is raised, the surface tension diminishes. At 20° C. (68° F.) the tension of the surface separating water from air is 81 dynes per centimeter. At this same temperature, the tension of the surface separating olive oil from air is 37 dynes per centimeter, while the tension of the surface separating olive oil from water is 20.6 dynes per centimeter. When the temperature is raised, the tension in all these surfaces is diminished, although that in the surface separating air from water diminishes much more rapidly than the others.

In Fig. 122 let A represent a drop of olive oil placed on a surface of water. At the line of contact of air, oil, and water, there are three tensions, that in the surface separating water from air, T_1, that in the surface separating oil from air, T_2, and that in the surface separating oil from water, T_3. At ordinary temperatures, T_1 is greater than the sum of T_2 and T_3. Consequently the oil will spread out indefinitely on the surface of the water. If the temperature of the water is raised, T_1 will diminish in value faster than T_2 and T_3, with the result that equilibrium will be attained between the tensions in the surfaces. When this occurs, the drop will cease to spread. If the temperature is raised still higher, the drop will contract into a compact lenticular shape.

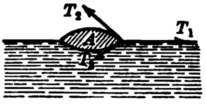

FIG. 122.

In order that the spraying of trees and plants may be effective, the drops of liquid must wet the foliage. Consequently, the surface tension between the drop and air (T_2, Fig. 122), must be small. To lower this surface tension soapy substances are commonly added to spraying solutions.

If a small object, such as a sewing needle, be coated with a thin film of oil and be then carefully placed on the surface of water, it will float, even though its density be considerably greater than water. As in the case considered in Fig. 123, at the line of contact of air, oil, and water there are three tensions, T_1, T_2, and T_3. In addition, the small body B is acted upon by its weight downward and a force upward equal to the weight of the water displaced When the weight of the water displaced, plus the sum of the vertical components of T_1 and T_2, equals the sum of the weight of the body and the vertical component of T_3, the body will float. In this manner insects derive their support when they walk on the surface of water.

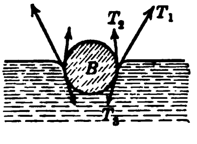

FIG. 123.

Rain-proof fabrics consist of cloth each fiber of which is coated with a microscopic film of a substance of such small surface tension that it is not wet by water. Water drops falling on such a fabric will roll off like mercury from a clean glass plate.

Water spreads over a clean glass surface, but not over a greasy surface. For this reason, air particles in water contained in a clean glass vessel will not cling to the sides of the vessel. But if the glass be greasy, bubbles will cling to the sides.

Certain solids are more readily coated with a film of oil than others. For example, if a mixture of particles of sand, metals and metallic sulphides be shaken with water and a very little oil, the particles of metal and of the sulphides will become coated with a film of oil whereas the sand will not. The shaking will cause air bubbles to cling to the greasy particles thereby buoying them up sufficiently to cause them to rise to the surface. By skimming off the foam, the metal and sulphide particles are separated from the sand and other particles. This is the basis of the flotation process of separating certain ores.

The fact that the surface tension of an oil-air surface is much less than that of a water-air surface is utilized in reducing the waves in a storm at sea. A wave involves changes in the surface. This stretching and thinning of the oil film will be greater at some portions than at others. Where the oil film is thinnest the surface tension will be the greatest. Hence pulls will be exerted on the less stretched portions of the surface by the more stretched portions. These pulls in the surface entail an absorption of the energy of the wave and a diminution of the amplitude of the wave.

Owing to the reduction of surface tension by an increase of temperature the oil film on a heated bearing is drawn away from the rubbing surface toward the surrounding cooler parts. This may cause the lubrication to become more and more defective until finally a "hot box" is produced.

129. Pressure Produced by Surface Tension.—Due to surface tension the surface of a water drop or an air bubble will contract till stopped by a counteracting force produced by pressure within. We shall now find the value of the pressure produced by surface tension within a spherical drop.

If the surface tension be T, the edges of the two halves of the surface of a drop of radius r will be pulled together with a force $2\pi rT$. This force is counteracted by a force due to pressure developed within the sphere. If the pressure be denoted by P, then across the equatorial section of the sphere of radius r, the total force will be $P\pi r^2$. And since the counteraction equals the action,

$$P\pi r^2 = 2\pi rT.$$

Therefore, the pressure within a spherical surface of radius r produced by a surface tension T has the value

$$P = \frac{2T}{r}. \qquad (94)$$

Since the internal pressure required to hold in equilibrium the tension of a curved surface is inversely proportional to the radius of curvature, it follows that vapors will more readily condense on points and edges than on flat surfaces. Smoke and dust particles facilitate the condensation of water vapor in the air. In the region of manufacturing centers and of great battles fogs and rains are common.

If the surfaces of solid bodies be uniformly wetted and then placed in contact, there will be a great outward pull in the film at the points of contact. This pull will develop a diminished pressure within the film at those points. Consequently, the liquid will be drawn into the spaces between the bodies. The particles of a moist, not wet, soil are enclosed by thin films of water held in place by surface tension. The greater part of the water, however, is collected in the angles between the particles, Fig. 124.

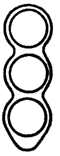

Fig. 124.

In the case of a vertical column of soil particles, the weight of the film will cause a downward flow and a consequent thinning above and thickening below. This action will continue till the surface tensions and internal pressures are balanced at all points. If now, water be removed from the contact angles at the upper end of the column, by evaporation for example, the internal pressures in these angles will be diminished and water will ascend from below till equilibrium is again attained. An increase of temperature, by decreasing the surface tension, diminishes the amount of water that would be raised. But, on account of the diminution of viscosity thereby produced, an increase in temperature will increase the rate of rise of the water.

130. Capillarity.—The tension of a liquid surface in contact with a solid is very marked when the latter is a capillary tube, i.e., one of very small bore. As this case has been considerably studied, the phenomenon of surface tension is often termed capillarity.

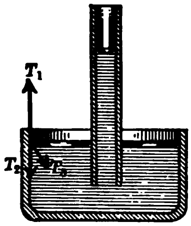

Fig. 125.

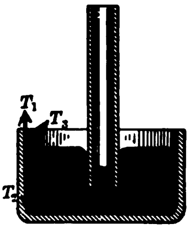

Fig. 126.

Consider a liquid in contact with a vertical solid surface. At the line of contact of air, liquid and solid, there are three tensions—that in the surface separating air and solid, T_1, that in the surface separating liquid and solid, T_2, and that in the surface separating air and liquid, T_3. If T_1, is greater than T_2, the liquid in this region will be drawn up above the level of the general surface

(Fig. 125). Water and glass is an example of this case. If, however, T_2 is greater than T_1, the liquid in this region will be depressed below the level of the general surface (Fig. 126). Mercury and glass is an example of this case.

If the solid be in the form of a capillary tube, these effects will be greatly magnified. Due to surface tension, water will rise in a previously moistened fine glass tube several inches above the general surface, and mercury will be correspondingly depressed below the level of the general surface.

The action of wicks in feeding oil to a flame is an example of capillarity. Again, the pores or minute spaces between particles of fine soil act as small tubes in bringing moisture to the surface of the ground. Anything that will enlarge these spaces will serve to diminish the capillary action. For this reason cultivation retards the evaporation of moisture from the ground.

131. Solutions—Solution Pressure.—If some sugar be placed in water, the solid will disappear. The sugar is said to dissolve in the water and form a solution of sugar. A *solution* is defined as a homogeneous mixture of two or more substances, incapable of separation by mechanical means, in which the relative quantities of the components can vary continuously between certain limits. The *solvent* is the substance in largest proportion, and a *solute* is a substance present in less proportion. As more and more solute is added to the solution, a state may finally be reached such that if more be introduced it will not dissolve at that particular temperature. A solution that has dissolved all it can at its present temperature is said to be *saturated*. The ratio of the mass of solute to the mass of solvent in a saturated solution at a given temperature is called the *solubility* of the solute at that temperature. On going into solution, the molecules of some substances divide into parts called *ions*.

In most cases, the solubility of solids increases when the temperature is raised, while the solubility of gases decreases when the temperature is raised. When the pressure is increased, the solubility of solids increases slightly and that of gases increases considerably. The solubility of a given material is usually less in a solution of another substance than in the pure solvent. On adding a liquid to a solution with which it can mix, the solute will be precipitated from the solution to some extent if it be insoluble in the liquid added.

If a saturated solution is slowly cooled, some of the solute is slowly deposited or crystallized. To start the crystallization it is necessary to have a nucleus such as a particle of dust, a sharp point protruding into the solution from the side of the dish, or a crystal of the solute. If no such nucleus is present, and the solution is cooled, it will become supersaturated, that is, will contain more solute than the given quantity of solvent can hold in stable equilibrium at the given temperature. A supersaturated solution is in unstable equilibrium with respect to the internal forces acting upon the molecules composing it. A slight disturbance, such as the introduction of a crystal of the solute, or a sudden jar, is sufficient to start crystallization. A solution can be rendered supersaturated by evaporation, by partial freezing, and by various other means.

The fact that some substances dissolve more readily in a given solvent than do others is described by the statement that every substance has a definite solution pressure in a given solvent. The *solution pressure* of a substance is the measure of its tendency to go into solution.

132. Adsorption.—All solids tend to condense upon their surface any gas or vapor with which they may be in contact. Cocoanut shell charcoal condenses on its surface such a large amount of any poisonous gas with which it may be in contact that this substance is much used in gas masks. The phenomenon of the concentration or condensation of one substance upon the surface of another is called *adsorption.*

A given material adsorbs different liquids and gases in unequal degree. The substance that is adsorbed in greater degree will displace a substance that is adsorbed in less degree. If a solid adsorbs a liquid more than air, a liquid film will adhere to the solid—that is, the liquid will wet the solid. Oil will displace water, while alcohol will displace oil, when in contact with metal. Gasolene will run through a fine-meshed metal sieve that water will not run through. A cloth wet in alcohol is used to wipe the oil from a metal surface.

If a solution is in contact with either a solid, or a liquid in which both components of the solution are insoluble, and if the surface tension between the solution and the other substance is different than that between the solvent and the other substance, it will be found that in the layer between the solution and the other substance the concentration of the solution is not the same as in the remainder of the solution. If the solute lowers the surface tension between the solvent and the other substance, the concentration in the surface layer is greater than in the body of the solution. On the other hand, if the solute raises the surface tension, the concentration is less than in the remainder of the solution, that is, adsorption does not occur.

Due to adsorption, the first portion of a solution passing through a filter is less concentrated than the subsequent portion. Due to adsorption, certain soils retain soluble salts which prevent their being carried away by rain. The property depends upon the kind and size of the particles of soil and also upon the kind of soluble salt.

Coffee is clarified by adding egg albumen before boiling. The solid particles adsorb the albumen from the solution of albumen in water. The subsequent heating, by coagulating the albumen, separates the solid particles from the liquid. Similarly, wines are clarified by the addition of gelatine. The solid particles adsorb the gelatine from the solution of gelatine in water, and the tannin in the wine coagulates the gelatine.

The first stage of dyeing is an adsorption process. Dyestuffs lower the surface tension of their solvents. Hence the concentration becomes great in the surface layers between the fabric and the dye. In fact, the concentration becomes so great that precipitation occurs within the fiber of the fabric being dyed. If this precipitation is accompanied by a chemical action between the dye and the substance of the goods by which a stable insoluble compound is formed, the dye is fast, that is, cannot be washed out by water.

133. Diffusion.—When two or more fluids which do not react chemically upon one another are placed in contact, the liquids may either gradually mix until the whole mass is homogeneous, or they may form distinct layers. The first operation is called diffusion. Diffusion is due to the motion of the molecules constituting the liquids. Gases diffuse more rapidly than do liquids.

When the liquid forms into layers, the mutual attractions of like molecules is so much greater than the attractions between unlike molecules that very few molecules of one kind succeed in breaking away from their fellows and migrating amongst the molecules of the other sort.

134. Osmotic Pressure.—A gas tends to distribute itself throughout the space in which it is situated. This tendency can be measured by inclosing the gas and observing the pressure exerted on the inclosing wall by the impact of the moving molecules. In a similar manner a solute tends to distribute itself throughout the solvent in which it is situated. When this process of diffusion is hindered, the motion of the molecules of the solute may manifest itself in the form of a pressure analogous to the pressure developed by an expanding gas. That which causes a substance to diffuse through a given solvent and thus occupy more space is called the *osmotic pressure* of the solute in the given solvent.

On account of its solution pressure a substance goes into solution. When it is dissolved, it has a certain osmotic pressure which causes it to diffuse throughout the solvent and to press upon the part which is not yet dissolved. This osmotic pressure of the part that has dissolved acts in opposition to the solution pressure of the part that is not yet dissolved. As more of the substance dissolves the osmotic pressure of the dissolved part increases until, when a certain amount has dissolved, the osmotic pressure equals the solution pressure. The solution is now saturated, that is, if any more substance dissolves, an equal amount will go out of solution.

Osmotic pressure plays an important part in the production of currents of electricity by galvanic cells.

135. Semipermeability.—The osmotic pressure of a solute in a given solvent can be made evident, and its magnitude measured, by separating the solution from a portion of the pure solvent by a partition which prevents the passage of molecules of the solute but which permits the passage of molecules of the solvent. Such a partition is called a semipermeable membrane.

Consider a mass of pure solvent A, Fig. 127, separated from the solution B by a membrane M that is permeable to the solvent and impermeable to the solute. Let the black dots in the figure represent molecules of the solvent, and the small circles molecules of the solute. Both sides of the partition M are bombarded by molecules,—the inside by molecules of the solvent, and the outside by molecules of both solvent and solute. If a molecule of the solvent strikes the partition, it will be absorbed and will pass through. Since molecules of the solute cannot pass through, those of its molecules in contact with the partition screen a portion of the partition from the impact of molecules of the solvent. Since one surface of the partition is in contact with molecules of the solvent only, while the other surface is in contact with molecules of both solvent and solute, it follows that more molecules of solvent will traverse the partition from the pure solvent A to the solution B than will traverse it in the opposite direction. Thus, the solution becomes less concentrated. In other words, the solute diffuses through more solvent. The tendency of the solute to diffuse through more solvent can be counteracted by applying a pressure on the solution by some device P, Fig. 127. The pressure necessary to prevent the solution from altering in concentration equals the pressure with which the solute tends to expand through the solvent, that is, equals the osmotic pressure of the solute in the given solution. The osmotic pressure can be experimentally determined by adjusting the plunger P until the level of the solute in the index tube I remains constant, and then observing the pressure gauge G. It is found that the magnitude of the osmotic pressure of any solute in any solvent varies directly with the sum of the number of molecules and of ions of the solute contained in unit mass of the pure solvent.

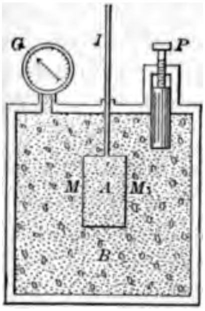

Fig. 127.

There is a number of substances which act as semipermeable partitions for certain solutes in given solvents. For example, copper ferrocyanide is semipermeable for sugar in water. Water is semipermeable for benzene in ether. That is, ether will pass through a layer of water, but benzene will not. Red hot palladium transmits hydrogen but not carbon monoxide. India rubber will transmit in a given time 13.6 volumes of carbon dioxide and but one volume of nitrogen.

Examples of partial semipermeability are very common. Sugar placed on strawberries causes the juice to come out. If raisins be placed in water they will become filled with water till spherical. If pears that are somewhat hard, due to being insufficiently ripe or cooked, are preserved in concentrated sugar syrup they will become dry and tough. Whereas if they are first placed in a weaker syrup for a couple of days and later in the stronger syrup they will not become so tough.

Semipermeability appears to be a phenomenon of selective solubility. Copper ferrocyanide dissolves water, but not sugar. Consider a film of copper ferrocyanide separating water from an aqueous solution of sugar. On account of the dissolved sugar on one side of the partition, this side dissolves less water than does the side of the partition facing the pure water. The partition with its contained water constitutes a solution of greater concentration on one side than on the other. Consequently water diffuses from the region of greater concentration to the region of less, i.e., toward the sugar solution. The sugar molecules unite with these water molecules as soon as they reach the surface of the partition. The amount of water dissolved by the partition on the solution side can be increased by compressing the solution. This makes it possible to counterbalance the decrease of solubility of water due to the presence of sugar, and, by equalizing the solubility on the two sides of the partition, to stop the flow of water into the solution. The pressure required to stop the flow equals the osmotic pressure. Osmotic pressures as great as 500 lb. per sq. in. have been measured by means of semipermeable membranes.

The passage of water in one direction through plant and animal tissues with greater facility than in the opposite direction is due to osmosis and semipermeability. The action of certain saline cathartics in causing water to pass from the blood through the intestinal walls is an example.

The crystalloids, such as sugar and chemical salts, pass readily through wet parchment paper, whereas the colloids such as starch, albumen and the gums, pass very slowly. This fact is the basis of the process of separating crystalloids from colloids called *dialysis*. In case a person is suspected to have died from arsenic poisoning, a parchment paper tray containing water may be floated on the wet contents of the stomach. After a time, the water within the tray will contain any crystalloids such as salts of arsenic which the stomach may have contained.

Questions

1. A glass tube has such a small bore that water will rise in it three inches. A two-inch length is bent as in Fig. 128. Why cannot a wheel be operated by water falling from the upper end?

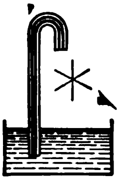

Fig. 128.

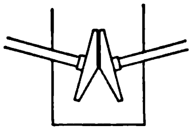

Fig. 129.

2. In Fig. 129 is represented a horizontal section of two wheels dipping in water. The shafts are inclined to one another so that the faces of the wheels are not parallel. Water will rise toward the reader in the space between the faces of the wheels, but higher on the side where the faces are closer together. One side of the apparatus will thus be weighted more than the other. Neglecting friction, show why continuous motion would not be produced.

3. Which of the following are cases of osmotic pressure and which of solution pressure? (*a*) The pressure that tends to make salt dissolve in water; (*b*) the pressure that tends to drive the dissolved salt through the solution, thus making the concentration uniform; (*c*) the pressure that tends to make the salt crystallize from a water solution; (*d*) the pressure that tends to make water mix with alcohol.

4. A 10% solution of boric acid and water is often used as a poultice for inflamed eyes. The conjunctiva of the eyes acts as a semipermeable membrane. The blood is 0.9% salt solution. Explain how osmotic action may decrease the inflammation.

5. The human blood is a 0.9% salt solution. The tissues act as a semipermeable membrane. What would be the osmotic action on a wound if a surgeon bathed it with distilled water? With a strong salt solution?

6. Carbon dioxide has a greater density than any of the other constituents of the atmosphere. Explain why it does not collect at the surface of the earth and destroy all animal life.

CHAPTER X

THE MOTION OF A BODY UNDER THE ACTION OF A VARIABLE FORCE

§ 1. *Simple Harmonic Motion*

136. Simple Harmonic Motion of Translation Defined.—Consider a small block resting on a smooth horizontal table and attached to two light similar horizontal springs, as shown in the figure. When at rest, the block will be at some point C. If the block be displaced in the direction of the axis of the springs to some point B, a force will act upon the block causing it to move towards its position of equilibrium. During its motion toward this point, the potential energy of the system due to the distortion of the springs diminishes until, when at the point C, it becomes zero. At this point the entire energy is kinetic, and the body passes through its position of equilibrium and attains some position as at A. This action will then be repeated, the body vibrating back and forth between A and B.

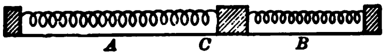

FIG. 130.

According to Hooke's Law (Art. 116), when a spring or other elastic body is distorted through a small range, there is developed a restoring force F which, at any instant, is directly proportional to the displacement d of the end of the spring or other elastic body. That is,

$$F = c'd, \quad \ldots \ldots \quad (95)$$

where c' is a positive constant. Since, $F = ma$, where m is the mass of the block against which the springs press and a is the acceleration with which the block moves, it follows that

$$ma = c'd,$$

or,
$$a\left[=\left(\frac{c'}{m}\right)d\right] = cd, \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

where c is a positive constant.

This means that the acceleration with which the block moves is greatest when the block is furthest from its position of equilibrium, and is zero when the block is at the mid-position. At C, the middle of its path, its velocity is greatest, but is neither increasing nor decreasing, for at that point its acceleration is zero. As it moves toward either end of its path its velocity decreases, and according to (95) decreases more and more rapidly until the block reaches the end of its path, where for an instant only it does not move at all. But at this point its acceleration is changing most rapidly—in fact, is changing from a velocity in one direction to a velocity in the opposite direction.

The motion above described is that which occurs in the case of the prongs of a tuning fork, a plucked stretched string, the air in a sounding organ pipe. It is called simple harmonic motion of translation. *Simple harmonic motion of translation* is that reciprocating motion which has at every instant an acceleration which is directed toward the center of its path and which varies directly with the distance of the moving body from that point.

Not every reciprocating motion is a simple harmonic motion. If a body moves back and forth with an acceleration which is not proportional to the displacement, the motion is not simple harmonic.

The time which elapses between two consecutive passages of the oscillating body in the same direction through any given point of its path is called the *period* of the simple harmonic motion. The maximum distance attained by the oscillating body from its position of equilibrium is called the *amplitude* of the simple harmonic motion.

137. Relation between Uniform Circular Motion and Simple Harmonic Motion.—Let a particle P', Fig. 131, move with uniform speed in the circumference of a circle $P'A'B'$, and let P be the projection of this point on any right line AB in the plane of the circle. As P' moves with uniform speed in the circumference of the circle, its projection P oscillates back and forth through a middle position C between two extreme positions A and B. The sort of motion described by P along the line AB will now be investigated.

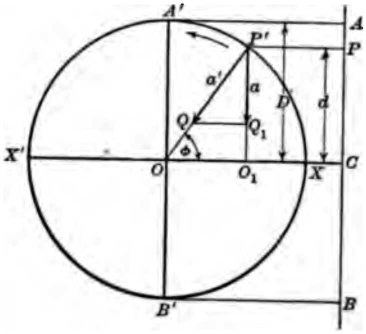

FIG. 131.

As the particle P' moves with uniform speed in the circumference of a circle there is a constant acceleration directed toward the center of the circle (Art. 61). In Fig. 131, this radial acceleration a' is represented by the line $P'Q$. From the principle of the resolution of linear accelerations, it follows that the acceleration a of the point P is the component, in the direction AB, of the acceleration of the particle P'. Thus,

$$a = a' \sin \phi.$$

Represent the constant linear speed and angular speed of P' by v and w, respectively. Let t represent the time since P' was last at X. Then,

$$\phi = wt, \ (24); \quad a' = \frac{v^2}{r}, \ (40); \text{ and } v = wr, \ (31).$$

On substituting these values in the above equation, we obtain

$$a[= a' \sin \phi] = \frac{v^2}{r} \sin wt. = w^2 r \sin wt. \quad . \quad . \quad . \quad (97)$$

Representing by d the displacement PC $(= P'O_1)$ of the point P from the middle of its path,

$$d = r \sin \phi = r \sin wt. \quad . \quad . \quad . \quad . \quad . \quad (98)$$

On substituting for r in (97) the value given in (98), we find that

$$a=w^2d. \qquad \text{. . . . . . .} \qquad (99)$$

Since w is constant, it follows from this equation that the acceleration of P is proportional to its distance from the center of its path. That is, *if a point moves with uniform speed in the circumference of a circle, the projection of the point on any straight line in the plane of the circle moves with simple harmonic motion of translation.*

138. The Period of a Simple Harmonic Motion.—The fact that "if a point moves with uniform speed in the circumference of a circle, the projection of the point on any straight line in the plane of the circle moves with simple harmonic motion of translation," will now be used for the determination of the value of the constant c in the defining equation of simple harmonic motion (96).

A comparison of (96) and (99) shows that $c=w^2$. If the time of one revolution of P', that is, the time of one complete vibration of P, be denoted by T, we shall have

$$w=\frac{2\pi}{T} \text{ radians per unit of time.}$$

Consequently,

$$c[=w^2]=\left(\frac{2\pi}{T}\right)^2.$$

Hence, if a body of mass m is moving with simple harmonic motion of period T, then when the body is at a distance d from the middle of its path, there is an acceleration directed to the middle of the path of the value

$$a[=cd]=\left(\frac{2\pi}{T}\right)^2 d, \qquad \text{. . . . . . .} \qquad (100)$$

and a force toward the middle of the path of the value

$$F[=ma]=m\left(\frac{2\pi}{T}\right)^2 d. \qquad \text{. . . . . .} \qquad (101)$$

From (100), the period of a simple harmonic motion of translation is

$$T=2\pi\sqrt{\frac{d}{a}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

139. Simple Harmonic Motion of Rotation Defined.—If a body suspended by a vertical wire be rotated through a small angle about a line coincident with the axis of the wire, it is found by experiment that the restoring torque L is directly proportional to the angular displacement. That is,

$$L=k'\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

where k' is a positive constant.

Since, from (66), $L=K\mathbf{a}$, where K is the moment of inertia of a rotating body and $\mathbf{a}$ is the angular acceleration with which it moves, it follows that

$$K\mathbf{a}=k'\phi.$$

Whence,
$$\mathbf{a}=\left(\frac{k'}{K}\right)\phi=k\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

where k is a positive constant.

When a body rotates back and forth with a motion in which the angular acceleration is always directed toward a position of equilibrium and is always proportional to the angular displacement of the body from that position, the body is said to have a *simple harmonic motion of rotation.*

140. The Angular Acceleration and Period of a Body Moving with Simple Harmonic Motion of Rotation.—It will be noticed that (103) and (104) are perfect analogues of (95) and (96). The magnitudes of the angular velocity and the angular acceleration of a body moving with simple harmonic motion of rotation can be obtained by a simple transformation of the latter equations.

In Fig. 132 let ZZ' be the position of equilibrium of a certain line of a body which vibrates with simple harmonic motion of rotation about an axis through O normal to the plane of the diagram. While the given line vibrates about O with simple harmonic motion of rotation, every point in the line will oscillate about its position of equilibrium with simple harmonic motion of translation. Consider the linear motion of some point distant r from the axis of vibration passing through O. Denote the amplitude of the linear motion of the selected point, $CA[=CB]$ by the symbol D; let its linear displacement from the position of equilibrium when at some point P be denoted by d.

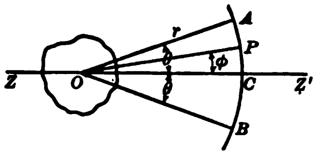

FIG. 132.

From (65) and (Art. 52) $a=\mathbf{a}r$ and $d=\phi r$, where a and $\mathbf{a}$ denote respectively the linear and angular accelerations of the given point, and d and ϕ the respective linear and angular displacements. On substituting in (100) these values for a and d,

$$\mathbf{a}=\left(\frac{2\pi}{T}\right)^2\phi. \qquad (105)$$

Since, from (66), $L=K\mathbf{a}$, the period of a simple harmonic motion of rotation is

$$T=2\pi\sqrt{\frac{\phi}{\mathbf{a}}}=2\pi\sqrt{\frac{K\phi}{L}}. \qquad (106)$$

Most substances increase in size when raised in temperature. When the balance wheel of a watch increases in size, the moment of inertia K increases and the time of vibration T increases, thereby causing the watch to lose time. Watches are now compensated for temperature by making the hair spring of an alloy having a Young's modulus which increases with rise of temperature, by the same amount as the moment of inertia of the balance wheel increases.

§ 2. *The Pendulum*

141. Galileo's Observation.—One day, while attending service in the cathedral at Pisa, Galileo was impressed by the observation that the great chandelier suspended from the ceiling by a long rod (Fig. 133), vibrated in apparently the same time whether the amplitude of vibration were large or small. In order to test the accuracy of this observation, the next day Galileo returned to the cathedral, determined to measure the period of oscillation of the chandelier, first just after it had been lighted, and again after the amplitude of swing had died down considerably. As there were no watches or other portable instruments for measuring time in those days, Galileo used his pulse-beats as the standard of time for determining the period of the swinging chandelier. The result of this measurement confirmed him in his opinion that the period of a swinging pendulum is independent of the amplitude of swing, so long as that amplitude is not excessive. The practicability of using a pendulum to mark off equal intervals of time now became evident, and the application to clocks quickly followed.

FIG. 133.

142. The Pendulum.—A compound or physical pendulum consists of a suspended rigid body free to oscillate about a horizontal axis. Consider the physical pendulum AC (Fig. 134), consisting of a body of mass m supported on an axis normal to the plane of the diagram and passing through the point A. Let the centroid of the pendulum, i.e., the point of application of the resultant of the weights of the particles composing the pendulum, be at C. Denote the distance AC by l. If the pendulum be deflected from its equilibrium position through an angle ϕ, it will be acted upon by a torque which tends to restore it to the equilibrium position and which has a value

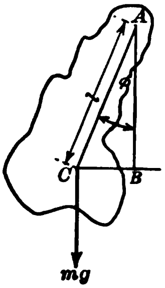

FIG. 134.

$$L = mg(BC) = mgl \sin \phi.$$

If the displacement from the equilibrium position is small, $\sin \phi$ is approximately equal to ϕ (Art. 52). In this case the above equation becomes

$$L \doteqdot mgl\,\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

Whence, at any instant, a pendulum displaced but a small distance from its equilibrium position is urged toward its equilibrium position by a torque nearly proportional to its angular displacement from that position. Consequently (Art. 139), the angular motion of such a pendulum is approximately simple harmonic motion of rotation.

The period of vibration of a physical pendulum oscillating through a small amplitude will now be determined. From (106), the period of a simple harmonic motion of rotation is

$$T=2\pi\sqrt{\frac{K\phi}{L}},$$

where K is the moment of inertia of the body with respect to the axis of rotation. Substituting in this equation the value of the torque acting on a compound pendulum displaced through a small angle ϕ from its position of equilibrium (107), we obtain for the value of the time occupied by one complete vibration of a compound pendulum,

$$T \doteq 2\pi\sqrt{\frac{K}{mgl}}. \qquad \qquad (108)$$

From this equation it is seen that when the amplitude of vibration of a compound pendulum is so small that $\sin\phi$ may be replaced by ϕ, the period of vibration of the pendulum is independent of the amplitude of swing.

143. The Simple Pendulum.—A simple or mathematical pendulum consists of a heavy particle suspended by a massless string. Since the moment of inertia with respect to the axis of oscillation of a simple pendulum of length l and mass m is, (68),

$$K=ml^2,$$

the period of vibration of a simple pendulum is, (108),

$$T_s \doteq 2\pi\sqrt{\frac{l}{g}}. \qquad \qquad (109)$$

A pendulum consisting of a small spherical bob supported by a very thin string approximates closely to a simple pendulum.

Equations (108) and (109) are the ones ordinarily used for the determination of the value of the acceleration due to gravity. The closeness of the approximation involved in these equations is such that if the angular amplitude of swing be one degree, the error introduced in the value of g will be less than one part in twenty-five thousand; if the amplitude be five degrees, the error will be less than one part in one thousand.

A pendulum which makes one-half of one complete vibration in one second of time is called a seconds pendulum. The period of vibration of a seconds pendulum is two seconds.

144. Sympathetic Vibration or Resonance.—The vibrations executed by a body which has been displaced from its position of equilibrium and then released are called *free* vibrations. The vibrations executed under the action of an external periodically varying force are called *forced* vibrations. Consider a number of pendulums of different lengths (Fig. 135) hung from the same support X. If one of the pendulums be set into oscillation, the support will receive a slight impulse every time the moving pendulum swings back and forth. The first impulse will start all of the pendulums to swinging slightly. If some one of the pendulums has the same period of vibration as the support, it will always be swinging in such a direction that the impulses tend to increase its motion. If another of the pendulums has a period of vibration somewhat different than that of the support, it will soon be swinging in such a direction that the impulses tend to decrease its motion. After a little time, then, those pendulums having natural free periods nearly the same as that of the support will be swinging with a considerable amplitude, while those of very different periods will be scarcely swinging at all. The vibration produced when a periodically varying force acts upon a body of nearly the same period of vibration is called *resonant forced vibra-*

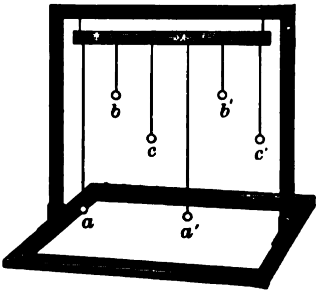

FIG. 135.

tion. The phenomenon of the production of resonant forced vibration is called *sympathetic vibration* or *resonance.* Resonance plays an important part in many phenomena of dynamics, sound, electricity and light.

If the mass of the receiving body is large and the opposition to motion is small, as in the case of a tuning fork, resonance occurs only when the period of the exciting force is almost exactly equal to the period of the receiving body. But if the mass of the receiving body is small and the opposition to motion is large, as in the case of air vibrating in a narrow tube, resonance will occur when the period of the exciting force is appreciably different than the free vibration period of the receiving body.

The above statements apply to the case in which the periodic exciting force varies harmonically. It should be noted, however, that if instead of varying harmonically, the impulses of the external periodic force start and stop suddenly, they will also set into resonant vibration a body whose free period is any submultiple of the period of the external force.

A bridge or other engineering structure has a definite natural period of vibration. If this should happen to be about the same as the step of a trotting horse, a bridge might be dangerously strained by horses trotting across it. Soldiers are required to break step when crossing a bridge. Well-built grand stands have failed under the strain of sympathetic vibration induced by the regular stamping of spectators at football games.

The rolling of a ship at sea is an example of resonance. If the natural free period with which the ship would roll when displaced out of the vertical in still water is nearly equal to the period of the waves, the angle of roll may become large even when the waves are not high. As the frequency with which the waves strike the ship depends upon the speed and course of the ship relative to the direction of the waves, the angle of rolling can be altered by changing either the direction or the speed of the ship.

Questions

1. What sort of a force is acting upon a body moving with, (*a*) uniform motion; (*b*) uniformly accelerated motion; (*c*) simple harmonic motion of translation?

2. A pendulum clock loses time. Show which way the pendulum bob must be moved, and show how you would calculate the distance it must be moved, in order that the clock may keep correct time.

3. State how the period of the pendulum is affected by, (*a*) increasing its length; (*b*) placing it in a mine; (*c*) elevating it above the earth; (*d*) altering the mass without changing the length.

4. In what forms is energy stored in a swinging pendulum? Is there any energy transformation during the swing? Explain why a pendulum bob is made heavy.

5. State Newton's law of gravitation and show how a clock would vary in rate as it is carried from New Orleans to Montreal.

6. Describe the phenomenon of resonance and give an example in which resonance is not desired.

7. Large and strong bridges have collapsed, due to the rhythmic tread of a company of soldiers marching over them. Where does the large amount of energy come from necessary to break down the bridges?

CHAPTER XI

WAVE MOTION

145. Waves.—A motion which goes through the same series of changes at regularly recurring intervals is called *periodic*. A periodic disturbance which is handed on successively from one portion of a medium to another is called a *wave motion*. The sort of wave motion easiest to imagine is that due to a simple harmonic vibration of the particles composing a medium such that each particle performs its vibration slightly later than the particles on one side of it and slightly sooner than the particles to the other side of it. For this reason, the illustrations of wave motion considered in the following articles are due to vibrations of portions of matter. But it should be kept in mind that wave motion is not limited to simple harmonic vibrations, nor to motions of matter. For example, we have light, electric and magnetic waves due to non-harmonic magnetic disturbances propagated by the ether instead of by matter. But it must be kept in mind that even in the case of waves through matter, it is a disturbance that travels through the medium, and not portions of matter that travel. Each portion of matter simply moves about its position of equilibrium.

146. Transverse Wave Motion.—Imagine a long piece of rubber tubing AZ to be laid in a straight east and west line on a frictionless horizontal plane, and let the west end of the tube A (Fig. 136) be moved back and forth in a north and south line with a simple harmonic motion. The motion of A will cause a neighboring small part of the tube B to move in a similar manner; but since the tube

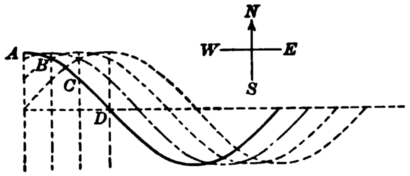

Fig. 136.

is not rigid, *B* will perform its motion slightly later than *A*. Thus when *A* has just reached the northernmost point of its path, *B* will not yet have quite reached the northernmost point of its path; and by the time that *B* does reach that position, *A* will already be a short distance on its way back, and so on. In like manner the motion of *B* will cause a neighboring part of the tube *C* to perform a simple harmonic motion of the same period and amplitude, but always lagging slightly behind *B*; *C* will in turn cause another small part of the tube to vibrate; that another; and so on. This sort of motion is illustrated in Fig. 136, in which the north-to-south dotted lines indicate the paths over which the particles at *A*, *B*, *C*, and *D*, respectively, move back and forth; the full curve indicates the position of the tube after the end *A* has made nine-twelfths of a vibration; the dashed curve, its position after *A* has made ten-twelfths of a complete vibration; the dashed and dotted curve its position after *A* has made eleven-twelfths of a vibration. This shows that as each particular part of the tube moves back and forth with harmonic motion, a wave motion passes to the east along the tube. The particular sort of wave motion in which the small parts of the vibrating substance move back and forth in a direction perpendicular to that in which the waves advance is called *transverse* wave motion.

Let Fig. 137 represent, at some particular instant, the position of a part of a length of rubber tubing along which a transverse wave is passing to the right. The arrows indicate the directions and magnitudes of the instantaneous velocities of the small parts of the tube. Small parts at *a* and *a'* which are in their position of

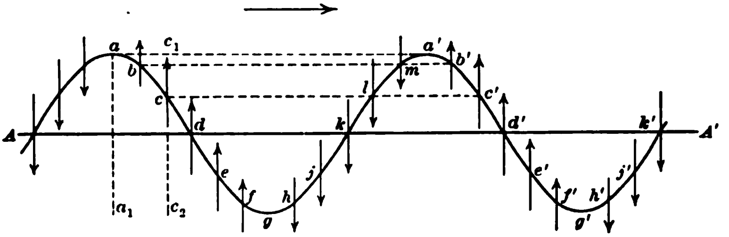

FIG. 137.

greatest displacement in one direction are said to be at the *crests* of waves, while small parts at g and g' which are in their positions of greatest displacement in the opposite direction are said to be at the *troughs* of waves. In any wave motion, the distance between two points in consecutive waves which are moving in the same direction with the same speed is called a *wave-length.* Thus, each of the dotted lines aa', bb', cc', represents one wave-length.

147. Longitudinal Wave Motion.—If one end of a long straight piece of rubber tubing be displaced in the direction of its length, then the various small parts of the tube throughout its length will be successively displaced in the same direction. And if one end be given a simple harmonic motion in the direction of the length of the tube, then the next small part will also execute a simple harmonic motion, lagging slightly behind the motion of the end. This second small part will then cause the third also to vibrate, but to vibrate a little later, the third a fourth, and so on. Thus a state will soon be reached in which, (*a*) each small part of the tube is moving back and forth with simple harmonic motion in the direction in which the waves are moving, and (*b*) the motion of each small part is performed slightly later than that of its neighbor on one side and slightly sooner than that of its neighbor on the other side. That sort of wave motion, in which the direction of vibration of the particles composing the medium is the same as the direction of propagation of the disturbance through the medium, is called *longitudinal* wave motion.

On a straight piece of rubber tubing, at rest, let a series of round spots be painted at equal distances apart throughout its length. Now cause a longitudinal wave motion to pass along the tube, and let $12t$ denote the period of the simple harmonic motion which each of these spots executes. In Fig. 138, the line L_0 shows the positions of the spots when no wave is passing, the line L_{36} their position $36t$ seconds after the first spot goes through its position of equilibrium, etc. In this figure it will be noticed that there are places where the spots on the tube are crowded closely together and other places where they are spread apart. A region where the crowding together is a maximum is called a *condensation;* a region where the spreading apart is a maximum is called a *rarefaction*. It will further be noticed that two condensations or two rarefactions remain always the same distance apart,

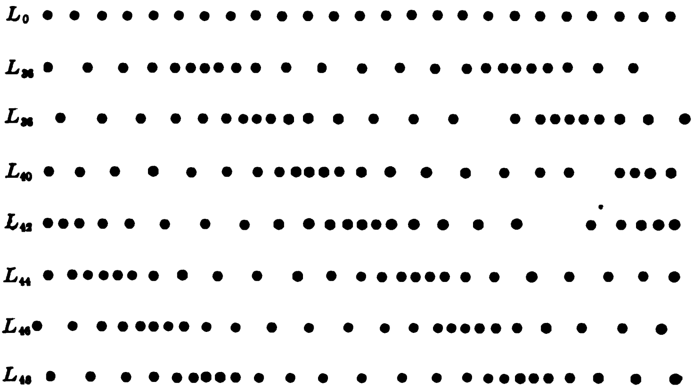

FIG. 138.

and that, although the particular spots merely move back and forth, yet the condensations and rarefactions move forward.

In Fig. 139 the consecutive spots on the rubber tube are numbered in order. The direction and size of each arrowhead represent the direction and magnitude of the velocity of the spot to

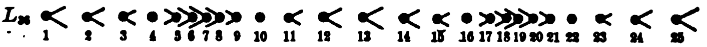

FIG. 139.

which it is attached $36t$ seconds after the first spot was in its position of equilibrium. At this instant spots 7 and 19 are in consecutive condensations and are moving with the same speed in the same direction. The distance between them is, therefore, one wave-length. Similarly the distances from 8 to 20, from 9 to 21, etc., are wave-lengths.

148. Wave Forms.—For longitudinal, as well as for transverse waves, it is customary to plot displacements perpendicularly to the line of propagation. For example, if displacements of the spots to the right of the equilibrium position in Fig. 139 be represented by ordinates above the axis of propagation, and displacements to the left of the equilibrium position by ordinates below the axis, we will have a displacement curve of similar appearance to the curve *Aaga'g'A'*, Fig. 137.

If the velocities of the particles be plotted as ordinates and either time or distance of propagation as abscissas, we will obtain a curve similar to *aga'g'A'*, Fig. 137.

If the pressures along the axis of propagation be plotted as ordinates, we will obtain another curve similar to the curve of velocities. Any one of these curves may be called the "form of the wave."

149. Speed of a Wave Motion.—In Fig. 137 it is seen that during the time required for the spot at a to execute a complete vibration, i.e., to move to a_1 and back, the spot at c has moved to c_1, thence to c_2, and back to c; and similarly every other spot on the tube has returned to the position it has in the figure. That is, every spot makes one complete vibration while the wave advances one wave length.

In Fig. 138 we see that while time increases from $36t$ to $48t$, i.e., during the time required for each spot to execute a complete vibration, the wave moves forward one wave-length.

Now during the time T in which a particle makes one complete vibration, the wave which it sets up advances one wave-length. If a particle vibrate n times per second, the wave which it sets up will advance in each second a distance $n\lambda$, where λ represents the wave-length. Therefore the speed of the wave is

$$v = n\lambda = \frac{\lambda}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

The speed of a wave depends upon the type of wave and upon the nature of the medium through which it is transmitted. A transverse wave will traverse a steel rod with greater speed than a hemp rope. And the speed of a longitudinal wave in steel is greater than the speed of a transverse wave in the same material.

If the period T of a vibration be increased, the disturbance will travel during that time a longer distance λ. In fact, the wave-length varies directly with the period for any given type of wave in any specified medium. Thus, from (110), the speed of propagation of a wave of given type through any selected medium is independent of the wave-length.

If waves of the same period and type traverse two media in which the speeds are different, the wave-lengths in the two media will also be different. When the periods are the same, (110) shows that the ratio of the speed to the wave-length is constant.

150. Flow of Energy. — (*a*) *With Transverse Waves.* — If a transverse wave is passing to the right along a piece of rubber tubing (Fig. 140), the force which tends to restore any small part of the tube to its position of rest in the line AA' is due to the tension in the tube. The forces acting on a small part of the tube at c are the pulls of the small parts of the tube on each side of it. The small part to the left is pulling it upward and to the left, and the small part to the right is pulling it downward and to the right. Since the wave is moving to the right, c is moving upward. That is, a component of its motion is in the direction in which the small part of the tube to the left is pulling it. This means that the small part of the tube to the left of the point considered is doing work upon it. Similarly it may be shown that, with the exception of b, e, g, i, etc., which are momentarily at rest, every small part of the tube is doing work upon the small part which lies adjacent to it on the right. That is, in transverse wave motion, energy is continuously moving in the same direction and with the same speed as the advancing wave.

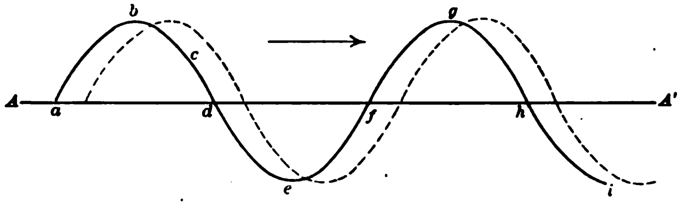

FIG. 140.

(*b*) *With Longitudinal Waves.*—In Fig. 139 it will be seen that where the spots are closer together than normal, they are moving to the right; and where the spots are farther apart than normal,

they are moving to the left. Now where the spots are closer together than normal, the tubing on which they are painted is compressed and each small particle of the tube is pressing both forward and backward against its neighbors; and where the spots are farther apart than normal, the tubing is stretched and each small part is pulling on its neighbors. This means that, except at the points 4, 10, 16, 22, which are momentarily at rest and where the tubing is neither stretched nor compressed, each small part of the tube is urging its neighbor on the right in the direction in which it is going, and its neighbor on the left in the direction opposite to that in which it is going. It follows that, except at the points where the tubing is momentarily at rest, each small part of the tube is doing work on the part just to the right of it. That is, in a longitudinal wave motion energy is continually moving in the same direction and with the same speed as the advancing wave.

Consequently, *each type of wave motion is accompanied by a flow of energy from one part of a medium to another. The direction of the energy flow is the direction in which the waves move, and the speed of the energy flow is the speed with which the waves move.*

151. Phase and Phase Angle.—In the case of vibrations along a horizontal line, displacements, accelerations, and forces directed from the position of equilibrium to the right are counted positive. In the case of vibrations along a vertical line, displacements, etc., from the zero position upwards are counted positive. The *phase* of a periodically vibrating body is the fraction of a whole period of vibration which has elapsed since the body last passed the zero position in the positive direction. Thus, if in Fig. 141 a body vibrates along the line AE with simple harmonic motion, and if at intervals of one-eighth of the period of vibration, the body is successively in the position C, D, E, D, C, B, A, B, etc., then when moving in the positive direction the phase of the body at C is zero, when at D it is $\frac{1}{8}$, when at E it is $\frac{1}{4}$, when moving in the opposite direction the phase at D is $\frac{3}{8}$, when at C it is $\frac{1}{2}$, etc.

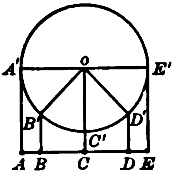

FIG. 141.

Or, since the projection, on any straight line, of a point moving with uniform speed in the circumference of a circle moves with simple harmonic motion of translation (Art. 137), phase may also be expressed in terms of the angle through which the radius of the reference circle has moved since the body last passed in the positive direction through its position of equilibrium. This angle is called the *phase angle*. Thus, in the above figure when the body is at D, and moving in the positive direction, its phase is $\frac{1}{8}$, and its phase angle is $\frac{1}{4}\pi$ radians, or 45°; when at E, its phase is $\frac{1}{4}$, and its phase angle is $\frac{1}{2}\pi$ radians, or 90°; when at D and moving in the negative direction the phase is $\frac{3}{8}$, and the phase angle is $\frac{3}{4}\pi$ radians, or 135°.

Two bodies having phase angles that differ by zero or by any integral multiple of 2π radians are said to be in the *same phase*. Thus in Fig. 137, a and a' are in the same phase, b and b' are in the same phase, etc. Two bodies having phase angles that differ by π radians are said to be in *opposite* phases. Thus in Fig. 137, a and g are in opposite phases; and in Fig. 139, the spots 5 and 11, and also 7 and 13, are in opposite phases.

152. Reflection of Waves.—Consider the passage of a disturbance from one medium to another in which the restoring force is different. To fix the ideas, take the case of a long spiral spring loaded for a part of its length with balls of large inertia and loaded for the remainder of its length with balls of smaller inertia. In the first part of the spring the restoring force is greater than in the second part. Let the ball a, Fig. 142, be displaced to the right a distance represented by the heavy arrow below a. The portion of the spring between a and b being now compressed more than that between b and c, the ball b will be displaced to the right. The ball c will, in turn, be displaced an equal amount. The motion

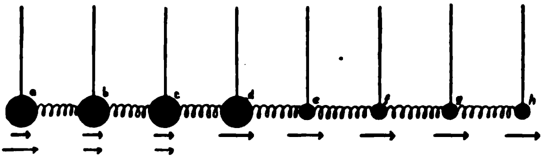

FIG. 142.

of the ball *d*, however, is opposed by the lesser inertia of the ball *e* and consequently *d* will be displaced a greater distance than any of the preceding balls. The succeeding balls will be displaced through distances represented by the heavy arrows below them.

When the large displacement of *d* has occurred, the portion of the spring between *c* and *d* is more extended than that between *b* and *c*. Hence the ball *c* will be displaced to the right through a distance represented by the light arrow below it. Thus, a pulse of compression from *a* to *d* is succeeded by a pulse of rarefaction from *d* to *a*. Similarly, if a pulse of rarefaction be sent from *a*, then at *d* a pulse of rarefaction will continue into the second portion of the spring, while a pulse of compression will be reflected back through the first portion of the spring. It follows that if a longitudinal wave originates in the left portion of the spring, then at the junction of the two parts of the spring a compression will be reflected as a rarefaction, and a rarefaction will be reflected as a compression.

When a longitudinal wave goes from one medium to another in which the restoring force is less, a pulse of compression is instantly reflected as a pulse of rarefaction, and a pulse of rarefaction is instantly reflected as a pulse of compression. And since in a wave, a pulse of rarefaction and a pulse of compression are separated by one-half of a wave-length, the wave on reflection is said to experience a loss of one-half wave-length. In going from water to air sound is reflected with loss of a half wave-length.

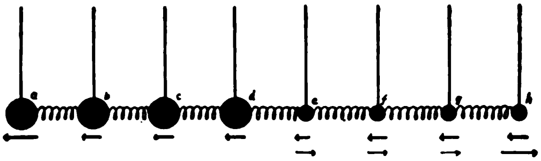

FIG. 143.

The case is somewhat different when a longitudinal wave proceeds from one medium to another in which the restoring force is greater. To fix the ideas, let the ball *h*, Fig. 143, be displaced to the left a distance represented by the heavy arrow below *h*. The balls *g*, *f* and *e* will, in turn, be displaced almost equal distances.

The inertia of d, however, is so great that d is displaced but slightly. The displacements of d, c, b, and a are represented by the heavy arrows below them.

When the small displacement of d has occurred, the portion of the spring between d and e is more compressed than the portion between e and f. Hence, the ball e will be moved to the right through a distance represented by the light arrow below it. Thus, a pulse of compression from h to d is succeeded by a pulse of compression from d to h. Similarly, if a pulse of rarefaction be sent from h, then at d a pulse of rarefaction will continue into the second portion of the spring, while another pulse of rarefaction will be reflected from d to h. When a longitudinal wave goes from one medium to another in which the restoring force is greater, reflection occurs without the loss of a half wave-length. In going from air to water or to a metal, sound is reflected without the loss of a half wave-length.

By vibrating a or h transversely, a transverse wave will be produced. When a transverse wave travels from one medium to another in which the restoring force is different, then at the boundary separating the two media part of the energy will be transmitted into the second medium and the remainder will be reflected back into the first medium. In going from air to glass or water, light is reflected with loss of a half wave-length.

In the above paragraphs we have considered waves limited to one direction. In a later Article it will be shown that for waves traveling in space there is a case in which reflection may occur without any energy being either transmitted or absorbed by the second medium.

When the restoring forces in two media are greatly different, nearly all of the energy will be reflected at the interface and but a small amount will enter the second medium.

153. Superposition and Interference of Wave Motions.—The force acting upon a body may be the resultant of two or more component forces. The motion of a medium may be the resultant due to the superposition of two or more wave motions. Consider three similar pieces of uniform rubber tubing, AB, $A'B$, BC, connected, as shown in Fig. 144, to the ends of a rod AA' capable

of oscillation about a horizontal axis. If the end A be moved up and down with periodic motion, waves will be started along the tubes AB and $A'B$. At B the two waves will be superposed. The motion of BC will be the result of the superposition of the two waves originating at A and A'. In the present case the component waves will necessarily be of the same period. And if the tubes AB and $A'B$ are of the same length and the distances from A and A' to the axis of rotation are equal, then the disturbances from A and A' reaching B at any instant will be of equal amplitude and opposite phase. Consequently, the displacement of B will equal zero—in other words, the motion of B will be zero, and no wave will be transmitted along BC. In general, the resultant due to the superposition of two waves which meet in opposite phases and which are of the same form, amplitude and wave length, is zero. Complete neutralization of the effect of one wave motion by another is called *total destructive interference.*

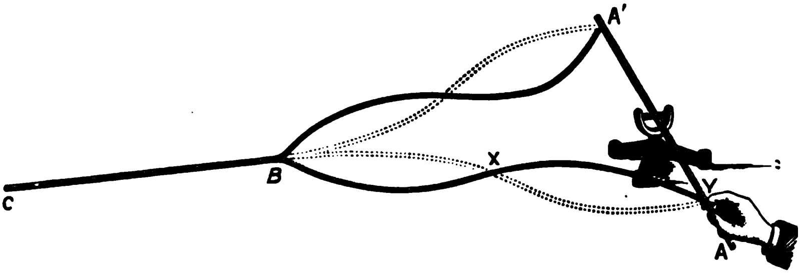

FIG. 144.

Interference is usually not total. If the forms, the amplitudes, or the wave-lengths of the two waves reaching B were not equal, or if the phases were not opposite, then the resultant displacement of B at every instant would not be zero. This point would then vibrate with an amplitude equal to the algebraic sum of the amplitudes of the two waves, and a wave would be transmitted along BC.

Interference is the distinguishing characteristic of wave motion.

Interference effects can be obtained with sound, light and electricity.

Let the plane of the page represent the surface of water in a large tank or pond. Let A and B, Fig. 145, represent the edges of two long boards which project through the surface. In A there is a narrow slit s, and in B there are two similar narrow slits, close together, parallel to and equally distant from s. If the water surface be slightly tapped with the finger at regular intervals at some point X, a circular wave will proceed from this point as a center. A narrow portion of the wave will proceed through the aperture s and portions of this wave will emerge through s_1 and s_2. Since s_1 and s_2 are arranged symmetrically with respect to s, the waves at s_1 and s_2 will at any given instant be in the same phase and have the same amplitude. The portions of the water surface to the right of B will now be traversed by waves that proceed as though they had been set up by two periodic disturbances at s_1 and s_2 of the same period, phase and amplitude.

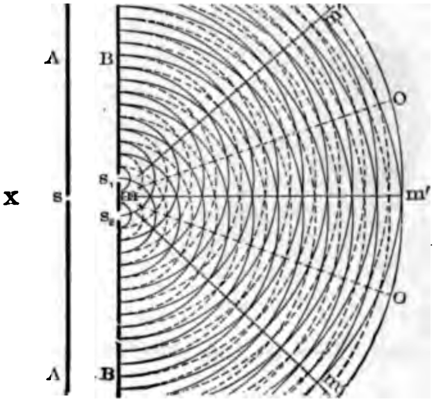

FIG. 145.

At some particular instant the crests of the waves proceeding from s_1 and s_2 will be in the positions marked by the circular arcs drawn in full lines, and the troughs of the waves by the circular arcs drawn in dotted lines. The elevation of the water above the undisturbed surface, at any point, equals the sum of the effects of the two waves at the given point. At a point where two crests coincide, the elevation is greater than that which would be produced by a wave from either of the sources. At a point where two troughs coincide the depression below the undisturbed level is greater than that which would be produced by a wave from either of the sources. At a point where a crest and a trough coincide the elevation is small—that is, the waves here tend to neutralize one another's effect and produce destructive interference. Since each particle of water moves up and down with the same frequency, it follows that the particles now displaced the maximum distance above the undisturbed level will, at an instant one-half period later, be displaced the maximum distance below the undisturbed level. The particles at one time undisturbed will remain undisturbed. Thus along the lines marked mm' the particles are in a state of maximum disturbance, and along the lines mo the particles are undisturbed.

Since the two sources of disturbance s_1 and s_2 vibrate in the same period and same phase, it follows that the condition for maximum disturbance at any given point is that at this point the two waves must be in the same phase. It follows that there will be maximum disturbance at any point whose distance from one source differs from its distance to the other source by an amount equal to any even number of half wave-lengths. Similarly there will be minimum disturbance at any point whose distance from one source differs from its distance to the other source by an amount equal to any odd number of half wave-lengths.

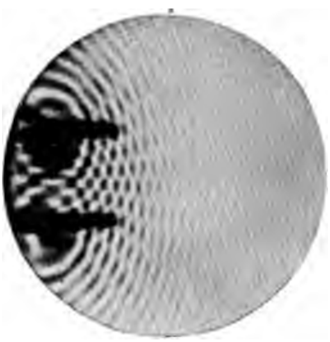

FIG. 146.

Fig. 146 was made from a photograph of the waves on the surface of mercury produced by the vibration up and down of two iron wires. The two vibrations were of the same period and in the same phase. Note the lines along which complete interference occurs.

154. Standing Wave or Stationary Undulation.—Consider a long uniform rubber tube *AB* which lies in a straight line on a frictionless horizontal plane. Let both ends, *A* and *B*, be given transverse simple harmonic motions of the same period and amplitude. Since the tube is uniform and the periods of the two vibrations are the same, the two waves thereby produced will not only have the same amplitude and period but will also move with the same speed toward the middle of the tube. Let the letter *R* denote the wave which is passing to the right from *A* toward *B*, and the letter *L* denote the wave passing to the left from *B* toward *A*. In Fig. 147 a section of the tube two wave-lengths long has been chosen, and the wave form for that particular part of the tube is represented at eight chosen instants by the curve marked *W*. The curve marked *R* is in each case the form the rubber tube would have if the tube were affected only by the wave advancing to the right, and the curve marked *L* is the form the tube would have if affected only by the wave advancing to the left. Since the displacement of any small part of the tube at any instant is the result of its displacement due to *R* and that due to *L*, the resultant wave form produced by the two simultaneous waves is obtained by taking for the ordinate of each point the algebraic sum of the ordinates of the two component curves, *R* and *L*. In this manner the curve *W* has been constructed for a series of successive instants one-eighth of a period apart.

At some moment the crests of *R* will coincide with the troughs of *L* and complete destructive interference will occur, as shown in (1), Fig. 147. At the instant one-eighth of a period later, *R* has advanced to the right, and *L* has advanced to the left, one-eighth of a wave-length. At this instant the condition is that represented in (2). It will be seen that at certain points, *a*, *b*, *c*, *d*, the two component displacements are still equal and opposite, but at other points they are not. At the end of a time interval equal to another one-eighth period we have the moment for which the curves in (3) are drawn. Here, the same points, *a*, *b*, *c*, *d*, are still undisplaced, while the other points are farther displaced. In the same manner we can follow through the other parts of the figure. As we do so we find that the points *a*, *b*, *c*, *d*, etc., a half wave-length apart, are never displaced; and that the parts of the tube between them swing out first on one side and then on the other in such a manner that all points of the tube pass through their positions of equilibrium at the same instant. In Fig. 148 the *W* curves of the previous figure are superposed,

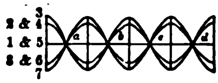

FIG. 148.

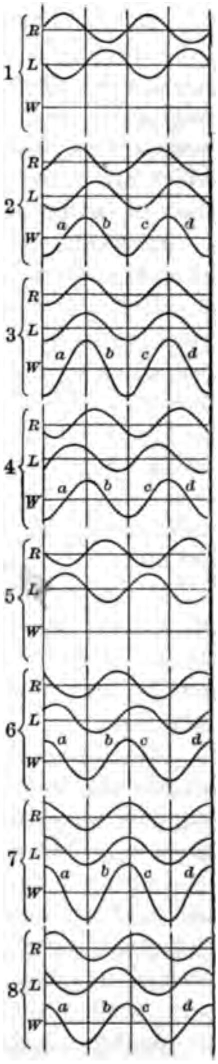

FIG. 147.

so that the eight curves indicate the positions of the tube at the end of successive eighths of a period.

Throughout the above reasoning transverse waves have been considered. Proceeding in a similar manner, it can be shown that if longitudinal wave motions of the same period, amplitude, and speed, meet when moving in opposite directions, all points of the resultant wave pass through their positions of equilibrium at the same instant; and that there are points a half wave-length apart which are never displaced and each of which is alternately in a state of compression and of rarefaction.

An oscillatory motion such that every portion of the medium affected goes through its equilibrium condition at the same instant is called a *standing wave*, or *stationary undulation*. A standing wave results from the composition of two waves of the same type, period, and amplitude, moving along the same path in opposite directions. The points, a half wave-length apart, which remain always at rest, are called *nodes*. The region between two nodes is called a *loop*, or *ventral segment*. The middle point of a ventral segment, i.e., the point midway between two successive nodes, is called an *antinode*. The maximum displacements occur at the antinodes.

At a node where the speed of the vibrating medium is a minimum, there is the maximum change of tension, pressure or density. At an antinode where the changes of tension, pressure or density are minimum, there occurs the greatest amplitude of vibration.

It is to be noticed that, since the two component wave motions are conveying energy at the same rate in opposite directions a standing wave effects no continuous transfer of energy from one place to another.

In the experiment adduced to illustrate interference (Fig. 144) the point of intersection of the rubber tubes remains at rest. At this point waves from A and A' are reflected. The motion of each of the tubes AB and $A'B$ is the resultant of two waves of the same type, period, and amplitude, moving along the same path in opposite directions. That is, the motion of each is a stationary undulation. There is no flow of energy into the tube BC.

155. Polarization.—Imagine a long horizontal rubber tube, one end fastened to the wall and the other end held in the hand. If the free end be moved back and forth in any straight line perpendicular to the length of the tube, each small part of the tube will, in succession, move back and forth in a direction parallel to the initial displacement. A wave motion in which the periodic motions are in straight lines perpendicular to the direction of propagation of the disturbance, is called a *plane polarized* wave motion.

If, however, the free end of the tube be moved in the circumference of a circle, a neighboring small part of the tube will be caused to move a little later in a similar manner; this will cause the next part to move in the same way, and so on. Thus, there will advance along the tube a wave motion in which each small part of the tube moves in the circumference of a circle. A wave motion in which the periodic motions are in circular paths in planes perpendicular to the direction of propagation of the disturbance, is called a *circularly polarized* wave motion. We can also produce elliptically polarized wave motion.

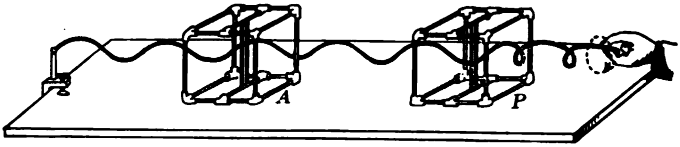

Fig. 149.

If an observer standing at one end of the tube looked along its axis and saw the tube as a straight line, a circle, or an ellipse, and knew that it was vibrating in some manner, he would be certain that the vibrations were transverse to the length of the tube. If, however, the tube were vibrating, but to the observer appeared as a point, he would be certain that the vibrations were in the direction of the length of the tube, that is, were longitudinal.

Let the tube just considered pass through two devices (Fig. 149), each of which will allow the tube to move freely in the direction of its length, and in one direction perpendicular to its length. Let the apparatus *P* be so turned that the only transverse waves which it will transmit are those having the vibrations in a vertical plane. Suppose that the right end of the tube be given a periodic motion in a vertical plane. For instance, suppose that it be moved in the circumference of a vertical circle in a plane normal to the length of the tube. The tube will be given a motion which will advance to *P* in the form of a helix. But since within *P* the tube can move only in a vertical line, the wave emerging from *P* will be the vertical component of the incident wave. It will be a wave plane polarized in the vertical plane. Any device that will produce plane polarized waves is called a *polarizer*.

If the apparatus *A* be so placed that the only transverse waves which it can transmit are those having the vibrations parallel to those transmitted by *P*, then the waves transmitted by *P* will also be transmitted by *A*, Fig. 149. If now, either *P* or *A* be rotated 90° about the length of the tube as an axis, Fig. 150, then the plane polarized waves transmitted by *P* cannot pass through *A*. But if the wave incident on *A* were not plane polarized, a component would be transmitted. Thus the device *A* serves to distinguish plane polarized from unpolarized waves. A device which serves to detect plane polarized waves is called an *analyzer*. An apparatus that can be used as a polarizer can be used as an analyzer, and conversely.

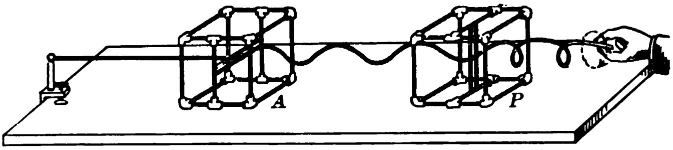

FIG. 150.

If the original wave had been longitudinal, that is, if the vibrations had been in the direction of the tube, then with the polarizer and the analyzer in either position above considered, the wave would have been transmitted by *P* and also by *A*.

By means of a polarizer with an analyzer we can distinguish a longitudinal wave from a transverse wave. If a given wave motion is transmitted by two devices when arranged in a certain position relative to one another, is not transmitted when one of them is rotated 90°, is again transmitted when the rotation is increased to 180°, and is not transmitted when the rotation is increased to 270°, then the given wave motion must be transverse. This phenomenon of polarization is the distinguishing characteristic of transverse wave motion.

Ordinary light passes readily through a slice of tourmaline crystal in any direction. Suppose two slices are cut parallel to the axis of the crystal. It is found that ordinary light passes through these slices separately, and also passes through the two when placed one behind the other with their axes parallel as in *A* and *B*, Fig. 151. If, however, the two slices are placed one behind the other with the axis of one at 90° with the axis of the other (Fig. 151 *C*), no light will be transmitted. This indicates that light waves do not have the same properties in all directions about the line of propagation. Hence, the vibrations constituting light are not in the direction of the propagation of the wave. Therefore, light is a transverse wave motion. It is needless to say that the mechanism by which polarization is produced in tourmaline is not at all analogous to the slots in the frames, Fig. 150, used in the study of transverse waves in cords.

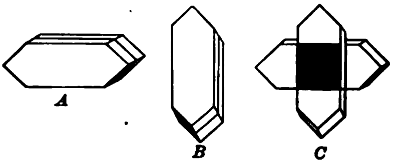

FIG. 151.

156. Isotropic and Anisotropic Media.—Any portion of a uniform specimen of glass or of quartz is similar in all respects to any other portion of the same specimen so long as the portion considered is larger than molecular dimensions. That is, uniform glass is homogeneous and uniform quartz is homogeneous. But glass differs from quartz in that light, heat and mechanical vibrations are transmitted with the same speed in all directions through uniform glass, whereas light, heat and mechanical vibrations are transmitted with unequal speed in different directions through uniform quartz.

A medium which at any point has the same properties in all directions is said to be *isotropic*. A medium which at any given point has different properties in different directions is said to be *anisotropic* or *aeolotropic*. Under ordinary conditions liquids and gases are isotropic, while crystals are anisotropic. Annealed glass is isotropic, but unannealed strained glass is anisotropic.

If a point in an isotropic medium be set into periodic motion, a wave will be produced which advances from the point in all directions with the same speed. The surface passing through all adjacent particles which are in the same phase of vibration is a sphere. The surface passing through all adjacent points at which the phase of the vibration is the same is called a *wave front.* The direction in which a wave is advancing is called a ray. In the case of a wave from a point source in an isotropic medium, the rays are perpendicular to the wave fronts.

If a point in an anisotropic medium be set into periodic motion, the wave produced will not be spherical. In anisotropic media, wave fronts in general are not perpendicular to the rays.

157. Construction of Wave Fronts.—Consider the wave motion due to a periodic disturbance at the point S in an isotropic medium of indefinite extent. If a wave front meets a screen AB, Fig. 152, containing an opening, a portion of the wave will be prevented from advancing beyond the screen. A portion *abc* of the advancing wave front will traverse the aperture and at some later instant will be in the position $a'b'c'$. The form $a'b'c'$ is determined solely by the form and position of the part *abc*, and would be the same even though the periodic disturbance at S had ceased after originating the wave. Consequently, the new wave front $a'b'c'$ is due solely to the position *abc* of the previous wave front, and the energy in *abc*.

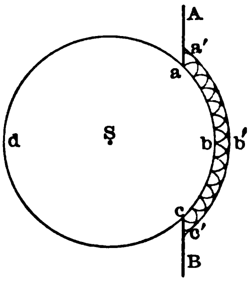

FIG. 152.

These considerations lead to the conclusion that every point in a wave front traversing an isotropic medium is a center of disturbance from which spreads a spherical wave. This result is called Huyghens' Principle.

For example, in the above figure consider the disturbance sent out by each point of the wave front *abc*. At some given instant the spherical wave fronts produced by the disturbances at these points will have reached the positions indicated by the little circles. On the surface $a'b'c'$ tangent to all of these wave fronts the disturbance is very great, and it can be shown that at points back of this enveloping surface the waves from the different points of *abc* interfere destructively. Consequently, at any instant the wave front of a disturbance is the envelope of all the secondary wave surfaces which are due to the action as separate sources of all the points that at some previous instant constituted the wave front.

This method of determining the form of a wave front at any point after some previous position is called Huyghens' Construction. For example, wave fronts of different forms, *abc*, Fig. 153, 154, and 155, advancing to the right in an isotropic medium, would at some later instant have the forms $a'b'c'$ as shown.

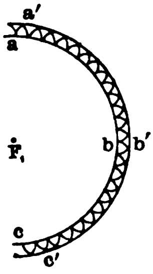

FIG. 153.

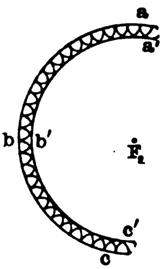

FIG. 154.

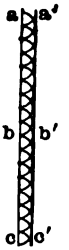

FIG. 155.

A wave which has a front of continually increasing radius, as in Fig. 153, is called a *diverging* wave. A wave which has a front of decreasing radius, as in Fig. 154, is called a *converging* wave. A *plane wave*, Fig. 155, may be considered to have a wave front of infinite radius.

The point from which a wave diverges (F_1, Fig. 153), or to which it converges (F_2, Fig. 154), is called the *focus* of the disturbance, or the *center* of the wave.

In the above discussion waves in isotropic media only have been considered. But Huyghens' Construction can be applied equally well to waves in anisotropic media. Since in this case the disturbance travels in different directions through the medium with

different speeds, a wave originating at a point source will not have a spherical front. Examples of the construction of wave fronts in anisotropic media will be considered in a later Article.

When a wave passes through an opening, the emergent wave front will extend but a slight distance into the geometric shadow of the sides of the opening if the wave-length is short, Fig. 156.

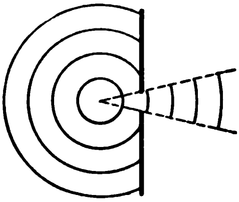

Fig. 156.

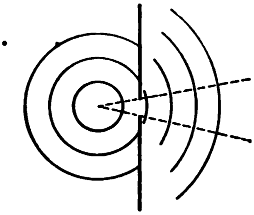

Fig. 157.

But if the wave-length is long, the wave will extend for a considerable distance into the geometric shadow, Fig. 157.

Similarly, in the case of an obstacle, there will be a distinct shadow if the wave-length of the disturbance is short, Fig. 158, whereas there will be very little shadow if the wave-length is long, Fig. 159. Light gives distinct shadows, whereas sound produces indistinct shadows.

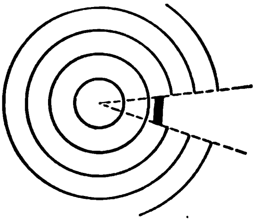

Fig. 158.

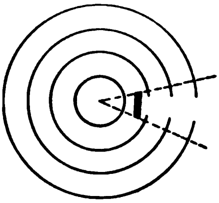

Fig. 159.

158. The Laws of Reflection from Mirrors.—If a wave be incident on a rough surface, the incident wave will be broken up and scattered in all directions. If, however, the distance between the elevations of the surface be less than a quarter wave-length, the incident wave will be reflected without scattering. A surface which reflects waves without scattering is called a *mirror*.

Consider a wave diverging from a point source S and incident on a plane mirror MM', Fig. 160. It will be assumed that the medium surrounding the mirror is air or some other isotropic substance. In this case the wave front of the wave from S will be spherical. If the mirror had been absent, the wave front would at some instant occupy the position MQM'. With the mirror in place, each element of the mirror struck by the wave becomes a center of disturbance from which energy is propagated in every direction. Energy that is propagated back into the first medium is said to be reflected; the energy that is propagated through the second medium—in this case the substance constituting the mirror—is said to be transmitted; while the energy that at the mirror is transformed into heat is said to be absorbed. In general, when a wave is incident upon a mirror some of the energy is reflected, some is transmitted, and the remainder is absorbed.

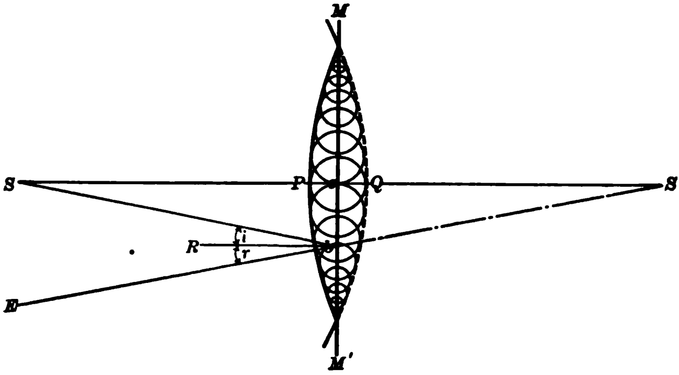

FIG. 160.

If the mirror were absent, at a certain time after the wave reached the point c the disturbance would have progressed a distance cQ. But the mirror being present, energy is turned back into the first medium, and at the same speed with which it came toward the mirror. Consequently, the reflected wave front is somewhere on a sphere of radius cQ, which has c as a center. It will also be seen that the reflected wave front touches every sphere tangent to the surface MQM' that can be described about centers on the mirror surface. From Huyghens' Construction the envelope of all these secondary spherical surfaces constitutes the reflected wave front MPM'.

The above construction shows that the incident wave front MQM' and the reflected wave front MPM' have equal radii of curvature, and that the line SS' joining the centers of curvature of the incident and reflected waves is normal to the mirror. Consequently, a plane mirror reverses the direction of the curvature of the incident wave without altering the amount of the curvature.

Since the reflected wave appears to originate at S', this point is called the *virtual source* or *virtual focus* of the wave.

The relation between the magnitude of the angle of reflection and the angle of incidence will now be obtained. In Fig. 160, consider the wave that travels from the source S to the mirror along any line Sb. From b draw the line bE normal to the reflected wave front. This line being the path of the wave reflected at b, is called the reflected ray at the point b. The angle SbR between the incident ray and the normal bR to the mirror is called the *angle of incidence.* Similarly, the angle EbR between the reflected ray and the normal to the mirror is called the *angle of reflection.*

In the triangles Scb and $S'cb$, cb is common, $S'c=Sc$ and SS' is perpendicular to MM'. Therefore the triangles Scb and $S'cb$ are concurrent. From the figure, the angles

$$bS'c=bSc,$$

and the alternate interior angles

$$bSc=SbR.$$

Therefore,

$$EbR=SbR.$$

That is, *the angle of reflection equals the angle of incidence.*

It will now be shown that the reflected ray, the incident ray, and the normal to the mirror at the point where reflection occurs, lie in the same plane. Since S was chosen in the plane of the paper, and since in an isotropic medium the wave front is normal to the ray, it follows that along an intersection, MQM', of the paper and the incident wave front, the wave front is perpendicular to the plane of the paper. Again, since when the plane of the mirror is normal to the plane of the paper, SS' is in the plane of the paper, it follows that the point S' is in the plane of the paper. Again, since S' is in the plane of the paper, and since in an isotropic medium the wave front is normal to the ray, it follows that the reflected wave front where it intersects the page is also perpendicular to the plane of the paper. Whence, the reflected ray lies in the plane of the paper. Consequently, *the reflected ray, the incident ray, and the normal to the mirror at the point where reflection occurs, lie in the same plane.*

159. Change in the Form of a Wave Front Produced by Reflection from a Curved Surface.—The mirrors in most common use have either plane or spherical surfaces. The center C of the spherical surface MPM_1, Fig. 161, is called the *center of curvature* of the mirror. The middle point P of the reflecting surface is called the *pole* of the mirror. The right line CP joining the center of curvature and the pole is called the *principal axis* of the mirror. The diameter MM_1 of the circular boundary of the mirror is called the *linear aperture.*

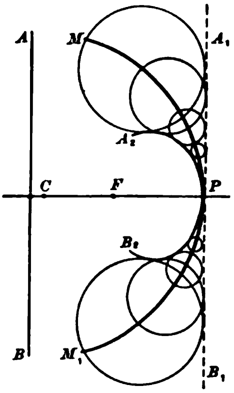

FIG. 161.

First, consider the reflection of a plane wave from a concave spherical mirror. By Huyghens' Construction (Art. 157), the reflected wave front is determined by considering the disturbance set up at each point of the mirror that is struck by the incident wave. If the mirror were absent, then at a given instant the wave front would have reached the plane A_1PB_1. But the mirror being present, the original wave is interrupted, and each point of the mirror that is struck by the wave becomes a center of disturbance and originates a secondary spherical wave which goes back into the first medium with the same speed with which the original wave approached the mirror. Consequently, if with each point of the mirror as a center, spheres be constructed tangent to the plane A_1PB_1, the tangent surface A_2PB_2 enveloping these spheres will be the reflected wave front at the given instant.

In general, when a spherical wave is incident on a spherical surface (a plane is a sphere of infinite radius), the reflected wave will not be spherical. When a spherical wave is incident on a spherical mirror, the deviation from a spherical form of the reflected wave is called *spherical aberration.*

The portion of the wave reflected from the mirror near the pole is approximately spherical. Thus, if the aperture of the mirror is small compared with the radius of curvature of the mirror, the reflected wave front is nearly spherical and advances with diminishing radius of curvature until the entire wave front shrinks to nearly point dimensions at F. In all of the following consideration of curved mirrors, small apertures will be assumed.

A converging spherical wave will shrink to point dimensions. The point to which a plane wave that is advancing toward a concave mirror parallel to the principal axis converges after reflection is called the *principal focus* of the mirror. The distance of this point from the pole of the mirror is called the *principal focal length* of the mirror.

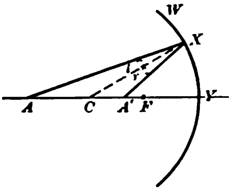

FIG. 162.

160. The Position of the Principal Focus of a Spherical Mirror.—Let A, Fig. 162, be a point source on the principal axis of a concave mirror having the center of curvature at C. The wave traveling along the ray AX will be reflected along the path XA'. Since $CXA' = AXC$, we have,*

$$\frac{AC}{CA'} = \frac{AX}{A'X}.$$

---

* In any triangle, the bisector of an angle divides the opposite side into segments proportional to the adjacent sides.

If the angle XCY is small, then $AX = AY$ (nearly), and $A'X = A'Y$ (nearly). In this case, the above equation may be written,

$$\frac{AC}{CA'} \doteq \frac{AY}{A'Y}.$$

Putting this equation into the form

$$\frac{CA'}{A'Y} \doteq \frac{AC}{AY},$$

we see that so long as the angular aperture WAY is small, the position of the point A' depends only upon the position of the source and the curvature of the mirror. That is, under this condition, all the light from A that strikes the mirror will be reflected to the point A'. Consequently, A' is the focus of the light from A.

It will be convenient to represent the radius of curvature of the mirror by r, the distance of the source A from the pole Y by u, and the distance of the focus A' from the pole by v. Using this notation, the last equation assumes the form

$$\frac{r-v}{v} \doteq \frac{u-r}{u},$$

or

$$ur - uv \doteq uv - rv.$$

On dividing each term by uvr,

$$\frac{2}{r} \doteq \frac{1}{u} + \frac{1}{v}. \qquad \qquad (111)$$

This equation gives the relation between the distances of the source and of the focus from a concave mirror of small angular aperture in terms of the radius of curvature of the mirror. If the source be at infinity, that is, if $u\ (=AY) = \infty$, the above equation becomes

$$\frac{2}{r} \doteq \frac{1}{v},$$

$$v \doteq \frac{r}{2}. \qquad \qquad (112)$$

That is, the principal focus of a concave spherical mirror of small angular aperture is midway between the center of curvature and the pole of the mirror. The principal focal length is one-half the radius of curvature.

161. The Parabolic Mirror.—It is shown in Analytic Geometry that "the normal at any point P on a parabola bisects the angle between the focal radius FP and the line through P parallel to the axis of the curve." From this property, together with the laws of reflection, it follows that a wave from a point source at the focus of a parabola will, after reflection, emerge as a cylindrical beam parallel to the axis of the parabola. For this reason, a small intense light source at the focus of a parabolic mirror constitutes a simple and effective lantern or head light.

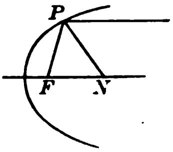

FIG. 163.

If the point source be between the pole of the mirror and the focus, the reflected beam will be divergent; if it be farther from the pole than the focus, the reflected beam will be convergent.

162. Change in the Form of a Wave Front Produced by Refraction at a Plane Surface.—Imagine a wave originating at a point S, Fig. 164, to pass from one isotropic transparent medium to another in which the velocity is greater than in the first. Every point of the plane surface NN' separating the two media that is struck by the wave will be a new center of disturbance. Suppose that in the second medium the velocity of the wave is 1.5 times as great as in the first medium. If with the second medium absent the wave front at some given instant were NdN', the actual wave front in the second medium could be constructed as follows: With various points a, c, e, etc., of the interface as centers, construct a number of spheres of radii 1.5 (ab), 1.5 (cd), 1.5 (ef), etc. The surface NPN' enveloping these spheres is the wave front in the second

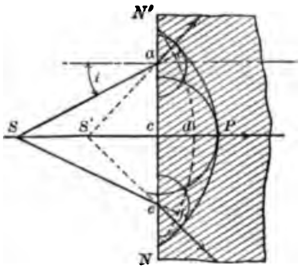

FIG. 164.

medium. If the distance NN' is small compared with SN, it can be proved that the new wave front is nearly spherical in form and that the wave in the second medium advances as though it originated at a point S'. The real source S and the virtual source S' are on a line normal to the surface separating the two media. Consequently, a wave traveling from the first medium to the second in any direction except the one normal to the interface will be bent out of its original direction at the interface separating the two media. The phenomenon of the breaking or bending of a ray at the surface separating two media in which the wave travels with different speeds is called *refraction.*

When a wave passes obliquely from a medium in which the speed is less to a medium in which the speed is greater, the ray is bent away from the normal to the surface separating the two media.

By proceeding in exactly the same manner as above, it can be shown by means of Fig. 165 that when a wave passes from a point source in a transparent isotropic medium to another in which the speed is less, the two media being separated by a plane surface, the wave front in the second medium is nearly spherical in form and advances as though it originated at a point S' at a greater distance from the interface than the real source S. Also, when a wave passes obliquely from a medium in which the speed is greater to a medium in which the speed is less, the ray is bent toward the normal to the surface separating the two media.

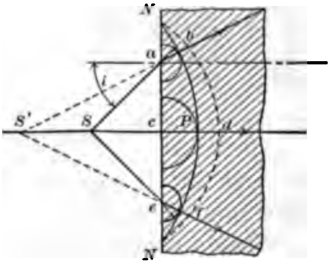

FIG. 165

163. Change of Wave Front Produced by a Convex Lens.—A lens that is thicker at the center than at the edges is called a *convex lens,* while one that is thinner at the center than at the edges is called a *concave lens.* The line joining the centers of curvature of the two faces of a lens is called the *principal axis* of the lens. The points where the principal axis intersects the faces of a lens are called the *poles* of the lens. The diameter of the uncovered part of a lens is called the *aperture* of the lens.

Consider the change in the form of a wave front produced by passage through a convex lens. In Fig. 166 is represented a plane wave AB proceeding toward a convex lens parallel to the principal axis. To simplify the present construction, the first face of the lens is taken to be plane. On entering the first surface the speed of the wave is changed, but not the form of the wave front. To fix the ideas assume that the lens be of glass and that the speed of the wave in glass is two-thirds the speed in the surrounding air. If the glass extended indefinitely to the right, then at some instant the wave front in the glass would be A_1B_1. But when the glass is bounded by the surface represented in the figure, we find by Huyghens' Construction, that at this instant the wave front is A_2PB_2.

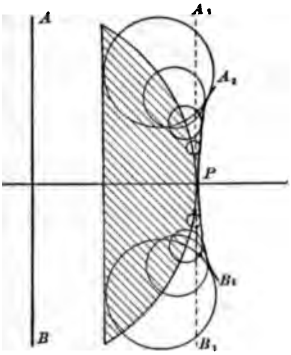

Fig. 166.

When a spherical wave is incident on a lens of large aperture bounded by spherical surfaces (a plane is a sphere of infinite radius), the emergent wave will in general not be spherical. If, when a spherical wave is incident on a lens, the emergent wave front is not spherical, the deviation from the spherical form of the emergent wave is called *spherical aberration by refraction.*

If the incident wave is spherical and proceeds parallel to the principal axis of the lens, and if the lens is covered, except a small area about the pole, the emergent wave will be practically spherical. That is, the spherical aberration will be negligible. In the remaining diagrams of the present article, spherical aberration is neglected.

The change in the form of a wave front produced by passage through a convex lens made of a material in which the speed of the wave is less than in the surrounding medium may be illustrated by the four following diagrams. A point toward which a wave converges is called a *real* focus; a point from which a wave diverges is called a *virtual* focus. In Fig. 167 a plane wave advancing parallel to the principal axis converges after transmission to the real focus F. The point to which a plane wave advancing parallel to the principal axis converges after emergence, or from which it diverges after emergence, is called the *principal focus* of the lens or system of lenses. · A lens has a principal focus on each side.

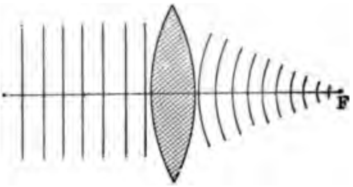

FIG. 167.

Fig. 168 illustrates the fact that if the wave originates at a point S farther from the lens than the principal focus, the emergent wave will converge to a focus F_1, beyond the other principal focus.

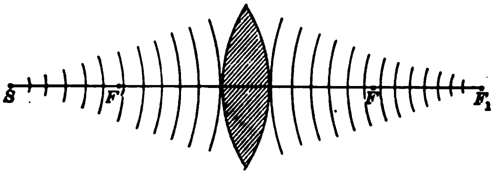

FIG. 168.

Fig. 169 illustrates the converse of Fig. 167, namely, that a wave originating at a principal focus will after transmission by a convex lens be a plane wave.

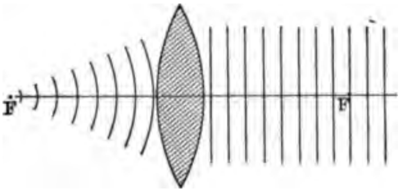

FIG. 169.

A wave originating at a point between a convex spherical lens and a principal focus, Fig. 170, will after transmission advance as though it had originated at a point S' beyond the principal focus.

There are also lenses having faces which are surfaces of two cylinders with parallel axes. A convex cylindrical lens converges a plane or spherical wave to a line focus. By means of a convex cylindrical lens a wave that is converging toward a line focus can be brought to convergence at a point focus. This is the purpose for which cylindrical lenses are most often used. An astigmatic eye causes light from points of an object to converge to line foci. This error is corrected by eyeglasses consisting of cylindrical lenses (Art. 419).

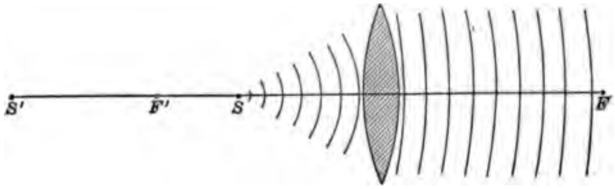

Fig. 170.

164. Change of Wave Front Produced by a Concave Lens.—It is left as an exercise for the student to construct by Huyghens' method the wave front emerging from a concave lens.

A plane wave advancing along the principal axis of a concave lens made of a material in which the speed of the wave is less than in the surrounding medium will, after transmission, become a diverging wave that appears to have originated at a point on the principal axis. This virtual source, *F*, Fig. 171, is one of the principal foci of the concave lens.

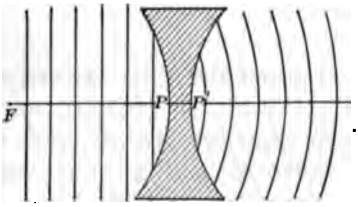

Fig. 171.

Lenses, like mirrors, imprint on a wave a new curvature.

It is left as an exercise for the student to construct by Huyghens' method the wave front emerging from convex and concave lenses made of a material in which the speed of the wave is greater than in the surrounding medium.

165. The Laws of Refraction.—The amount that a wave is deviated out of its course when it passes from one isotropic medium to another will now be determined. Let *DG*, Fig. 172, be the front of a plane wave advancing toward the plane NN_1, separating two transparent isotropic media. As each point of the advancing wave strikes the interface separating the two media a disturbance is there set up which is transmitted back through the first medium and also one that is transmitted forward through the second medium. The reflected wave has been considered in Art. 158. The wave transmitted through the second medium will now be considered.

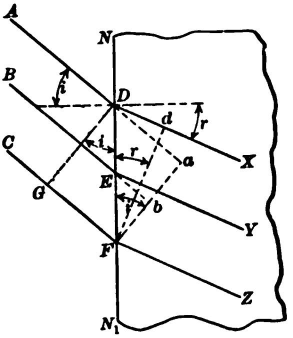

Fig. 172.

On entering the second medium, the wave that has traveled along *AD* will be bent out of this ray. If the velocity be less in the second than in the first medium, the ray in the second medium will be bent toward the normal to the surface separating the two media. If the interface between the two media be plane, all rays in the first medium parallel to *AD* will, at this interface, be bent toward the normal by the same amount. That is, rays that are parallel before incidence on the interface will be parallel after refraction. Since the interface is plane and the velocity in the second medium is uniform, the wave front in the second medium will be plane and normal to the refracted rays.

The angle between the incident ray *AD* and the normal to the surface separating the two media is called the *angle of incidence.* It will be seen that the angle of incidence equals the angle between the incident wave front, *DG*, and the surface separating the two media. The angle between the refracted ray *DX* and the normal to the surface separating the two media is called the *angle of refraction.* It will be seen that the angle of refraction equals the angle between the refracted wave front *dF* and the surface separating the two media.

The ratio of the speeds of a wave in two media is called the *relative index of refraction* of the two media for the particular wave. The *absolute index of refraction* of a medium is the ratio of the speed of the wave in vacuum to the speed in the given medium. If the speed in the first medium be represented by v_1, and the speed in the second medium by v_2, then from definition, the index of refraction of the second medium relative to the first is

$$\mu = \frac{v_1}{v_2}. \qquad (113)$$

Now since the wave travels in the first medium the distance GF $(=Da)$ during the time t, it follows that

$$Da = v_1 t.$$

During this same time the wave travels from D into the second medium a distance Dd such that

$$Dd = v_2 t.$$

Substituting these values of v_1 and v_2 in (113) we obtain

$$\mu = \frac{Da}{Dd}.$$

But from the figure, $Da = DF \sin i$, and $Dd = DF \sin r$. Therefore, the index of refraction

$$\mu = \frac{\sin i}{\sin r}. \qquad (114)$$

Consequently, *the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction.* This is called Snell's Law of Refraction.

It can also be shown that *when both substances are isotropic, the refracted ray, the incident ray, and the normal to the refracting surface at the point of incidence, lie in the same plane.*

The index of refraction of a substance is different for waves of different wave-lengths.

SOUND

CHAPTER XII

THE NATURE OF SOUND

166. Sound is Propagated by Matter.—The word "sound" is used in two different senses. In Physics the word refers to the form of energy that is capable of producing the sensation of hearing. In Physiology and Psychology it is often employed to indicate the sensation itself.

If a bell be struck under water, a person with his ears under water will hear the sound, even at a considerable distance. If the bell and the person be in the air, the same is true. All kinds of matter propagate sound. But if the bell be within a jar devoid of matter, no sound will be heard. Sound requires matter for its propagation.

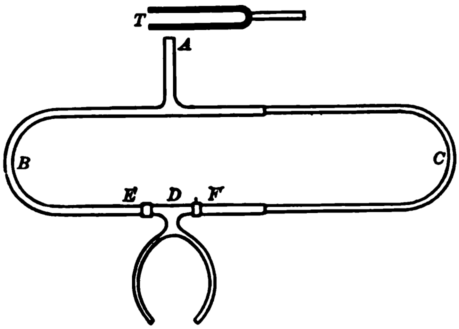

Fig. 173.

167. Sound is a Wave Motion.—A tube *A* (Fig. 173), divides into two branches *B* and *C* which reunite at *D*. The length of the branch *ACD* can be altered by drawing in or out a sliding section similar to the slide of a trombone. Attached to *D* are two flexible tubes provided with tips that can be placed in the ears. At *E* and *F* are sliding doors which can close the tubes at those points.

If a sounding tuning fork *T* be placed at *A*, the sound heard at the ear pieces will be the sum of the sound traveling by the paths *ABD* and *ACD*. By adjusting the sliding section, a series of positions can be found such that no sound is heard, while with the slide in all intermediate positions sound is perceived. If, when the slide is in such a position that no sound is heard, either of the branches is closed by one of the sliding doors *E* or *F*, sound is heard clearly. This experiment shows that although sound travels to *D* along two different paths, the resultant sound at *D* is zero. That is, sounds from the two branches interfere destructively at *D*. This shows that sound is a wave motion. (Art. 153.)

In order that the waves may continuously interfere they must have the same wave-length and the crests of one wave must be superposed on the troughs of the other. This latter requirement means that one wave must be in advance of the other one-half a wave-length, or some odd number of half wave-lengths. Thus, in the above experiment, when destructive interference occurs, the difference in length of *ABD* and *ACD* is either one-half the wave-length of the sound produced by the tuning fork, or some odd multiple of this half wave-length. This furnishes a method of determining the wave-length of sound. In air at ordinary atmospheric temperature, the wave-length of sound of the pitch called " middle C " is about 1.3 meters.

168. Sound Waves are Longitudinal.—Sound is propagated by solids, liquids and gases. In a gas the only change that develops restoring forces is change of volume. Shearing stresses cannot exist in a gas. It follows that transverse waves cannot be propagated by a gas. Consequently, the only waves that can be propagated by a gas are longitudinal waves of compression and rarefaction.

Sound is a longitudinal wave motion consisting of alternating compressions and rarefactions. The distance between any point of a wave and the next point that is in the same phase is the wave-length. For instance, the distance between two succeeding maxima of compression is a wave-length.

169. Pressure Curves.—Suppose that a sound wave is advancing in the direction *AB*, Fig. 174. At various points on this line erect perpendiculars of a length proportional to the pressures at those points at a given moment. Let distances above the axis represent compression and distances below rarefaction. The curve drawn through the end points of these ordinates is called the pressure curve of the given sound wave. If the sounding body is vibrating with simple harmonic motion, the pressure curve will be a harmonic curve and the sound is said to be a *harmonic wave.*

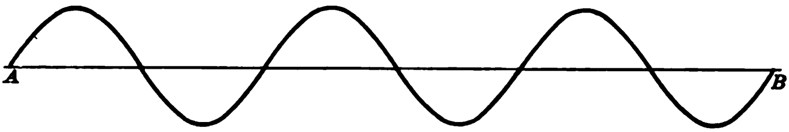

FIG. 174.

In the same manner we can construct displacement curves and velocity curves. Any one of these three curves may be called the *wave-form.*

170. Superposition of Sound Waves.—If two waves of the same kind traverse the same region, the motion of the medium will be the resultant of the two component waves. If in both components,

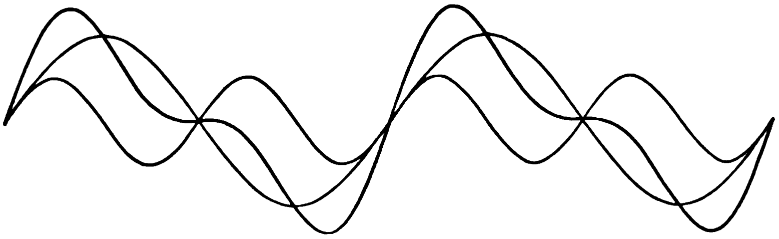

FIG. 175.

displacements of the particles from their equilibrium position are proportional to the restoring force, the ordinate at any point of the resultant pressure curve, displacement curve, or velocity curve, equals the sum of the ordinates at the given point of the corresponding curves of the component waves. This is called the Principle of Superposition of Waves.

For example, the heavy line in Fig. 175 represents the pressure curve which is the resultant of the two component pressure curves represented by the light lines.

171. Resonating Air Cavities.—If a sounding tuning fork be placed in succession over the mouths of a number of empty bottles of different shapes and sizes, some bottles will be found that will strongly reinforce the sound of the tuning fork. By pouring water into one of the larger bottles which when empty did not produce reinforcement, it can be caused to reinforce the sound of a given fork. If this bottle be now tilted so that the shape of the empty part is changed the reinforcement will not be altered. If the mouth be partly closed, the reinforcement will cease. It thus appears that reinforcement depends not upon the shape but upon the volume of the cavity and upon the size of the opening into it.

The cause of the reinforcement when the area of the opening is small and the volume of the cavity is large will now be considered. A pulse of air entering the cavity increases the pressure of the air inside. This increased pressure causes an outward flow. The inertia of the outgoing air causes the flow to continue until the pressure without the cavity is less than normal. Another inward flow is set up, and the same series of movements is repeated. The time of one cycle of movements, that is, the period of the vibration, depends upon the volume of the cavity and upon the ease with which the air passes in and out of the aperture. If the period of the sound in front of the aperture is nearly equal to the natural period of vibration of the air within the cavity, resonance will be produced (Art. 144). This resonance is the cause of the reinforcement of the sound. The pitch to which a cavity responds is raised by either decreasing the volume of the cavity or increasing the size of the aperture.

The case considered in this Article is that in which a mass of air within a cavity is alternately compressed and rarefied with very little motion of the air particles. It is quite different when motion is imparted to the particles of a column of air. In Art. 188 it is shown that the period of such a column of air depends upon the length of the column.

Helmholtz analyzed complex sounds by means of resonators.

He used resonators of fixed volume, *A*, Fig. 176, having a definite known frequency, and also others of variable volume, *B*, Fig. 176, whose frequency could be adjusted. To show when a resonator responded he attached to each a " manometric capsule," *C*, consisting of a small box divided into two compartments by a thin rubber partition. Illuminating gas enters the compartment not connected to the resonator and escapes from a small burner. When the air within a resonator is set into vibration, the diaphragm moves back and forth, and the gas flame jumps up and down. The vibration of the gas flame may be rendered evident by means of a slowly rotating four-sided mirror *M*. If the mirror be rotated when the gas flame is stationary, a streak of light is seen in the mirror: when the flame is vibrating, a row of saw teeth is seen. By substituting for the rotating mirror a camera with moving film, a permanent record can be obtained and the frequency of the resonator determined.

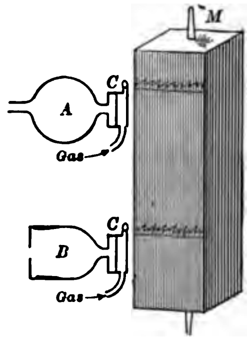

Fig. 176.

In using this method for determining the frequencies of the components of a complex sound, the sound is produced in front of a large number of resonators of different periods. The periods of the resonators which are set into vibration are the periods of the component tones of the complex note.

172. Characteristics of Sound.—Suppose a card be held against the edge of a revolving toothed wheel. The card will vibrate as many times as it is struck by a tooth of the wheel. When struck less frequently than about 30 times per second, one hears each separate blow. When struck more frequently than this number, one hears a continuous note. If the speed of the revolving wheel be gradually increased, thereby increasing the frequency of vibration of the card, the pitch of the sound will become higher and higher. Whatever the source of the sound, it is found that for the same pitch the sounding body makes the same number of vibrations per second. Whence, in sound waves, pitch corresponds to frequency of vibration. When the frequency exceeds about 20,000 vibrations per second, no impression is produced on the human ear.

Toothed wheels may be used having either small or large spaces between the teeth. But so long as the card is struck the same number of times per second, the pitch of the sound will be the same. It is quite otherwise, however, with the loudness. When the amplitude of vibration of the card is great, the loudness will be greater than when the amplitude of vibration is small. This illustrates the fact that the loudness of sound is a function of the amplitude of vibration. It can be shown, though the proof will not here be given, that loudness, or sound intensity, is determined by the square of the amplitude of the sound wave. It might here be mentioned that it is found that the intensity of the impression produced on the organ of hearing is not proportional to the loudness, but to the natural logarithm of the loudness.

It is a matter of common observation that notes even of the same pitch and loudness produced by different sorts of instruments do not produce the same impression on the ear. That characteristic of sound which causes notes of even the same pitch and loudness to produce different impressions on the ear is called *quality*. By means of resonators Helmholtz found that most sounds are not due to pure harmonic wave motions, but are due to the resultant of several harmonic waves of different pitch and loudness; and that two sounds of different quality consist of different components. By setting into simultaneous vibration tuning forks having the frequencies of the resonators which respond to a given sound, he found that the given sound could be matched in pitch, loudness and quality. Hence, quality is a matter of the frequencies and amplitudes of the components of the sound.

Each of the components of a complex note is called a *tone*. The wave or tone of lowest pitch is called the fundamental. The accompanying waves or tones of high pitch are called *overtones*. Overtones whose frequencies are exact multiples of the fundamental are called *harmonics*. The overtones produced by musical instruments are harmonics. The difference between a note produced by a piano and the same note produced by a cornet is due to the difference in the pitches and loudness of the various harmonics accompanying the dominant fundamental tone. In the case of a note composed of the small number of tones that are used in music, a competent ear is able to distinguish the separate tones, that is, to analyze the complex note.

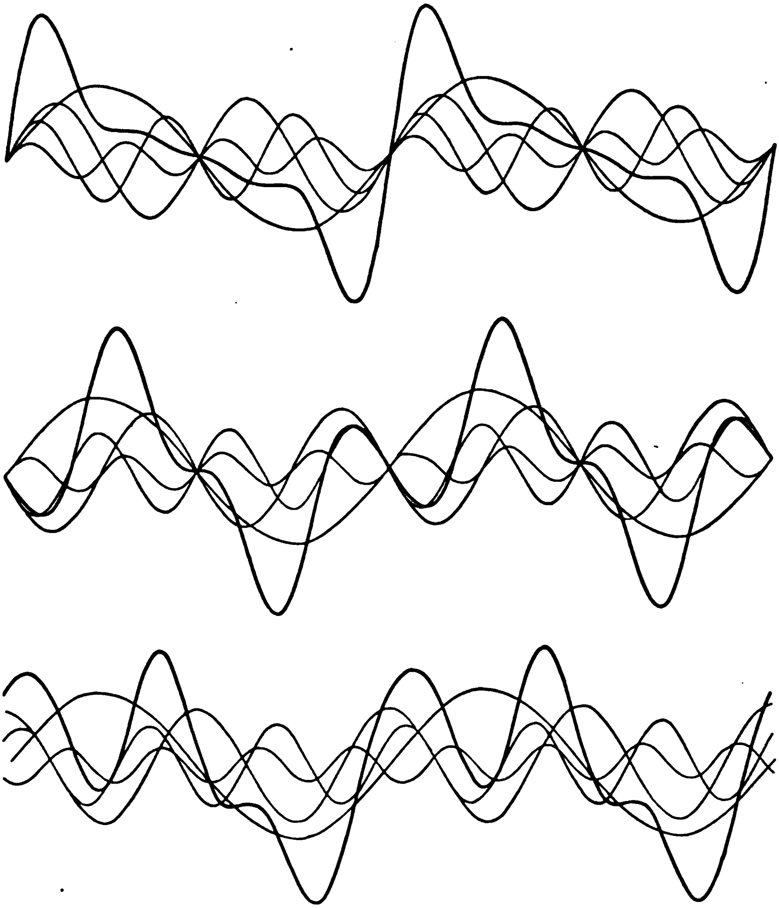

FIG. 177.

Each of the curves in Fig. 177 is the resultant of the same three harmonic curves. The difference in form is due solely to the phase difference between the components. Notes represented by these three resultant curves produce the same impression on the ear. Quality is not a matter of wave form but of the particular components in the wave.

To sum up: Pitch is determined by the frequency of the fundamental wave; loudness by the square of the amplitude of the resultant wave; quality by the waves superposed on the fundamental.

173. Determination of Sound Direction by Binaural Hearing.—A sound either directly in front of or directly behind a person produces a different impression than a sound to one side. This is due to the fact that in the latter case the sound reaches the two ears in different phases, and also to the fact that the loudness is slightly different at the two ears on account of the head partially screening one ear. Our estimation of the direction of sound depends partly upon the differences in the phase and partly upon the relative loudness of the sounds entering the two ears.

One method for warning ships against danger of rocks and collision depends upon these facts. Attached to the inside of the hull, below the water line, are two iron boxes, one on either side of the bow, Fig. 178. Each box is filled with water and contains a telephone transmitter connected to a separate receiver on the officer's bridge. When the ship is pointing toward a submerged sounding bell an observer with the two receivers at his ears will perceive equal sounds. When the ship is pointing to one side of the submerged bell, the sound from the transmitter on the side toward the source will be louder than that from the other.

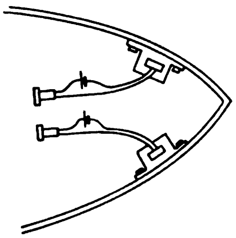

Fig. 178.

If a person turns his head from one side of the line of direction of a sound source to the other, he will have the impression of sound first in one ear and then in the other. At the moment when the sound seems to change from one ear to the other, the line joining the two ears is perpendicular to the line of direction of the sound. The degree of precision in locating sound direction would be greater if the distance between the ears were greater.

This binaural effect is the basis of various acoustic goniometers used in detecting and locating invisible submarines and aeroplanes. For submarine work, a simple form consists of two rubber bulbs *A* and *A'*, Fig. 179, each connected to a tube *B* or *B'* that can be inserted into the ears like the stethoscope used by physicians in listening to the sound of the heart and lungs. The receiving bulbs, separated by a distance of about 6 feet are mounted in a T-shaped frame capable of rotation about a vertical axis. Sound from the submarine traverses the water, strikes the two submerged receiving bulbs and is transmitted to the two ears of the listener on the deck of the chaser. By rotating the apparatus back and forth about a vertical axis, the position is found in which the cross-arm *AA'* is perpendicular to the line of direction of the sound.

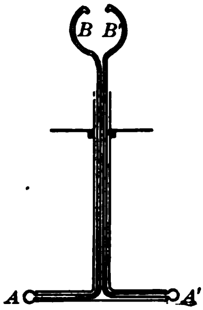

Fig. 179.

Instead of turning the apparatus back and forth, the same effect can be obtained when the apparatus is stationary by increasing and decreasing the length of one of the tubes. The direction of the sound source from the line of the arms of the *T* is then indicated by the difference in the lengths of the tubes when the sound appears to change from one ear to the other on increasing and decreasing this difference.

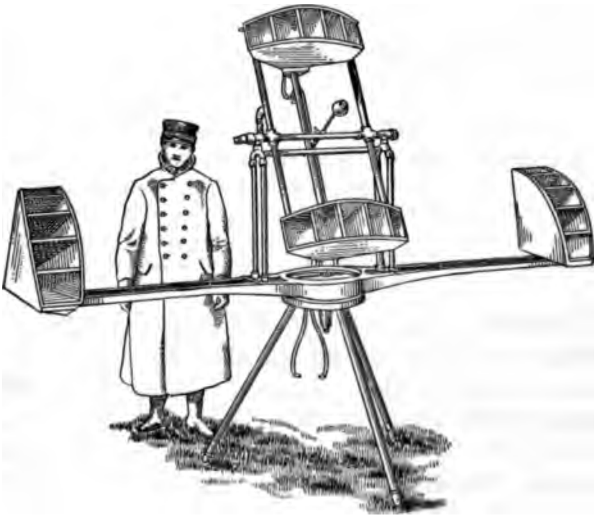

Fig. 180.

One form of acoustic goniometer for locating invisible aeroplanes is illustrated in Fig. 180.

174. Sound Ranging.—During the recent World War several highly useful methods were developed for the location of invisible sound sources. The method of locating large guns from the sound of the discharge will now be briefly outlined.

Suppose that a discharge of a gun somewhere near A, Fig. 181, sends out waves in all directions with the speed v. Suppose that an observer at any convenient station is provided with instruments which record the instants at which the sound of the discharge reaches three points B, C, and D. Let the interval of time from the moment when the sound reaches B and when it reaches C be denoted by t_1, and the interval from the moment when the sound reaches B and when it reaches D be denoted by t_2. Then, the source is farther from C than from B by a distance $x_1 = vt_1$, and is farther from D than from B by a distance $x_2 = vt_2$.

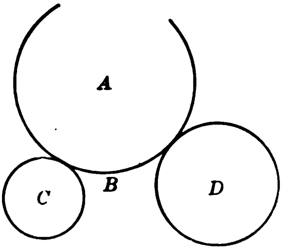

Fig. 181.

On the map, with C as a center, describe a circle of radius x_1; and with D as center describe a circle of radius x_2. From the construction, the sound source must be equally distant from B and from these two circles. Therefore, it must be at the center of a circle passing through B and tangent to these circles. The source can now be located by simple geometrical methods.

In one method of sound ranging, the sound is received at B, C and D on the diaphragms of large telephone transmitters. Wires from these transmitters extend to the central station and connect to highly sensitive galvanometers provided with devices for recording the variations of air pressure acting on the transmitters. Since different guns give characteristic pressure variations, it is possible to identify the record made by the particular gun desired.

175. Doppler's Principle.—If one stands beside a railway track while a rapidly moving whistling locomotive is passing, he will observe that at the moment of passing there is a lowering in the pitch of the whistle. Though not so readily observed, it is also true that to a stationary observer, the pitch of the whistle of a stationary locomotive is lower than the pitch of the same whistle when the locomotive is approaching the observer, and higher than the pitch when the locomotive is departing from the observer. The modification in the frequency of a wave produced by motion of the source relative to the receiver is called Doppler's Principle.

The value of the change in the frequency can be readily found as follows. Let the number of vibrations per second made by the source be represented by n, and the number of vibrations per second received by the observer by n_0. Let the velocity, relative to the earth, of the wave in a stationary medium be represented by v; that of the medium, relative to the earth, by v_m; that of the observer by v_0; and that of the source by v_s all velocities being measured in the direction from the source to the observer.

The velocity of the wave relative to the earth in a medium moving with velocity v_m toward the observer equals $v+v_m$. When the source is moving toward the observer with velocity v_s, the velocity of the wave relative to the source is $v+v_m-v_s$. Whence, the wave-length is, (110)

$$\lambda\left[=\frac{\text{velocity}}{\text{frequency}}\right]=\frac{v+v_m-v_s}{n}. \quad . \quad . \quad . \quad (115)$$

Again, the velocity relative to the observer is $v+v_m-v_0$. Consequently, the frequency at the observer is

$$n_0\left[=\frac{\text{velocity}}{\text{wave-length}}\right]=n\frac{v+v_m-v_0}{v+v_m-v_s}. \quad . \quad . \quad (116)$$

176. Reflection of Sound.—At a point where there is a sudden change of the restoring force acting on a wave, part of the energy of the incident wave will be reflected (Art. 152). The energy that is not reflected will be either absorbed or transmitted by the medium.

A sound wave traveling along the air within a tube moves with less ease than in the free air. Consequently, on reaching the end of the tube, or an abrupt enlargement, the wave will suddenly expand and part of the energy will be sent back as a reflected wave. At a sudden constriction in a tube, reflection will also occur.

The laws of reflection of waves have already been derived (Art. 158). On account of their great wave-length, sound waves bend around the edges of obstacles to such an extent that mirrors and lenses for sound must be of very great size.

When a wave goes from one medium to another in which the restoring force is much different, nearly all of the energy will be reflected and only a small part will enter the second medium. For this reason, between water and air, or between a metal and air, the reflection of sound is nearly complete. When a sound is produced under water, by a submarine for example, almost no sound emerges into the air. Also, when sound is produced in the air, almost no sound enters the water. A fish is not disturbed by the noise of conversation in a boat but is frightened by the noise produced by the scuffling of feet on the bottom of the boat. The propagation of sound long distances through speaking tubes is due to the nearly complete reflection at an air-metal surface. For the same reason, sound can be heard a much greater distance in shallow water than in either deep water or in the open air.

Suppose that a moving submarine is at *A*, below the air-water surface *XZ*, Fig. 182. Sound will travel from *A* to *B* along the path *AB* and also along the path *AYB*. Since the speed of sound in water is greater than in air, there will be reflection at the interface with loss of a half wave-length (Art. 152). It follows that if the length of the two paths *AB* and *AYB* were equal, all waves traveling these paths would arrive at *B* in condition to produce total destructive interference. But the lengths of the two paths being unequal, the interference at *B* will not be total except for waves of particular wave-lengths. There will be total destructive interference only for waves of wave-length λ such that λ, 2λ, 3λ, etc., equals the difference in path. Waves of all other lengths will produce sound at *B*.

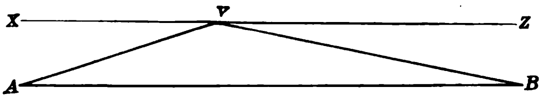

Fig. 182.

When the difference of path produces a phase difference, the direct and the reflected wave will not totally interfere at *B*. The loudness increases with the phase difference which is due to difference of path. When the distance *AB* is great, the phase difference at *B* is greater for short waves than for longer waves. Consequently, if a distant submarine produces vibrations of various frequencies, the shorter waves will be more prominent at *B*. When the distance *AB* is less, the phase difference at *B* is greatest for longer waves. In this case the longer waves will be more prominent in the sound heard at *B*.

Therefore, when the distance between a submarine and a listener is changing, the pitch and quality of the observed sound will change. As a submarine approaches, the pitch becomes lower.

177. Reflection of Sound at the End of a Pipe.—If the end of a pipe be closed by a solid, sound waves will here be reflected without change in sign of the condensation. At a solid end a condensation will be reflected as a condensation and a rarefaction will be reflected as a rarefaction.

If, however, the end of the pipe be open, a pulse of either condensation or rarefaction on emerging into the free air will meet with less opposition to motion than within the pipe. On emergence the speed will be greater than when within the pipe. Consequently, there will be reflection. At the open end a condensation will be reflected as a rarefaction, and a rarefaction will be reflected as a condensation.

178. Echoes and Reverberation.—Sound waves, in air, are not only reflected by mountains, buildings and other solid objects, but also by clouds of water vapor, and layers of either cooler or warmer air. The repetition of a sound in air, caused by reflection, is called an *echo*. When the sound is reflected many times, we have a multiple echo. Thunder is the multiple echo, due to reflection from clouds or mountains, of the sound produced by the sudden collapse of the electrically heated air-column constituting a lightning flash.

A multiple echo in which the individual echoes follow one another so closely that they cannot be separated by the ear is called a *reverberation*. When the distances between the reflecting objects are great, the individual echoes are separate. But when the distances are small, reverberation occurs.

An auditor in a large auditorium receives from the source three trains of sound waves—one directly from the source, a second after reflection from some large flat or curved surface of walls or ceiling, and a third due to multiple reflection. The second may give distinct echoes, and the third may give rise to reverberation. The existence of echoes or reverberation renders hearing difficult.

In order that an auditorium may be acoustically satisfactory, (*a*), there must be no echoes; (*b*), any reverberation must be of such brief duration that a syllable received after reflection shall not obscure the following syllable received directly from the speaker. Experiments show that if the duration of the reverberation is greater than one second, a speaker must articulate slowly, distinctly, and without too much energy, or he will not be understood.

The duration of the reverberation depends upon the volume of the room, together with the absorptive powers of the sides of the room and of the audience. From an elaborate series of experments Sabine has found that the duration of reverberation may be expressed by the equation

$$t=\frac{CV}{a+b},$$

where V represents the volume of the room; a is the absorptive power of the empty room; b is the absorptive power of the audience; and C is a constant depending upon the pitch, loudness and quality of the original sound together with the intensity of the reverberation. For the average human voice, in a large number of auditoriums, the constant C has a value of about 0.17.

Many halls that are acoustically quite poor when empty are entirely satisfactory when filled by an audience. Halls that are acoustically unsatisfactory can be greatly improved by covering the ceiling and all the walls except behind the speaker with sound absorbing materials such as draperies, cloth, felt and asbestos.

Soft porous materials are our best sound absorbers or sound deadeners. Their effectiveness is due partly to their low elasticity, and partly to the damping produced by the friction offered to the passage of waves that are long relative to the size of the apertures.

179. Beats.—If two sound waves of frequences n_1 and n_2 vibrations per second travel the same path in the same direction, then during one second the two waves will be in the same phase (n_1-n_2) times. That is, during one second of time, the intensity of the resultant sound will rise to a maximum and fall to a minimum

$(n_1 - n_2)$ times. If the difference of frequency be greater than about 30 per second, the variations of intensity are indistinguishable by the ear. We then have a note of constant loudness. If the difference of frequency be between 10 and 30 per second, the note is rough and harsh. If the difference of frequency be less than about 10 per second we have a note that rises and falls in loudness. The maxima of loudness arising from two notes of nearly the same pitch are called *beats*.

In the lower part of Fig. 183 the light lines represent the displacement curves of two waves of nearly the same frequency. The heavy line is the resultant displacement curve. The line L represents the variations in the loudness of the resultant sound.

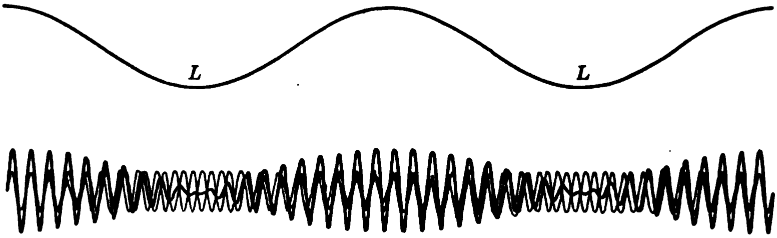

Fig. 183.

The ordinary method of determining the difference between the frequencies of two notes of nearly the same pitch is to count the number of beats which occur per second when the two notes are sounded together. Again, two notes may be brought into unison by varying the pitch of one till there are no beats when the two are sounded together.

180. Refraction of Sound.—Whenever a wave goes from one medium to another in which the speed is different, the curvature of the wave front is changed (Art. 162). Also, except when the incidence is normal to the interface separating the two media, the direction of the wave is changed on entering the second medium. Sound obeys the ordinary laws of refraction (Art. 165).

Since the speed of sound in warm air is greater than in cold air, refraction will occur when sound goes from air at one temperature into air at a different temperature.

If air be at rest and of uniform temperature, the wave front of sound from a point source will be spherical. Fig. 184 illustrates the propagation of sound from a point source *S* at the surface of the ground. Air being isotropic, the energy will be propagated in directions normal to the wave fronts. Sound will travel along the surface of the ground as in other directions.

The air at the surface of the earth is usually warmer than at higher altitudes. In this case the speed of sound is greater at the surface of the earth than at higher altitudes. In still air, sound from a point source will have wave fronts as illustrated in Fig. 185. At the ground, the wave fronts will be inclined to the vertical. And since the direction of propagation is normal to the wave front, the direction of propagation at the ground will not be horizontal, but will be directed upward. That is, when the air at the ground is warmer than the air at higher altitudes, sound does not travel far along the surface of the ground, but is deflected upward.

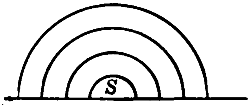

Fig. 184.

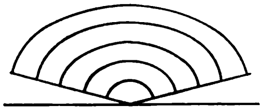

Fig. 185.

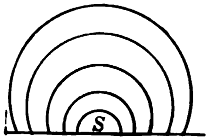

Fig. 186.

In the evening after a hot day, the heated ground sometimes loses heat so rapidly that the air near the ground becomes cooler than the air at higher altitudes. When this condition occurs and the air is still, sound will travel more slowly near the ground than in the region above. The wave fronts due to a point source will then have forms something as represented in Fig. 186. Since the direction of propagation is normal to the wave front, sound from points above the earth will be deflected downward. On a lake or on level land after a hot day one often remarks the great distances that sounds are distinctly heard.

181. Effect of Wind on the Form of Wave Fronts of Sound.—In still homogeneous air over smooth ground the wave fronts of sound from a point source are spherical. Where there is a horizontal wind, the speed of sound relative to the earth is diminished in the direction opposite to the wind, and increased in the direction with the wind. Again, at higher altitudes wind moves more rapidly than at lower altitudes. Consequently, where there is a horizontal wind, the wave fronts of sound from a point source are distorted.

On the windward side of a sound source (*W*, Fig. 187), the speed of the sound being greater at the ground than at higher altitudes, the wave fronts near the ground are inclined to the vertical. And since the direction of propagation is normal to the wave front, the sound is there directed upward leaving the earth in almost silence. If wind is blowing from a sound-ranging base toward an enemy battery, the sound of the discharge will be deflected upward and may not reach the sound-ranging instrument On the side of the source toward which the wind is blowing (*A*, Fig. 187), the speed of the sound near the ground is less than at higher altitudes. On this side, the wave bends over toward the ground, thereby distributing the sound to considerable distances in the direction along the ground. A moderate wind from an enemy battery toward the sound-ranging base is favorable to accurate results.

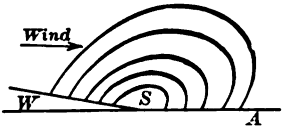

Fig. 187.

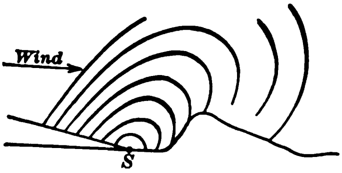

Fig. 188.

If the wind blow against a hill or other large object, there will be a space free of sound behind the obstacle, beyond which there will be sound, Fig. 188.

It is left as an exercise for the student to show that if a sound be produced at one end of a tunnel, then the loudness at the other end will be greater when a wind blows through the tunnel in the direction opposite to the sound than when a wind blows in the direction of the sound.

CHAPTER XIII

SOUNDING BODIES

182. Vibrating Rods.—By sprinkling sand upon a vibrating rod, fixed at one end, one or more places will be found on which the sand remains, that is, which are almost at rest. The reason is not far to seek. If the free end of the rod be pulled aside and released, a wave will travel to the fixed end, there be reflected, travel to the free end, there be reflected, and so on. The resultant condition produced by the component waves in opposite directions is a stationary undulation (Art. 154). The modes of vibration of a rod fixed at one end when giving the fundamental and the first two overtones are shown in Figs. 189, 190 and 191, respectively. The distance from the point at rest to the far end of the rod is less than one-third the length of the rod, and the distance *bc* is less than *cd*. The relative frequencies of the fundamental, and the first two overtones are not 1, 3, 5, as in the case of a closed pipe, but are approximately 1, 6.25, 17.5. The frequencies of the transverse fundamental vibration of rectangular rods of the same material, fixed at one end, are proportional to the thickness in the plane of vibration, and inversely proportional to the squares of their length. The motion of a vibrating rod is usually the resultant of the fundamental vibration and several overtones.

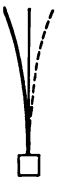

FIG. 189.

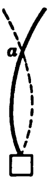

FIG. 190.

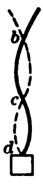

FIG. 191.

The tuning fork is essentially a rod fixed at the middle point, Figs. 192 and 193. When the prongs vibrate, the stem moves up and down with the same period. The prongs set so little air into vibration that the loudness due to the vibrating prongs is not great. By resting the stem on a table, the whole table top and a large mass of air are set into vibration, and the loudness of the sound is greatly increased.

Tuning forks are often attached to the top of a resonator, Fig. 194, consisting of a box open at one end and containing a volume of air having a natural period of vibration nearly the same as the fundamental of the fork. The overtones of a tuning fork are weak and of short duration. As the resonator reinforces only the fundamental, the note from a tuning fork on a resonating box is very pure, that is, free of overtones.

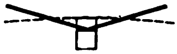

Fig. 192.

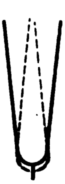

Fig. 193.

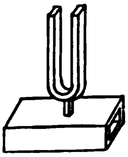

Fig. 194.

183. Vibrating Plates.—If the center of a telephone diaphragm be displaced and then released, the diaphragm will vibrate back and forth,—the center with the greatest amplitude,—with a frequency of from 750 to 900 vibrations per second. For voice frequencies not greater than the natural frequency of a telephone diaphragm, the diaphragm vibrates as a whole in one segment with the frequency of the voice vibrations. For voice frequencies greater than the natural frequency of the diaphragm, the diaphragm divides into two concentric segments separated by a circular nodal line. The range of the voice is from about 75 to about 3000 vibrations per second.

The presence of nodal lines can be shown by sprinkling the plate with lycopodium powder or fine sand. The sand or powder will remain on the parts of the plate at rest and will roll away

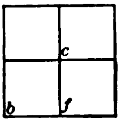

Fig. 195.

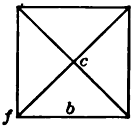

Fig. 196.

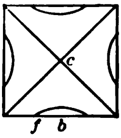

Fig. 197.

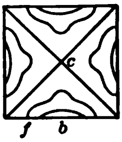

Fig. 198.

from the parts in motion. By this means Chladni first studied the modes of vibration of plates. Figs. 195-198 show the positions of the nodal lines of a plate clamped at *c*, when bowed at *b* and touched by a finger at *f*. The more segments into which a vibrating plate is divided by nodal lines, the higher the pitch of the note emitted.

184. Vibrating Wires and Strings.—If a stretched flexible string be plucked, the plucked element will vibrate transversely with periodic motion. This periodic motion, by being handed on successively from one element of the string to the next, will produce a transverse wave. This wave is reflected from the ends of the string. The two superposed oppositely directed waves set the string into stationary undulation.

If the string be plucked at the middle point, the whole string will vibrate in one segment, Fig. 199*a*. But if the middle point of the string be lightly touched with a match and at the same time a point be plucked midway between this point and one end, the string will vibrate in two segments with double the frequency of the fundamental vibration, Fig. 199*b*. On removing the match, the node at the middle of the string will persist as may be shown by dropping on the string several little Λ shaped pieces of paper. All of these "riders" will be thrown off except the one at the node. In a similar manner a string may be set into stationary undulation with three, four, etc., segments and frequencies of three, four, etc., times that of the fundamental.

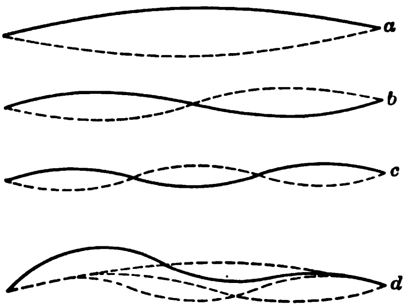

FIG. 199.

By means of resonators it can be shown that the note from a vibrating stretched string usually consists of a fundamental together with several harmonic overtones. It follows that the stationary undulation of the string must be the resultant of the superposition of a fundamental and several harmonics as illustrated in Fig. 199*d*.

Certain tones when sounded together give an unpleasant sound. If it be required that a certain harmonic of a vibrating string shall not be produced, the string is plucked at a point at which a node would be formed if the undesired harmonic were present. To prevent the formation of undesired harmonics, piano strings are struck at points from one-eighth to one-ninth of their length from one end.

It can be proved that the frequency n of the fundamental vibration of a flexible string of length l, and mass per unit length m, when stretched by a force f, is

$$n=\frac{1}{2l}\sqrt{\frac{f}{m}}. \qquad \qquad (117)$$

In stringed musical instruments the required pitch can be obtained by varying the length of the string, the tension, or the mass per unit length. The mass per unit length may be increased, without any great decrease in flexibility, by wrapping the string with one or more layers of wire.

As the piano and zither have strings of fixed length, tension, and mass per unit length, these instruments must have a separate string for each note that is to be produced. In such instruments as the violin and guitar, each string can be changed in length so as to produce a number of different notes. Consequently these instruments require but few strings.

185. Sound Boards.—A vibrating string cuts through the air without setting much of it into vibration. Consequently, the loudness of the note emitted by a string vibrating in the open air is very small. But by stretching the string over one or more bridges that rest on a broad thin board, the vibrations of the string will be propagated through the bridges to the board, and the latter will be set into forced vibration of the same frequency as the string. The large mass of air thereby set into vibration greatly increases the loudness of the note.

It should be noted that the sound-board is not set into vibration by resonance. The same sound-board is forced into vibration by any note whatever the frequency.

186. The Production of Vocal Sounds.—Vocal sounds are due to waves in the air produced by the vibration of the so-called vocal cords. These consist of two membranes, *cc*, Fig. 200, across the trachea or windpipe at the enlargement called the "Adam's apple." In men the vocal cords have a length of about 1.8 cm., and in women about 1.2 cm. In ordinary conversation the wave-lengths of a man's voice are from three to four meters, while those of a woman are about one-third as great.

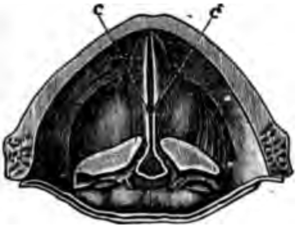

Fig. 200.

The tension, and to a slight extent the length, of these membranes, as well as the width of the slit between them, is controlled by attached muscles. By these changes the pitches of the fundamental and accompanying notes are altered. The intensity of the overtones is altered by resonance in the air passages of the throat, mouth and nasal passages. When the vocal cords are in a completely relaxed condition, the breath passes between them without setting them into sonorous vibration. In whispering, the vocal cords are under such slight tension that they produce almost no sound.

187. Physical Characteristics of Vowels.—Speech sounds which can be continuously intoned without change so long as breath is supplied are called *vowels*. Vowels are continuous sound waves produced by vibration of the vocal cords and modified by resonance, but not by audible friction, in the air passages above the cords. The Century Dictionary distinguishes nineteen vowels in the English language. Unfortunately, we do not have a different symbol in our alphabet to represent each vowel.

Consonants are vocal sounds which either come to a definite stop (as *p*) or which are modified by audible friction (as *f*). In the case of some consonants there is no vibration of the vocal cords (for example *p* and *f*), while in the case of others (for example *b* and *v*) there is vibration of the vocal cords.

A permanent record of the pressure variation of a sound wave may be made by a phonograph and other instruments of even greater sensitivity. A study of such records of wave forms shows that the wave produced by the vibration of the vocal cords consists of a fundamental accompanied by many overtones. As many as 24 overtones have been found in some vocal sounds. By changing the size and shape of the various cavities and openings of the mouth and throat, different overtones can be strengthened by resonance. For example, if the vocal cords be set into vibration when the mouth is wide open and the tongue is low, the sound of *a* in m*a* will be emitted. If now the lips be nearly closed, everything else remaining as before, the sound of *oo* in m*oo* will be produced. The pitch of the overtone that is strengthened is independent of the loudness of the sound or the pitch of the fundamental. An analysis of vowel sounds shows that each vowel is characterized by one or two overtones of definite pitch independent of the pitch of the fundamental.

D. C. Miller* has found that the distinguishing characteristic of the vowel *a* in m*a* is an overtone of pitch of about 922 vibrations per second, that of *a* in m*a*w by an overtone of about 732; that of *o* in m*o*w by an overtone of about 461; and that of *oo* in m*oo* by an overtone of about 326 vibrations per second. Other vowels are characterized by two overtones of constant pitch. The characteristic overtones of *a* in m*a*t have frequencies of about 800 and 1843 vibrations per second; those of *e* in m*e*t of about 691 and 1953; those of *a* in m*a*te of about 488 and 2461; and those of *ee* in m*ee*t have frequencies of about 308 and 3100 vibrations per second.

188. Vibrating Air Columns.—If a vibrating tuning fork be held at the end of a column of air whose length can be varied, Fig. 201, it will be found that as the length of the column is gradually increased from zero, there will be strong reinforcement of the

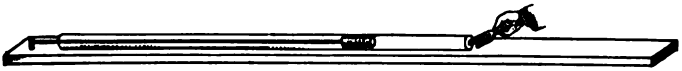

FIG. 201.

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* "The Science of Musical Sounds," p. 237.

sound when the air column has a certain length. If this length be slightly increased or decreased, the loudness will diminish. But by gradually increasing the length of the air column, another length can be found at which strong reinforcement again occurs. If the pipe be of sufficient length, several such points of reinforcement can be found. This effect will now be studied in more detail.

When the prong of a tuning fork moves toward the mouth of the pipe, a pulse of compression is sent down the column of air. When the prong moves away, a pulse of rarefaction is sent down the column of air. At the closed end of the pipe a pulse of compression is reflected as a pulse of compression, and a pulse of rarefaction is reflected as a pulse of rarefaction. On reaching the open end, a pulse of compression is reflected as a pulse of rarefaction, and a pulse of rarefaction is reflected as a pulse of compression. At the open end, the part of the energy that is not reflected emerges into the surrounding air.

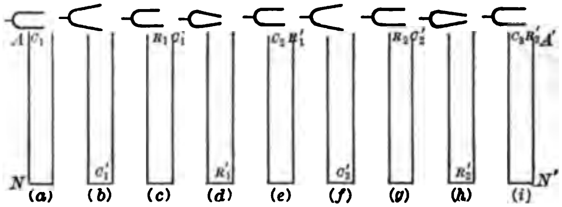

FIG. 202.

In the separate diagrams of Fig. 202 and 203 are indicated the positions of the maxima of compression and rarefaction at instants separated by an interval of one-quarter of the period of the fork. Successive pulses of compression are represented by the symbols C_1, C_2, etc., and successive pulses of rarefaction by R_1, R_2, etc. When a pulse is going downward, the symbol is placed on the left side of the pipe: when the pulse is going upward, the symbol is placed on the right. When a compression, C_1 for example, is reflected as a rarefaction, the symbol is changed to R'_1. On subsequent reflections with change in the sign of the compression, the primes and subscripts are omitted.

In Fig. 202 the pitch of the tuning fork is such that while the prong makes one-quarter of a complete vibration a pulse will travel the length of the pipe. When the prong has moved from the upper end of its path to the mid-position, Fig. 202*a*, the maximum of the pulse of compression is at the mouth of the pipe. At the end of the next quarter period of the fork, this pulse of compression has reached the closed end of the pipe and has there been reflected as a condensation, Fig. 202*b*. During the next quarter period this pulse of compression has gone to the top of the pipe, Fig. 202*c*, and is on the point of being reflected as a pulse of rarefaction. At the same instant, the fork is producing at the mouth a maximum of rarefaction R_1. The two pulses conspire to produce a pulse of greater rarefaction. On the return to the open end of the pipe after reflection at the closed end, this pulse of rarefaction is reflected as a compression, and at the same time there is superposed upon it another pulse of compression due to the fork. In this way, the air in the pipe continues to receive small increments of energy from the fork until the vibrations emitted from the open end of the pipe are of sufficient amplitude to produce a loud sound.

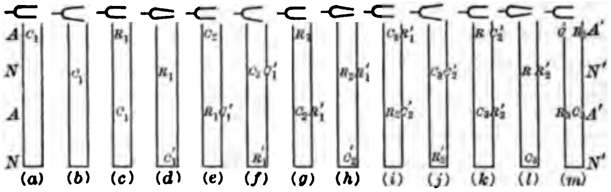

Fig. 203.

The same column of air will respond to notes of certain other frequencies. In Fig. 203 is represented the progress of pulses of compression and rarefaction when the pitch of the tuning fork is such that while the fork makes one-quarter of a complete vibration, a pulse travels one-third of the length of the pipe.

An examination of these figures shows that after a few pulses have returned to the open end, there are certain fixed points within the pipe at which a state of condensation alternates with a state of rarefaction. At one moment air rushes toward these points from both directions, and at another moment rushes away in both directions. At these points, situated on the horizontal lines NN' in the figures, the density of the air changes periodically but the velocity of the air particles remains zero.

There are other fixed points at which the density is always the resultant of a condition of condensation and an equal rarefaction. At these points, marked AA' in the figures, the density of the air is constant but the velocity of the air particles is first in one direction and then in the other.

Since, in the vibrating air column there are stationary points at which the velocity of the air particles is nearly zero, alternating with other stationary points at which the velocity changes periodically, the air is in a state of stationary undulation (Art. 154). The points at which the velocity of the air particles is nearly zero and the density changes periodically are called *nodes*. The points at which the velocity changes periodically and the density remains constant are called *antinodes*.

In the case of a stationary undulation in a pipe closed at one end, there is a node at the closed end and an antinode near the open end. The distance between two nodes, or between two antinodes, equals one-half wave-length; and the distance between a node and an adjacent antinode equals one-quarter wave-length (Art. 154). Hence, where there is a stationary undulation in a pipe closed at one end, the length of the pipe must be nearly equal to an odd number of quarter wave-lengths. That is, the wave-lengths of the air waves within a closed pipe are, respectively, $4l$, $\frac{4}{3}l$, $\frac{4}{5}l$, $\frac{4}{7}l$, etc., where l represents the effective length of the pipe. Consequently, the wave-lengths of the air waves within a closed pipe are in the ratio of 1, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc. It follows that a closed pipe of fixed length will respond to notes having frequencies in the ratio 1, 3, 5, etc.

It is left as an exercise for the student to discuss stationary undulations of air in pipes open at both ends and to show that, in this case, the length of the pipe must equal an even number of quarter wave-lengths. That is, the wave-lengths are, respectively,

$\frac{4}{2}l$, $\frac{4}{4}l$, $\frac{4}{6}l$, $\frac{4}{8}l$, etc. Consequently, the wave-lengths of the air waves within an open pipe are in the ratio of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc. An open pipe of fixed length will respond to notes having frequencies in the ratio 1, 2, 3, 4, etc.

A standing wave of sound may be shown by the following experiment. *AB* (Fig. 204) is a sheet-iron tube closed at *A* by a metal head and closed at *B* by a draw-tube provided with a thin rubber diaphram *D*. The upper element of the tube is provided with a row of gas jets extending from end to end and supplied with illuminating gas which enters through the inlets I_1 and I_2.

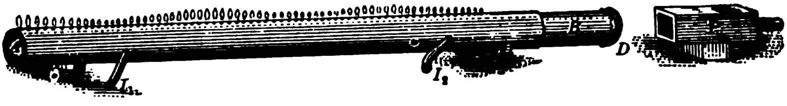

Fig. 204.

If a loud note be produced in front of *D*, a sound wave will traverse the column of gas to the end *A* and there be reflected back along the same path. If the length of the tube has been properly adjusted, these two waves will set up a standing wave of sound in the gas. At the nodes the particles of gas vibrate hardly at all, but the pressure at the nodes changes very considerably. This is because the particles of gas in the ventral segments on both sides of a node rush toward the node, thus increasing the pressure at the node, and then on both sides rush away from the node, thus decreasing the pressure there. So that during every period of vibration the node is once in the midst of a condensation and once in the midst of a rarefaction. When the pressure at a node is greatest, the flame above it burns highest; and when the pressure is least, the gas is hardly forced out of the tube at all and the flame is low. Thus during every vibration of the gas the flame above each node rises and falls, whereas the flame above an antinode, where the pressure does not change, burns steadily. The period of vibration is so short that our eyes cannot follow the motions of the flames, and so flames at the nodes appear to be all the time burning higher than those at the antinodes. If a person moves his eyes rapidly past the flames, he can see that they really are moving up and down.

189. Determination of the Velocity of Sound.—From (110) the velocity of a wave of frequency n and wave-length λ is given by

$$v = n\lambda. \qquad (118)$$

In the case of a stationary undulation in any medium, the distance between two nodes, or between two antinodes, equals one-half wave-length. Consequently, if the frequency of the vibration be known, the velocity can be determined. By this method the velocity of sound in different media and at different temperatures has been determined.

The velocity of sound in free still air at the temperature of melting ice is 331.29 meters, (1086.93 ft.) per second. At $t°$ C., the velocity is

$$v_t = 33129\sqrt{1+\frac{t}{273}}\text{cm. per sec.}, \quad . \quad . \quad . \quad (119)$$

and at $t'°$ F., the velocity is

$$v_t' = 1086.93\sqrt{1+\frac{t'}{459}}\text{ ft. per sec.} \quad . \quad . \quad . \quad (120)$$

If the air is moving, the speed of sound is affected. For instance, the sound of the explosion of a gun is greater than normal near the muzzle. In the case of a twelve-inch gun, the speed of the sound of the explosion does not become normal till after about five hundred feet have been traversed. Winds may increase or decrease the speed of sound.

The speed in sea water at the freezing point is about 4530 feet per second. Up to atmospheric temperatures, a rise of temperature increases the speed in sea water about three feet per second, per degree Fahrenheit. In pure air-free water at 66° F., the speed is 4794 feet per second.

In other substances the speed, expressed in feet per second, has the following approximate values—steel, 16,500; granite, 12,900; ash wood, along the fiber, 4670; ash wood, across the fiber, 4570.

By means of (118) we can determine the length of any wave of assigned frequency and known speed. For example, the note C_3 (" middle C ") of an instrument tuned to International pitch has a frequency of 258.65 vibrations per second. Consequently, when traversing still air at the temperature of freezing water, the wave-length of sound of this pitch has the value

$$\lambda_a\left[=\frac{v}{n}\right]=\frac{1086.93}{258.65}\doteq 4.2\text{ ft.}$$

In sea water, at the same temperature, the wave-length of the same note would be

$$\lambda_w = \frac{4530}{258.65} \doteq 17.5 \text{ ft.}$$

190. The Flue Organ Pipe.—The sounds of most wind musical instruments are due to the resonant vibration of a mass of air set into motion by a mouthpiece.

In the case of the flue organ pipe, Fig. 205, a sheet of air is blown against the sharp edge of the mouth of the pipe. If a part of the sheet enters the pipe, a pulse of condensation will travel to the farther end and there be reflected. If the farther end be closed, the incident pulse of condensation will be reflected as a pulse of condensation, which, proceeding to the mouth, will there produce a rise of pressure and a consequent outward deflection of the air sheet. This outward deflection of the air sheet at the mouth produces a rarefied condition within, which, proceeding to the farther closed end, is reflected as a pulse of rarefaction. On reaching the mouth, this rarefied condition, by drawing the air sheet inward, again produces a pulse of condensation, and the cycle of events is repeated. It thus appears that the period of vibration of the air sheet at the mouth is determined by the period of vibration of the column of air within the pipe. For a pipe closed at one end, the period is the time occupied by a pulse to travel the length of the pipe four times. Thus, the wave-length of the fundamental tone emitted by a closed pipe is four times the length of the pipe. In addition to the fundamental, there will be overtones of wave-lengths $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc., that of the fundamental (Art. 188).

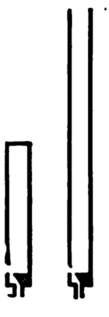

FIG. 205.

In an open organ pipe the wave-length of the fundamental tone will be two times the length of the pipe. There will be a series of overtones of wave-lengths $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., that of the fundamental.

The note of a closed organ pipe consists of the fundamental and the odd harmonic overtones. The note of an open organ pipe consists of the fundamental and both odd and even harmonics (Art. 188).

The frequency or pitch of the fundamental of a vibrating air column is determined by the length of the column and the velocity of sound. From (118) the frequency

$$n=\frac{v}{\lambda}.$$

A change of temperature, by changing the velocity, will change the pitch.

SOLVED PROBLEM

PROBLEM.—A certain flue organ pipe at 20° C. has a pitch of 512 vibrations per second. Find the pitch at 10° C.

SOLUTION.—From (118) the frequency of a wave

$$n=\frac{v}{\lambda},$$

where v is the velocity of the wave, and λ is the wave-length, which in this case is determined by the length of the pipe.

Due to the changing temperature, the velocity of the sound changes as given by (119). The length of the pipe is altered by a negligible amount. From (118) and (119), the frequency at 10° C., is

$$n_{10}\left[=\frac{v_{10}}{\lambda}\right]=\frac{33129}{\lambda}\sqrt{1+\frac{10}{273}}=\frac{33129}{\lambda}\sqrt{\frac{273+10}{273}},$$

and at 20° C., the frequency is

$$n_{20}[=512]=\frac{33129}{\lambda}\sqrt{\frac{273+20}{273}}\text{ vibrations per second.}$$

Dividing each member of the former equation by the corresponding member of the latter, we obtain

$$\frac{n_{10}}{512}=\sqrt{\frac{273+10}{273+20}}$$

191. The Flute.—The air column within a flute, Fig. 206, is set into vibration by a current of air directed against the thin edge of an elliptical hole near one end of the instrument. The tube is pierced with a number of holes which can be opened or closed at the will of the performer. When all of the holes are closed and the instrument is blown with a certain pressure, the pitch is that corresponding to the length of the instrument. When one of the holes is opened the pitch is raised. The pitch depends not only upon the pressure with which this instrument is blown and the distance from the mouthpiece to the hole uncovered, but also upon the size of the hole uncovered (Art. 171).

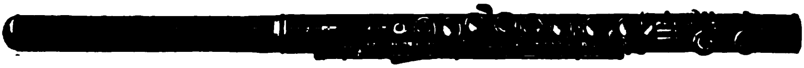

FIG. 206.

192. The Brass Wind Instruments.—In an important class of wind instruments the vibrations of the air column are induced by the breath of the player setting into vibration the tightly stretched lips pressed against a bell-shaped mouthpiece, Fig. 207. The pipe containing the vibrating air column is usually so long that for convenience of handling it is coiled.

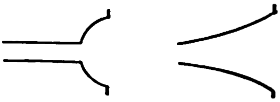

FIG. 207.

The trumpet, Fig. 208, has an air column of fixed length. The only notes that can be produced are those that can be formed by changes in the pressure of the air and the tension of the lips.

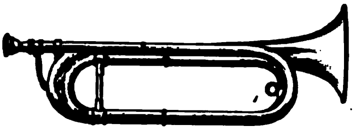

FIG. 208.

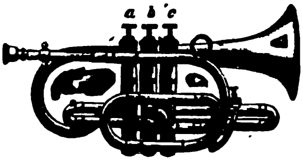

FIG. 209.

The length of the air column within the cornet and certain other instruments can be changed by fixed amounts by means of pistons *a*, *b*, *c*, Fig. 209.

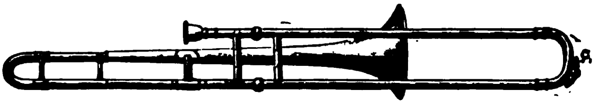

FIG. 210.

The length of the air column within the trombone can be changed by means of a sliding section *cd*, Fig. 210.

Questions

1. If one end of a tube be placed in the ear and the other end be moved along a train of stationary vibrations, at what points would be heard sound of maximum loudness?

2. If a sounding fog-siren be at a considerable altitude above the sea, there will be a "zone of silence" where no sound will be heard. At points either nearer or farther from the source the sound will be heard. Explain.

3. By placing against a stretched string the end of the stem of a vibrating tuning fork which has either the same frequency as the string or any exact multiple, the string will be set into vibration. Explain.

4. When a regiment is marching behind a band, the men are not all in step with each other although each man is in step with the music as he hears it. Explain.

5. A distinct echo is produced of the whistle of a locomotive approaching a cliff. Will the pitch of the whistle be higher or lower than that of the echo heard by, (*a*), a person on the train, (*b*), a person standing on the ground?

HEAT

CHAPTER XIV

EFFECTS OF HEAT

§ 1. *Temperature and Quantity of Heat*

193. Heat is Energy.—If a lead bullet be fired against a steel target the kinetic energy of the bullet will disappear, and so much heat will be developed on impact that part of the lead will be fused. If the bearing of a freight car axle should run dry much energy will disappear at the rubbing surface, and so much heat will be developed that the oil-soaked cotton waste in the stuffing box may ignite. Again, if a gas be allowed to expand work will be done against the resisting force, and the gas will become cooler, showing that heat has been lost. These examples suggest that energy can be transformed into heat, and heat can be transformed into energy. From a study of such experiments as these we conclude that heat is a form of energy. Thus it is equally correct to say that a barrel full of lukewarm water can furnish a greater quantity of heat, or can furnish a greater quantity of energy, than a cup full of boiling water.

Many physical properties of a body are altered by the addition of heat to the body or abstraction of heat from the body. The effects of heat to be here considered are

(1) Change of the temperature of a body.

(2) Change of the physical state, from solid to liquid, liquid to gas, etc.

(3) Change of the size of a body.

194. The Sensation of Heat and Cold.—Though it is not a matter of general knowledge that besides the sense organs for seeing, hearing, tasting, smelling and feeling, we also possess specialized organs for the appreciation of heat and cold, yet that such is the fact can be easily shown as follows. Place the pointed end of a small metal rod in a flame for a few seconds and then very lightly trace a line on the back of the hand. As the tracing point passes over certain points of the skin a hot sensation will be perceived, whereas along the intermediate region there will be no such sensation. By marking these points with a pen dipped in red ink we can map out the positions of the nerve termini that are sensitive to heat. If the pointed metal rod is allowed to remain in the flame for a longer time, the same points on the hand will respond, but with a more intense sensation.

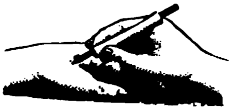

FIG. 211.

If, however, the pointed rod is placed against a piece of ice and passed over the skin as before, points will be found that give the sensation of cold. By marking these points with a pen dipped in black ink, it will be observed that these points do not coincide with those that give the sensation of heat, but constitute quite a different system. It will also be observed that neither the organs sensitive to heat nor those sensitive to cold coincide in position with those sensitive to touch. It thus appears that in addition to the organs of touch, the skin is supplied with specialized organs for the perception of heat and cold.

195. Distinction between Temperature and Quantity of Heat.—The fact that more ice can be melted by a gallon of boiling water than by a pint of boiling water is described by the statement that a greater quantity of heat can be furnished by the gallon of boiling water than by the pint of boiling water. Similarly, a barrel full of lukewarm water can furnish a greater quantity of heat than a cup full of boiling water.

If the cup full of boiling water be placed in contact with the barrel full of lukewarm water, the former will lose heat while the latter will gain heat. The condition which determines the flow of heat from one body to another is termed *temperature*.

If when two bodies are placed in contact and shielded from outside thermal disturbances, neither body gains nor loses heat, the two bodies are said to be in thermal equilibrium. According to the ordinary notions of temperature, two bodies in thermal equilibrium are at the same temperature. Two bodies in thermal equilibrium may possess either the same or different quantities of heat.

If when two bodies are placed in contact and shielded from outside thermal disturbances one body loses heat while the other body gains heat, the body that loses heat is ordinarily said to be at a higher temperature than the body that gains heat.

196. Comparison of Temperature.—Although we possess specialized organs for the perception of heat and cold, these organs cannot be depended upon to give quantitative indications of temperature. For instance, if the two hands be immersed, one in hot water and the other in cold water, and then both hands be plunged into a basin of lukewarm water, the sensation of temperature now perceived by the two hands will be different. To the hand that has been in hot water the lukewarm water will appear cool, while to the hand that has been in cold water the same lukewarm water will appear warm. Again, if the hand touch a piece of metal and a piece of wood that have been for some time in a room at ordinary temperature, the piece of metal will seem to be colder than the piece of wood.

Temperatures cannot be directly measured, but they can be indirectly compared by means of various easily measured effects. For example, if a circuit is formed of wires of two different metals, and if the two junctions of the metals are kept at different temperatures, an electric current is usually found to flow around the circuit. Since electric currents are readily measured, this thermoelectric effect can be used for the comparison of differences of temperature. We might call those differences in temperature equal which produce equal changes in current.

Again, since the electric resistance of a wire of any pure metal increases when its temperature is raised, we might call those differences of temperature equal that produce equal differences in the resistance of a wire made of any given pure metal.

Again, since when under constant pressure the length and volume of most substances increase as their temperature increases, we might call those temperature differences equal that produce equal differences in the length or in the volume of a specimen of any selected substance.

Again, since the pressure of a fixed mass of gas kept at constant volume increases when the temperature is raised, we might call those temperature changes equal which produce equal changes in the pressure of a fixed mass of gas kept at constant volume.

Again, since the total energy radiated per second by a hot body increases when the temperature is increased, we might measure temperature in terms of the rate of radiation of energy from unit surface of a body.

But if we define equal increments of temperature in terms of equal increments of thermal electric current, we are no longer at liberty to define as equal those increments of temperature which produce equal increments of electric resistance, nor those increments of temperature which produce equal increments in the length or volume of a given body, nor those increments of temperature which produce equal increments in the pressure of a fixed mass of gas kept at constant volume. In fact, we find that equal increments of thermal electric current are produced by increments of temperature which produce unequal increments of electric resistance, unequal increments in the length and volume of bodies, and unequal increments in the pressure of a fixed mass of gas kept at constant volume. Indeed, we find that any one of the above methods of defining equal increments of temperature leads to increments of temperature which would not be equal if any of the other methods of definition had been adopted.

197. The Constant Volume Hydrogen Temperature Scale.—If it be postulated that those temperature changes are equal which produce equal changes in any specified property of a definite substance, a scale of temperature can be constructed. The particular property that is adopted should be one which changes considerably with temperature, and which can be read throughout a wide temperature range. Now, the permanent gases, if maintained at constant volume, undergo a considerable pressure change when subjected to a change of temperature, and they do not change in either chemical constitution or physical state even when subjected to a wide variation of temperature. Consequently, the change in pressure of a fixed mass of some permanent gas maintained at constant volume has been adopted as the basis of the temperature scale used in experimental work. As hydrogen can be changed in temperature through a very wide range without any dissociation or change of state, and can be readily obtained in a pure condition, this gas has been adopted as the standard thermometric substance. According to the Constant Volume Hydrogen Temperature Scale, *when a fixed mass of hydrogen maintained at constant volume is changed in temperature, the change of pressure thereby produced is directly proportional to the change of temperature.*

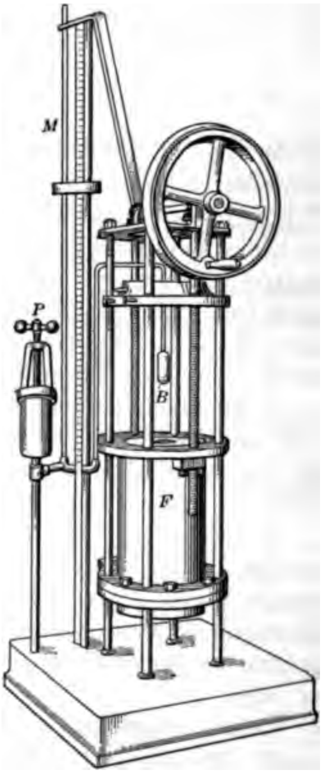

FIG. 212.

198. The Normal Thermometer.—The normal thermometer consists of a bulb *B*, Fig. 212, made of either quartz or an alloy of platinum and rhodium, joined by a capillary tube to an open manometer *M*. The bulb and the body whose temperature is desired are inclosed in an electrically heated furnace *F*. The volume of the gas in the bulb is kept at a definite value by adjusting the plunger *P* so that the mercury in the manometer is maintained at the index. In order that there shall be no transfusion of gas through the walls of the hot bulb, the space between the bulb and the walls of the furnace is filled with gas of the same kind as that inside the bulb.

While kept at constant volume, let the pressures of a given mass of hydrogen at temperatures t_1 and t_2 be denoted by P_1 and P_2, respectively, and let the pressure at some intermediate temperature t be denoted by P. Then since it is arbitrarily postulated that when a fixed mass of hydrogen kept at constant volume has its temperature changed, the change of pressure thereby produced is directly proportional to the change of temperature,

$$\frac{t-t_2}{t_1-t_2}=\frac{P-P_2}{P_1-P_2}. \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (121)$$

In order to determine t in comparison with t_1 and t_2, definite numerical values must be assigned to these latter temperatures. The particular temperatures selected for t_1 and t_2, as well as the particular numbers employed to represent these temperatures, are matters of arbitrary convention. But in order that the readings made from different thermometers may be comparable, physicists have decided upon two particular temperatures for the fixed points t_1 and t_2. For t_1 is used the temperature of steam rising from water boiling under a barometric pressure of 76 cm. of mercury, and for t_2 is employed the temperature of a mixture of pure ice and water when in thermal equilibrium and under a pressure of 76 cm. of mercury.

As the change of pressure of a fixed mass of gas kept at constant volume produced by a given change of temperature depends upon the original pressure of this gas, the International Bureau of Weights and Measures has specified that at the temperature of melting ice the pressure of the hydrogen shall be at 1000 mm. of mercury.

Various experimenters have used different numbers to represent these fixed points of the thermometer scale. Celsius assigned the number 0 to the temperature of melting ice, and 100 to the temperature of steam rising from water boiling under a pressure of 76 cm. of mercury. As the temperature interval between the two fixed points is divided into one hundred parts or degrees, this scale is usually called the *centigrade scale*. It is the thermometric scale ordinarily employed whenever the metric system of units is employed. Fahrenheit assigned the number 32 to the temperature of melting ice, and 212 to the temperature of steam rising from water boiling under a pressure of 76 cm. of mercury. The Fahrenheit scale is usually employed whenever the F.P.S. system of units is employed.

Replacing t_1 and t_2 by 100 and 0, respectively, and representing the corresponding pressures of the hydrogen in the constant volume thermometer by P_1 and P_2, then from (121), the number which represents on this thermometer the temperature of the hydrogen when at some intermediate pressure P is

$$t_c = \left\{ (100-0)\frac{P-P_2}{P_1-P_2}+0 \right\} \text{ degrees centigrade.} \quad . \quad . \quad (122)$$

This is the method by which temperatures are determined with the standard constant-volume gas thermometer.

Similarly, representing the fixed points of the thermometer by 32 and 212, respectively, the number which represents the same temperature as above is

$$t_f = \left\{ (212-32)\frac{P-P_2}{P_1-P_2}+32 \right\} \text{ degrees Fahrenheit.} \quad . \quad (123)$$

The relation between the number which represents on the constant volume hydrogen thermometer a given temperature according to the centigrade scale, and the number which represents the same temperature expressed in Fahrenheit degrees, can be obtained by dividing each member of (123) by the corresponding member of (122). Thus,

$$\frac{t_f-32}{t_c}=\frac{180}{100}.$$

Whence $$t_c=\tfrac{5}{9}(t_f-32), \quad . \quad . \quad . \quad . \quad . \quad . \quad (124)$$

and $$t_f=\tfrac{9}{5}t_c+32. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (125)$$

199. The Mercury-in-glass Thermometer.—For sensitivity of its indications, range of temperature for which it is applicable, and consistency with one another of the results obtained by it, the constant-volume hydrogen thermometer leaves little to be desired. On the other hand its large size, together with the care necessary for its manipulation and the amount of computation necessary in making a temperature determination render it inconvenient for ordinary commercial use.

The great majority of temperature determinations are made by means of the mercury-in-glass thermometer. This consists of a capillary glass tube terminating in a bulb. The bulb and a part of the stem are filled with mercury. In order to keep the mass of mercury in the instrument constant and also to prevent dirt from getting into the stem, the end of the stem opposite the bulb is usually hermetically sealed.

When such an instrument is raised in temperature, both the glass envelope and the contained mercury expand. The expansion of the mercury would cause the end of the mercury thread to rise while the expansion of the glass would cause it to fall. But since unit volume of glass expands less than unit volume of mercury for the same increase of temperature, the resultant effect of an increase of temperature is an increase in the length of the thread of mercury in the stem.

A mercury-in-glass thermometer, like every other commercial temperature measuring instrument, is standardized by comparison with the constant-volume hydrogen thermometer.

Since mercury freezes at $-38.8°$ C., the mercury-in-glass thermometer cannot be used for determining low temperatures. For such temperatures thermometers are used in which alcohol, toluene or pentane takes the place of the mercury.

If we denote by v the apparent increase in volume of the mercury in a mercury-in-glass thermometer when changed in temperature from 0° to 100°, we might denote by 200° the temperature that would produce an apparent increase in volume of $2v$. In the same manner, if we indicate by r the increase in electric resistance of a platinum wire when changed in temperature from 0° to 100°, we might denote by 200° the temperature that will cause the resistance to change by the amount $2r$. But on placing the mercury-in-glass thermometer in the space in which the platinum resistance thermometer indicates 200°, we would find that the mercury-in-glass thermometer would indicate 203°.

200. Units of Heat Quantity.—Heat being energy, it would be logical to measure quantities of heat in ergs or joules (1 joule $=10^7$ ergs). But since heat effects are more often connected with changes of temperature than with mechanical processes, it is usually more convenient to adopt as the unit of heat the quantity of heat required to raise the temperature of unit mass of water from $t°$ to $(t+1)°$.

It is shown by experiment that the number of ergs of work required to raise unit mass of water through one degree is different at different temperatures. It is found, however, that to raise the temperature of a given mass of water from freezing to boiling requires one hundred times as much heat as that required to raise it from 15° C. to 16° C. Consequently the unit of heat quantity is often taken to be the amount of heat required to raise one gram of water from 15° C. to 16° C. This is called the *gram calorie at* 15° *C.*, or the *mean gram calorie* or the *lesser calorie.* When the foot-pound-second system of units is employed, the unit of heat quantity is usually taken to be the heat required to raise the temperature of one pound of water from 60° F. to 61° F. This is called the *British thermal unit at 60° F.* In the following pages the former unit will be referred to as the "calorie" and the latter as the "B.t.u." Another frequently employed unit of heat quantity is the *greater* or *kilogram calorie.* The kilogram calorie equals 1000 gram or lesser calories.

The relation between the calorie and the erg can be found by various methods in which work is turned into heat. For example, if water be churned, the energy supplied can be measured in ergs, and the heat developed can be measured in calories. It is found that to develop one calorie of heat, there must be supplied 4.19 (10^7) ergs of work. Hence the *mechanical equivalent of heat*, that is, the number of work units in one heat unit, is 4.19 (10^7) ergs per calorie. In subsequent problem work we shall use the approximate value 4.2 (10^7) ergs per calorie.

SOLVED PROBLEMS

PROBLEM.—From the definitions of the two units of heat, find the number of calories in one B.t.u.

SOLUTION.—1 B.t.u. will raise 1 lb. water 1° F.
" " 454 gm. " "
" " 454×$\frac{5}{9}$ " " C.

Therefore, 1 B.t.u. equals 252 calories, nearly.

PROBLEM.—Assuming that the mechanical equivalent of heat is 4.19 (10^7) ergs per calorie, find the value of the mechanical equivalent of heat expressed in foot-pounds per British thermal unit.

SOLUTION.—41,900,000 dyne-cms. will raise 1 gram of water 1° C.

$\frac{41,900,000}{980}$ cm.-gm. " " " " "

$\frac{41,900,000}{980\times 30.5}$ ft.-gm. " " " " "

$\frac{41,900,000}{980\times 30.5}$ ft.-lb. " " 1 pound " "

$\frac{41,900,000(\frac{5}{9})}{980\times 30.5}$ " " " " " 1° F.

Therefore, 778 ft.-lb. of work will develop 1 B.t.u.

201. Thermal Capacity and Specific Heat.—If the same quantity of heat be imparted to equal masses of water and of mercury at the same temperature, it will be found that the temperature of the mercury will be raised very much more than that of the water. In general, different quantities of heat are required to change the temperatures of different bodies by equal amounts. The ratio of the heat required to change the temperature of a body to the change in temperature thereby produced is called the *mean thermal capacity of the body* for the given range of temperature. Thus, if H units of heat are required to change the temperature of a body from t_2°, to t_1°, then the mean thermal capacity of the body between t_2° and t_1° is

$$C=\frac{H}{t_1-t_2}. \qquad (126)$$

If $t_1=(t_2+1)$, this ratio is called the *thermal capacity of the body at* $t_2°$. The thermal capacities of bodies are different at different temperatures, and their mean thermal capacities are different for different temperature ranges.

If the mean thermal capacity of a body be divided by the mass of the body, the quotient obtained is called the *mean thermal capacity of the substance* of which the body is composed. Thus, representing by c the mean thermal capacity of a given substance,

$$c=\frac{C}{m}. \qquad (127)$$

In this equation, substituting for C the value obtained from (126),

$$c=\frac{H}{m(t_1-t_2)}. \qquad (128)$$

Whence, the mean thermal capacity of a substance numerically equals the number of units of heat required to change the temperature of unit mass of the substance one degree.

Solving the above equation for H, we have for the number of heat units required to raise the temperature of a mass m of a substance of mean thermal capacity c from temperature t_2 to t_1,

$$H=mc(t_1-t_2). \qquad (129)$$

The difference in the thermal capacities of various substances may be illustrated by means of the apparatus represented in Fig. 213. This consists of four cylinders of equal masses and external dimensions made of iron, brass, tin and lead. When all the cylinders are raised to the same temperature by being immersed in hot water they are placed on a cake of paraffin wax.

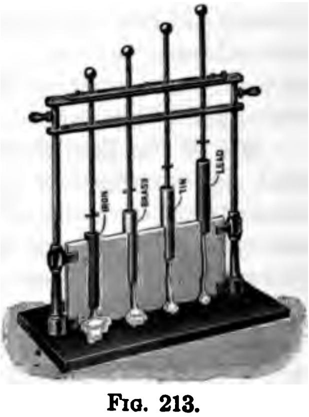

FIG. 213.

Each cylinder melts a hole in the wax until its temperature has fallen to the melting point of the paraffin. The iron sinks deepest into the wax, then brass, tin, and lead, in the order named. This experiment shows that in cooling through the same temperature range, equal masses of different substances give up different quantities of heat. That is, the thermal capacities of different substances are different; that of iron is greater than that of brass, that of brass is greater than that of tin, that of tin is greater than that of lead.

The ratio of the mean thermal capacity of a given substance for a given temperature range, to the thermal capacity of water at 15° C.,[*] is called the *mean specific heat* of the substance for the assigned range of temperature. Thus, representing by s the mean specific heat of a substance which for an assigned temperature range has a mean thermal capacity c,

$$s=\frac{c}{c'}$$

where c' is the thermal capacity of water at 15° C.[*]

From the definition of unit quantity of heat (Art. 200), the thermal capacity of water at 15° C. is one calorie per gram per degree centigrade. Consequently, the specific heat of a substance numerically equals the mean thermal capacity of the substance for the same temperature range. And since the thermal capacity of water is nearly the same at all temperatures, and the thermal capacity of other substances does not vary greatly with temperature, there is little error in taking the specific heat of any substance to be numerically equal to its mean thermal capacity for any ordinary temperature range.

202. Water Equivalent.—The mass of water which has a thermal capacity equal to that of a given body is called the *water equivalent* of the body. It is the mass of water which requires the same quantity of heat as the given body in order to change its temperature one degree.

The amount of heat required to produce a temperature change of one degree of a body of mass m and thermal capacity per unit mass c, is cm. If the mass of water which requires the same quantity of heat as the given body in order to produce the same change of temperature be denoted by e,

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[*] Or 60° F., if the F.P.S. system of units is employed.

$$1e = cm.$$

That is, the water equivalent of a body of mass m and thermal capacity per unit mass c equals cm.

If the mass of the body and the thermal capacity per unit mass be unknown, the water equivalent can be experimentally determined by the method described in the following Article.

203. Determination of Thermal Capacity by the Method of Mixtures.—Experiment indicates that when a number of bodies of different temperatures are brought together, the amount of heat lost by the bodies that fall in temperature equals the amount of heat gained by the bodies that rise in temperature.

In the Method of Mixtures the specimen whose thermal capacity is required is raised in temperature and then immersed in liquid at a lower temperature contained in a suitable vessel provided with a thermometer, stirrer and other accessories. An apparatus for the measurement of heat quantity is called a *calorimeter*. In the present case, the calorimeter includes the vessel in which the mixing occurs, the thermometer and the stirrer. After immersion, the specimen, the calorimeter, and the contained liquid attain a common temperature. The specimen loses heat, while the calorimeter and its contents gain heat. If the system neither gains heat from the surroundings nor loses heat to the surroundings, then the amount of heat lost by the specimen equals the sum of the quantities of heat gained by the liquid and by the calorimeter.

Consider a specimen of mass m, thermal capacity c and temperature t to be placed in a mass m_1 of liquid of known thermal capacity c_1, and temperature t_2. Let the common temperature of the system after the immersion of the specimen be t_1. Then the heat lost by the specimen is $cm(t-t_1)$, and the heat gained by the liquid in the calorimeter is $c_1m_1(t_1-t_2)$. Representing the water equivalent of the calorimeter by e, the amount of heat gained by the calorimeter is $1e(t_1-t_2)$.

Consequently, if the calorimeter and its contents neither gain heat from, nor lose heat to the surroundings, we have the equation

$$\left(\begin{array}{c}\text{Heat lost by}\\ \text{specimen}\end{array}\right)=\left(\begin{array}{c}\text{Heat gained}\\ \text{by liquid}\end{array}\right)+\left(\begin{array}{c}\text{Heat gained by}\\ \text{calorimeter}\end{array}\right)$$

$$cm(t-t_1)=c_1m_1(t_1-t_2)+1e(t_1-t_2). \qquad (130)$$

From this equation the value of c, the thermal capacity required, can be obtained when e, the water equivalent of the calorimeter, is known. If the masses and thermal capacities of the materials composing the calorimeter be known, the water equivalent can be obtained by computation as explained in the preceding article. Often, however, these data are unavailable. In this case the water equivalent may be obtained from another experiment as follows. Let a mass m' of hot water at temperature t' be poured into a calorimeter containing a mass m'_1 of colder water at temperatire t'_2. Let the temperature of the mixture be t'_1. If no heat is either lost to the surroundings or gained from the surroundings, then the heat lost by the hot water equals that gained by the cold water plus that gained by the calorimeter. That is, since without sensible error we may consider the thermal capacity of water for all temperature ranges equal to unity,

$$\left(\begin{array}{c}\text{Heat lost by}\\ \text{hot water}\end{array}\right)=\left(\begin{array}{c}\text{Heat gained by}\\ \text{cold water}\end{array}\right)+\left(\begin{array}{c}\text{Heat gained by}\\ \text{calorimeter}\end{array}\right)$$

$$1m'(t'-t'_1)\doteq 1m'_1(t'_1-t'_2)+1e(t'_1-t'_2). \qquad (131)$$

Since every quantity in this equation except e is known, the water equivalent of the calorimeter can be determined. On substituting in (130), the value of e thus obtained, the thermal capacity of the given specimen may be determined.

Approximate values of the mean specific heats of some ordinary substances are given below:

Aluminum	0.219	Lead	0.032
Alcohol (ethyl)	0.685	Nickel	0.113
Brass	0.093	Paraffin (solid)	0.560
Copper	0.093	Paraffin (liquid)	0.710
Granite	0.193	Silver	0.056
Hydrogen (const. press.)	3.4	Turpentine	0.467
Hydrogen (const. vol.)	2.4	Water	1.000
Iron (wrought)	0.108	Water, ice (−20 to 0° C.)	0.504
Iron (steel)	0.117	Water, steam (100° to 125° C.)	0.479

204. The Two Specific Heats of a Gas.—If a substance expands against a force, it will do work at the expense of either energy supplied from outside or the internal energy of the substance. If heat be imparted to a gas there will usually be an increase of temperature. If the volume increases, the gas thereby doing work, the rise in temperature will be less than it would be if the volume had remained constant. It follows that more heat is required to raise the temperature of a specimen of gas by a given amount when the volume changes than when the volume remains constant. Consequently, the specific heat of a gas at constant pressure is greater than the specific heat of the same specimen at constant volume. For example, the ratio of the specific heat of oxygen at constant pressure, to the specific heat at constant volume, at 0° C., is 1.4.

SOLVED PROBLEM

PROBLEM.—An audience of a thousand people is assembled in a church on an evening when the out-door temperature is 40° F. In order to ventilate the room efficiently air must be introduced at the rate of one-third of a cubic foot per second for each person present, and for the sake of comfort it must be warmed to 70° F. Find the amount of heat, expressed in British thermal units, that must be supplied in 1.5 hours, assuming that a cubic foot of air weighs 1.3 ounces, and its specific heat is 0.24.

SOLUTION.—The heat gained by m lb. of air of specific heat 0.24, in rising from 40° F. to 70° F.

$$[=cm(t_1-t_2)]=0.24(1000\times\tfrac{1}{3}\times\tfrac{1.3}{16}\times 1.5\times 3600)\ (70-40)=\text{B.t.u.}$$

QUESTIONS

1. What are the conditions which govern the choice of materials used in thermometers? Compare alcohol and mercury, each in glass, as to the above conditions.

2. Describe the mercury-in-glass thermometer and state its advantages and disadvantages, as compared with a hydrogen thermometer.

3. A mercurial and an alcohol thermometer have their scales divided into 100 equal spaces between the freezing and the boiling points of water. Does one division on the mercurial thermometer represent the same variation in temperature as one division on the alcohol thermometer?

4. In designing a mercury-in-glass thermometer, how would the following changes affect the reading?

(*a*)—an enlargement of the bore;
(*b*)—an enlargement of the bulb;
(*c*)—a downward displacement of the scale;
(*d*)—air above the mercury thread;
(*e*)—placing the thermometer in a horizontal position.

5. Two metals of equal mass but unequal thermal capacities are heated through the same range of temperature. How do the quantities of heat absorbed by each compare? Two metals of equal volume but unequal densities and specific heats are heated through the same range of temperature. How do the quantities of heat absorbed by each compare?

6. Equal masses of two liquids are placed in similar vessels, suspended in a room, and allowed to cool. If during the same time No. 1 falls through twice the difference of temperature through which No. 2 falls, how do the thermal capacities of the liquids compare? Explain fully.

7. With a piece of platinum, water, calorimeter, standard masses and scales at your command, how would you proceed to determine the temperature of a furnace? Explain fully.

§ 2. *Change of State.*

205. Transformation Points.—If heat be abstracted from a mass of steam, the temperature will fall until a certain temperature is reached. If the abstraction of heat be continued, the steam will gradually condense without any change of temperature until all of the steam is condensed. After all of the steam is condensed, a farther abstraction of heat will produce a fall in the temperature of the water until another certain temperature is reached. If the abstraction of heat be continued, the water will gradually solidify without any change of temperature until all of the water is frozen. After all of the water is frozen, a farther abstraction of heat will produce a fall in the temperature of the ice. The relation between the temperature and time for ordinary water, in cooling from above 100° C. to below 0° C., is given by the full line in Fig. 214. The points at which the temperature of a substance is unchanged

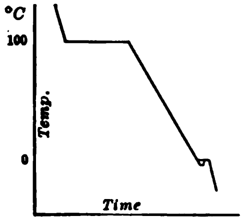

FIG. 214.

by an abstraction (or addition) of heat are called the *transformation points* or *transformation temperatures* of the given substance.

At the transformation temperatures a body that is falling in temperature emits heat, and a body that is rising in temperature absorbs heat. For this reason the transformation points obtained in cooling are called *recalescence points,* and those obtained on heating are called *decalescence points.* For some substances the recalescence points do not coincide with the decalescence points.

Water has two transformation points—one at the temperature at which steam condenses into water and another at the temperature at which water freezes. Many substances have more than two transformation points. For example, besides the freezing and vaporizing points, solid steel may have one, two or three other transformation points. The number and the temperature of these points depend upon the amount of carbon present. The transformation points on cooling steel of various carbon content are indicated in Fig. 215. At these points occur changes in the volume, hardness, crystalline structure and fineness of grain of the steel. These changes do not take place instantly but extend over an appreciable time.

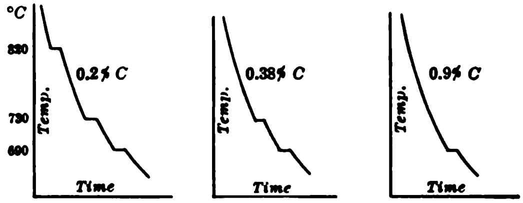

Fig. 215.

The physical properties of steel at any particular temperature can be rendered practically permanent by sudden cooling or quenching. This is the basis of hardening and tempering. The process of annealing consists in slowly heating the specimen to the temperature at which the desired size of grain occurs, and then cooling the specimen so slowly that there is sufficient time for the crystalline structure to change to that which is normal to iron at atmospheric temperatures.

The addition to steel of certain amounts of tungsten, chromium or vanadium causes an elevation of the transformation points to the temperature of red heat. Such steels will hold a cutting edge even when at a dull red heat. In machine shop practice these alloys are called "high-speed" steels.

206. The Melting Point.—If heat be added to a piece of ice at a temperature below 0° C., the ice will rise in temperature until it attains 0° C. A further addition of heat is accompanied by a change of part of the solid to liquid without alteration of temperature. If heat be added while part of the substance is solid and part is liquid, some of the solid will melt; while if heat be taken from the mixture, some of the liquid will solidify. But so long as part of the specimen is solid and part is liquid, and the mixture is well stirred, the temperature will remain constant however much heat is added to, or taken from the mixture. The temperature at which the solid and liquid states of a substance can exist together in equilibrium, is called the *fusion point* or *melting point* of the given substance. The melting point is a transformation point.

Every crystalline substance has a definite melting point. For such a substance the temperature at which melting begins is the same as that at which solidification begins. But non-crystalline substances, such as glass, iron and paraffin, soften and so pass gradually into the liquid state. These substances have no definite melting point. The temperature at which non-crystalline substances begin to turn from the solid to the liquid state is not the temperature at which they begin to turn from the liquid to the solid state.

The melting point of an alloy cannot be computed from the melting points of the components. For example, a mixture of four parts by weight of bismuth (melting point 269° C.), one part of cadmium (melting point 320° C.), two parts of lead (melting point 327° C.), and one part of tin (melting point 232° C.), makes an alloy that melts at 61° C.

A solution freezes at a lower temperature than the pure solvent. The depression of the freezing point is proportional to the sum of the number of molecules and of ions contained in unit mass of the pure solvent.

On melting and on solidifying, most substances abruptly change in volume. A few expand on solidifying as water, antimony, bismuth and gray cast iron.* The greater number, however, contract during solidifying, as wax, copper, lead and white cast iron. On account of this fact sharp castings can be taken of substances belonging in the first class, but not of substances in the second class.

207. Undercooling.—A liquid not in contact with any of the solid substance can be cooled below the melting point without solidifying. For example, by very gradually cooling air-free water in sealed capillary tubes the temperature can be reduced to −16° C., before freezing occurs. The addition of a fragment of the solid to the undercooled liquid will induce equilibrium and the liquid will quickly solidify. When a substance solidifies it gives out heat. When an under-cooled liquid solidifies the heat thereby given out raises the temperature to the ordinary solidifying point. The undercooling of water is indicated by the dent in the curve, Fig. 214.

When a piece of red-hot steel is cooled, the temperature falls somewhat below a transformation point before the physical and chemical transformations occur. When these transformations begin, there is a sudden evolution of heat and rise of temperature. The undercooling and recalescence of steel are strikingly shown by heating a wire to a red heat by means of an electric current and then switching off the current. As the wire cools, it will become almost black, when suddenly, as the transformation point is reached, it will glow brightly.

Ordinary water freezes at 0° C. Air-free water must be undercooled before it will begin to freeze. But as soon as solidification begins, it proceeds much more rapidly than in the case of ordinary water.

It is a matter of common remark that when the plumbing of a residence freezes, it is the pipes from the hot water tank that first burst. The reason is that as soon as the temperature becomes sufficiently low, the air-free boiled water in the hot water pipes quickly freezes thereby suddenly expanding outward with sufficient force to burst the strongest pipes. Though the water in the cold water pipes will begin to freeze at a higher temperature, the ice will form more slowly at the surface of the pipes and will expand inward. On account of the air in the water, this ice will be porous. Not only will there be a smaller bursting force developed by the freezing of ordinary water, but if the pipes are of equal diameter, the ordinary water will not all be frozen till after the air-free water has solidified.

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* Gray cast iron is a mechanical mixture of impure iron with uncombined graphite. White cast iron is a solid solution or alloy of iron carbide and impure iron.

208. Change of Melting Point with Pressure.—If by the application of pressure a substance that expands on solidification is prevented from expanding, more heat must be taken from the substance to cause it to solidify than if the body were allowed to expand freely. For example, to cause water to freeze when subjected to a pressure of 160 atmospheres, there must be an abstraction of heat sufficient to lower the temperature to −1° C. Conversely, ice at −1° C. will melt if subjected to a pressure of 160 atmospheres. This phenomenon is described by the statement that pressure lowers the freezing point of substances that expand on solidification. This is a special case of the general principle enunciated by Le Chatelier—*when a system in equilibrium is subject to a constraint by which the equilibrium is altered, a reaction takes place which opposes the constraint.*

This affords an explanation of the fact that two pieces of ice can be caused to freeze firmly together by simply pressing them into contact. Suppose two pieces of ice are forced together. At the points of contact the pressure is so great that at these points the melting point of the ice is appreciably lowered. That is, at the points of contact the actual temperature of the ice is above the temperature at which ice can exist under the present pressure. Consequently fusion occurs at the points of contact.

Melting requires heat. The heat that melts the ice at the points of contact is furnished by the surrounding ice. As a consequence, the ice in the neighborhood, and the water formed by the melting, are at a temperature lower than the normal melting point. When the melted ice is squeezed out from between the points of contact, it is relieved from the extra pressure. Being at a temperature below the melting point of ice under the present pressure, the water refreezes. In freezing, it gives out heat just sufficient to raise its temperature to the original temperature of the ice. The phenomenon of the refreezing of water from ice melted by pressure, when the pressure is relieved, is called "regelation."

To regelation is due the compacting of dry snow into a snowball, and the compacting of dry snow into ice under the pressure of a horse's hoofs. The flow of a glacier down a valley is dependent upon regelation. If the valley filled with snow and ice be considerably inclined to the horizon, there will be an enormous force tending to move the entire mass down the incline. This motion is hindered by the crookedness of the valley and the unevenness of the bed and sides. But if at each obstacle the ice yields under the force due to the weight of the mass farther up the valley, and then refreezes, the entire mass will gradually advance down the incline.

On the other hand pressure is favorable to the freezing of a substance that contracts on solidification. Thus pressure raises the melting point of substances that contract on solidification. For example, under a pressure of 100 atmospheres the melting point of paraffin is 3° C. higher than at atmospheric pressure.

209. Heat Equivalent of Fusion.—When heat is imparted to a solid body at the melting point, fusion is produced without change of temperature. When heat is abstracted from a liquid at the freezing point, freezing occurs without change of temperature. In the case of an under-cooled liquid, the temperature rises to the freezing point. In melting, heat is absorbed. In freezing, heat is given out.

In order to melt a unit mass of any substance there must be imparted to it a definite quantity of heat which is different for different substances. Thus, to melt one gram of ice at 0° C., requires the expenditure of 80 calories of heat, and to melt one gram of gray cast iron at its melting point requires about 25 calories. The ratio of the heat used in melting a substance at its melting point, to the mass melted, is called the *heat equivalent of fusion* of the given substance.

Thus, if H units of heat are required to melt a mass m, the value of the heat equivalent of fusion is

$$L_f=\frac{H}{m}. \qquad \ldots \ldots \qquad (132)$$

The heat equivalent of fusion is measured in calories per gram, B.t.u. per pound, etc.

From the fact that the heat absorbed by a body during fusion does not change the temperature of the body, at the time when heat was considered to be a form of matter, it was supposed that the heat absorbed during fusion exists in the melted body in a hidden or latent form. Thus, heat absorbed during fusion was then called the "latent heat" of fusion. But since it has been proved that heat is a form of energy, we believe that the heat absorbed by the body during fusion does not exist in the melted body as heat, but as some form of potential energy. Consequently, the expression "latent heat of fusion" is now out of date and has given place to the term "heat equivalent of fusion."

SOLVED PROBLEM

PROBLEM.—Assuming that 80 calories of heat will melt one gram of ice at the melting point, find the number of B.t.u. required to melt one pound of ice at the same temperature.

SOLUTION.—80 gm.° C. units of heat will melt 1 gm. ice.
80 lb.° C. units of heat will melt 1 lb. ice.
80 ($\frac{9}{5}$) lb.° F. units of heat will melt 1 lb. ice.
∴ 144 B.t.u. are required to melt one pound of ice at the melting point.

210. Determination of the Heat Equivalent of Fusion by the Method of Mixtures.—Let the specimen be placed in a liquid of such a mass and at such a temperature that it will be melted and the melted substance raised to a temperature above the melting point. If the initial temperature were below the melting point, the thermal changes are as follows. The specimen is raised in temperature to the melting point; it is melted without change of temperature; the melted substance is raised to a temperature above the melting point. The calorimeter and contained liquid fall in temperature to the final temperature of the melted specimen.

If the solid specimen were of mass m, temperature t and thermal capacity c, then in rising to the melting point t' it would absorb the quantity of heat $cm(t'-t)$. If the heat equivalent of fusion of the substance were L_f, then in melting, the specimen would absorb the quantity of heat mL_f. If the final temperature of the specimen were t_2 and the thermal capacity of the substance in the liquid state were c', then in rising from the melting point to the final temperature, the specimen would absorb the quantity of heat $c'm(t_2-t')$. The total quantity of heat gained by the specimen would then equal

$$cm(t'-t)+mL_f+c'm(t_2-t').$$

If the calorimeter were of water equivalent e, and initially was at the temperature t_1, then in falling to the final temperature t_2 the heat lost by the calorimeter would be $1e(t_1-t_2)$. If the liquid in the calorimeter were of mass m_1 and thermal capacity c_1, then in falling from its initial temperature t_1 to the final temperature t_2, it would lose the quantity of heat $c_1m_1(t_1-t_2)$.

If the system neither lost heat to the surroundings nor gained heat from the surroundings, the quantity of heat gained by the specimen must equal the quantity of heat lost by the calorimeter and contained liquid. Therefore,

$$cm(t'-t)+mL_f+c'm(t_2-t')=1e(t_1-t_2)+c_1m_1(t_1-t_2),$$

from which the value of the heat equivalent, L_f, can be obtained.

211. Heat of Solution.—A solid can pass into the liquid state by the action of a solvent. To change a solid into a liquid by the process of solution heat is required. If heat is not supplied from without, the mixture will fall in temperature. For example, by dissolving ammonium nitrate in an equal mass of water, both at 0° C., a solution is produced that has a temperature of −15° C. In some cases, however, the cooling action of solution is masked by the generation of heat due to chemical action between the solvent and solute.

A saturated solution in contact with undissolved solute at the same temperature is in stable equilibrium. In most cases the addition of heat will cause more solute to dissolve and the abstraction of heat will cause some of the dissolved substance to go out of solution. By gradually evaporating or cooling a saturated solution which contains no undissolved solute, an unstable condition can be produced that is analogous to the state of unstable equilibrium existing in an under cooled liquid (Art. 207). In the case of a supersaturated solution, as in the case of an undercooled liquid, equilibrium can be induced by dropping into the solution a fragment of the solid solute. The excess of solute will quickly solidify, and the heat thus given out will raise the temperature of the solution. This phenomenon of supersaturated solution can be shown with any highly soluble substance. An aqueous solution of sodium acetate is especially suited for the exhibition of the fall of temperature during solution and the rise of temperature during crystallization.

212. Freezing Mixtures.—The freezing point of a solution is lower than that of the pure solvent. If a dilute solution be gradually cooled, a temperature will be reached at which some of the pure solvent will freeze out. As the solution becomes more concentrated a lower temperature will be required to freeze out more of the solvent. If the cooling be continued, a temperature will be attained at which the remaining concentrated solution will freeze—solvent together with solute. This complex solid mixture is called the *cryohydrate* of the two substances. The temperature and concentration of the solution when freezing occurs are definite for any particular pair of substances.

For instance, the cryohydrate of common salt in water consists of 23.8 parts of salt and 76.2 parts of water, and its temperature of equilibrium is −22° C. If salt and ice in this proportion be intimately mixed the cryohydrate is formed. If this be at a temperature above −22° C., it will melt until by the abstraction of the heat thereby required, the temperature of the mixture becomes −22° C. This phenomenon is the basis of the action of many of the so-called "freezing mixtures."

A mixture of 143 parts of crystallized calcium chloride and 100 parts of snow, gives a temperature of −50° C.

213. Vaporization.—At some temperatures a gaseous substance can be liquefied by the application of pressure, while at other temperatures the addition of pressure will not produce liquefaction. When a gaseous substance can be liquefied by pressure alone it is called a *vapor*. When it cannot be liquefied by the addition of pressure alone, it is called a *gas*. The conversion of a liquid or solid into vapor is termed *vaporization*. Four different sorts of vaporization can be distinguished. These are evaporation, sublimation, boiling and the spheroidal state.

214. Evaporation and Condensation.—If heat be added to water at a temperature below 100° C., the water will rise in temperature, that is to say, the molecules will be given greater kinetic energy. A molecule within the liquid will be struck from all sides by neighboring molecules, but a molecule at the free surface will be struck only from below and on the sides. Consequently, at the surface there will occasionally be molecules moving upward so rapidly that in spite of the attraction of the molecules below them, they will escape into the space above. The conversion of a liquid into a vapor is termed *evaporation.* The rate of evaporation depends upon the nature of the liquid, the temperature of the liquid, the temperature of the space above the liquid, the area of the free liquid surface, and the vapor pressure on the free liquid surface.

While in the vaporous condition, the molecules continue to move and hit one another. On account of these impacts, and on account of the weight of the molecules, some of these molecules will find their way back and become entangled within the liquid surface. The reduction of a vapor to a liquid is termed *condensation.*

If evaporation takes place in a closed space there may be a time when the rate of condensation equals the rate of evaporation. When the liquid and vapor are thus in equilibrium, the vapor is said to be *saturated.*

215. Vapor Pressure of a Liquid.—The force which causes a liquid to vaporize is called the *vapor pressure* of the liquid. The pressure of the saturated vapor of a liquid equals the maximum vapor pressure of the liquid at the given temperature. The maximum vapor pressure of a liquid increases when the temperature of the liquid is raised. The maximum vapor pressures, in centimeters of mercury, of a few substances, at 0° and 100° C., are given below.

	0° C.	100° C.		0° C.	100° C.
Mercury	0.00004	0.028	Ethyl Alcohol	1.27	169.75
Water	0.458	76.000	Ethyl Ether	18.44	495.33

216. Saturated and Unsaturated Vapors.—If the volume of an unsaturated vapor be diminished while the temperature is maintained constant, the pressure will increase and the vapor become more nearly saturated. If the volume be sufficiently diminished the vapor will become saturated, and the pressure will become equal to the maximum vapor pressure of the given liquid corresponding to the temperature of the vapor.

If the volume of a saturated vapor be diminished while the temperature is maintained constant, some of the vapor will condense, the remainder will remain saturated, and the pressure will remain unchanged. *For a saturated vapor maintained at constant temperature, the pressure is constant.*

If the volume of a saturated vapor, not in contact with any of the liquid, be increased while the temperature is maintained constant, the vapor will become unsaturated and the pressure will become less. If a saturated vapor, not in contact with any of the liquid, be allowed to expand without doing work and without receiving or losing heat, it will become unsaturated and lower in temperature.

In order that equilibrium may be maintained between a liquid and its saturated vapor, any increase of pressure must be accompanied by an increase in temperature, and any decrease of pressure must be accompanied by a decrease in temperature. *There is but one temperature at which a given liquid and its vapor, under an assigned pressure, can exist together in equilibrium.*

217. Critical Temperature.—If the temperature of a gas be above a certain value which is characteristic of the particular substance, the gas cannot be condensed by the application of any pressure however great. The temperature above which it is impossible to liquefy a gas by any pressure however great is called the *critical temperature* of that gas. The pressure of a gas at the critical temperature is called the *critical pressure.*

The critical temperature and the corresponding critical pressure of some familiar substances are given below:

Substance	t° C.	Atmospheres
Air	−140	39
Ammonia	130	115
Carbon dioxide	31	73
Water	374	195

218. Sublimation.—If solid camphor, arsenious oxide, ammonium carbonate or mercuric chloride be left for some days in a closed bottle, the sides of the bottle will become encrusted with crystals of the substance. These are examples of a solid turning directly into a vapor and back to a solid without passing through the liquid state. The phenomenon of the formation of a vapor from a solid and its condensation into a solid without passing through the liquid state is called *sublimation.*

Substances that sublime have a higher vapor pressure in the solid state than at the same temperature in the liquid state. A solid will sublime at atmospheric pressure if its vapor pressure exceeds the pressure on its surface. The temperature at which the vapor pressure of a solid equals the atmospheric pressure is called the *sublimation point.* Substances that sublime at atmospheric pressure cannot be melted except when under a pressure greater than the atmospheric pressure.

At temperatures below 31° C., carbon dioxide can be liquefied by the application of sufficient pressure. If liquid carbon dioxide be allowed to expand quickly, heat will be abstracted from it at such a rate that the liquid will become a snow-like solid at −78° C. At −78° C., and atmospheric pressure, solid carbon dioxide sublimes. Subliming carbon dioxide affords a convenient means for producing temperatures as low as −78° C. Sulphuric ether is often mixed with the solid carbon dioxide, thereby forming a slush which will come into better contact with the object which it is desired to cool.

219. Boiling.—If sufficient heat be supplied at the bottom of an open vessel filled with ordinary water, bubbles will form at the bottom and will increase in size due to evaporation from the enclosing liquid. The first bubbles will rise till they are condensed by the colder liquid above. The sound produced by the condensation of these bubbles constitutes the "singing of the tea kettle." Later, bubbles will rise to the surface and burst. The phenomenon of the formation of bubbles of vapor within a liquid, their rise and bursting at the surface, is called *boiling.*

The quantity of heat necessary for the expansion of a bubble depends directly upon the pressure of the vapor within the bubble. For a bubble of vapor to form within a liquid, the vapor pressure within the bubble must equal the sum of the pressure on the free liquid surface, the hydrostatic pressure due to the liquid above the bubble, the surface tension, and the force of cohesion of the liquid. The surface tension is very small, and the force of cohesion may be reduced to zero by the presence of gas or solid particles uncombined with the liquid.

The temperature of a bubble within the liquid equals the temperature at which the given liquid and its saturated vapor can exist together in equilibrium under the pressure to which the bubble is subjected. As the bubble rises through the liquid the pressure on its surface decreases and the temperature falls. When the bubble bursts at the surface, the temperature is that at which the liquid and its vapor can exist together in equilibrium under the pressure existing at the free surface. The temperature at which a liquid and its vapor can exist together in equilibrium when under a pressure equal to that supported by the free liquid surface is called the *boiling point* of the given liquid at the given pressure.

There is a sharp distinction between evaporation and boiling. Evaporation occurs at the free surface, occurs at almost any temperature, and ceases when the pressure of the vapor above the free surface equals the pressure of saturated vapor at the given temperature. In boiling, the vapor is formed within the liquid, the temperature must have a definite value and boiling ceases when the sum of the pressure of the vapor and of the air above the free surface exceeds the pressure of the saturated vapor of the liquid.

A liquid boils when the pressure of its vapor equals the external pressure. If the pressure does not change, the temperature of the boiling liquid remains constant as long as there is any liquid to vaporize.

The temperature of any boiling liquid is higher than the boiling point of the liquid. For a boiling liquid in which the cohesion has not been diminished by the presence of uncombined gas or solid particles, the temperature may be several degrees above the boiling point. Pure water that is air and dust free can be raised to 105° C. in a clean glass dish. A globule of water 1 cm. in diameter suspended in an oil of the same density can be heated to 110° C. without boiling.

The boiling point of a given liquid depends only on the total pressure to which the free surface of the liquid is subjected, whereas the temperature of the boiling liquid depends upon the material and roughness of the containing vessel, the presence of a dissolved substance, and the presence of uncombined gas or dust particles.

At the low atmospheric pressure on Pike's Peak, Colo., water boils at 86° C. At such a low temperature, the cooking of foods is very slow, and in the case of some foods, is impossible. The required temperature, however, can be readily obtained by means of a kettle having a tightly fitting lid provided with a pressure gauge and safety valve. Such pressure cookers are in common use in both factories and private homes for hastening the cooking of certain foods and for securing the temperatures necessary for the rapid sterilization of products that are to be canned. Some of the bacteria always present on meats and vegetables can be killed by subjection to a temperature of 100° C. for hours, or by a treatment to a higher temperature for minutes.

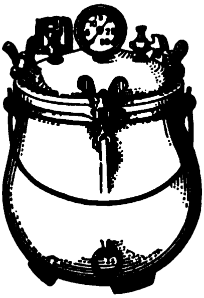

FIG. 216.

The physical properties of rubber mixed with sulphur are profoundly modified by moderate temperature changes. Depending upon the sulphur content and the temperature to which the mixture is subjected, the product may be suitable for making elastic bands or for hair combs. The vulcanizing temperature is usually between 135° C. and 145° C. The definite desired temperature is obtained from the steam of water boiling under the corresponding pressure.

The mercury-in-glass thermometer cannot be used for the measurement of temperatures as high as the boiling point of mercury. But by filling the space above the mercury with gas under high pressure, the boiling point of the mercury can be raised so much that the instrument is available for the measurement of temperatures up to 900° F.

When a liquid is boiling under a low pressure the rate of vaporization is greater than when boiling under a higher pressure. The amount of heat required to raise a liquid to its boiling point and to vaporize it at that temperature is less when the pressure is low than when the pressure is high. Therefore, vaporization can be produced more rapidly and more economically by boiling the liquid under low pressure. In the manufacture of condensed milk, the pressure is reduced till boiling occurs at about 55° C. The heat is supplied by means of coils of steam pipe within the vacuum pan. In addition to the economy of evaporating at low temperature, the low temperature avoids the possibility of certain undesirable chemical changes in the product which are apt to occur at higher temperatures.

In the manufacture of sugar, the cane juice or beet juice is boiled under diminished pressure. Uusually the vaporization is carried on successively in three or more vacuum pans under progressively lower pressures. In the case of three-stage vaporization, the juice is first boiled under a vapor pressure of about 26 inches of mercury. The steam rising at about 205° F. is passed through coils within the second vacuum pan containing juice which has been previously boiled in the first pan. The pressure in the second pan is about 13 inches of mercury. The steam rising from the juice in the second pan, at about 172° F., is passed through coils within the third pan which contains juice previously boiled in the second pan. The pressure in the third pan is about 3 inches of mercury, and the steam rising from the contents is at about 115° F.

220. Boiling with Bumping.—If ordinary water be boiled in a smooth glass vessel, small bubbles of vapor will form, rise quietly to the surface and there burst. This constitutes ordinary boiling. But after the boiling has been continued for some time, fewer bubbles will form and these will expand with almost explosive violence. This phenomenon is called "boiling with bumping."

Boiling with bumping is due to the cohesion of the liquid molecules. Ordinary water contains gas and dust particles uncombined with the liquid molecules. The presence of these particles between the water molecules diminishes the cohesion. But by continued boiling these particles are either carried off by the escaping vapor or are caused to combine with the liquid molecules. On account of the increase in the force now to be overcome, the temperature necessary to form a bubble of vapor is much greater than before. But when started, the elastic force of the bubble will overcome the cohesion of the liquid in its surface and thus make the pressure on its surface less than it was when the bubble was formed. Being now subjected to a less pressure than when formed, the bubble will expand so rapidly as to suggest an explosion.

Bumping can be prevented by supplying the liquid with particles of some foreign substance that will not readily combine with the liquid. Bits of pumice stone will prevent the bumping of boiling water. Bumping can also be diminished by the addition of sharp cornered fragments of any insoluble substance. Bubbles form more readily about the sharp points than on the smooth surface. If there are many such points many small bubbles will be formed at one time instead of a few large ones. The effect of projecting points is largely due to the greater quantity of heat supplied in a given time to the small element of volume of the liquid surrounding a point than to an equal volume of liquid at a smooth surface.

221. The Temperature of a Boiling Solution.—In order to separate the solvent from the solute thermal energy must be supplied. Consequently, to boil a solution of a nonvolatile substance a higher temperature is required than to boil the pure solvent. The elevation of the temperature of a boiling solution above the boiling point of the solvent is directly proportional to the sum of the number of molecules and of ions of the solute contained in unit mass of the pure solvent.

The temperature of the steam escaping from a boiling solution is that of the upper layer of the solution. But if this steam condenses on the bulb of a thermometer, the latter will indicate the boiling point of the pure solvent.

222. The Spheroidal State.—If a small quantity of water be dropped on a polished surface maintained at a temperature above 200° C., the liquid will not spread over the heated surface nor come into contact with it, but will assume a spheroidal shape and remain poised on a thin cushion of vapor. The cushion of vapor supporting the spheroidal mass is maintained by rapid vaporization of the liquid. The temperature of the spheroidal mass remains several degrees below the boiling point of the liquid, while the temperature of the supporting vapor is nearly that of the hot plate.

A striking experiment illustrating the fact that in the spheroidal state the temperature of liquids remains much below the temperature of the containing dish is the following due to Faraday. A mixture of ether and solid carbon dioxide is placed in a red-hot crucible. Though the heat supplied to the mixture causes rapid evaporation, the temperature of the mixture remains so low that if some mercury is poured into the mixture the mercury will freeze.

It is probable that many boiler explosions have been due to the water being so low that the boiler plates have become sufficiently heated to cause the water to assume the spheroidal state. On the sudden introduction of more water the boiler plates will become so cooled that the water and metal will come into contact, thereupon causing such a rapid evolution of steam as to produce an explosion.

223. Heat Equivalent of Vaporization.—Although a liquid boiling under constant pressure does not change in temperature, heat must be supplied to maintain the boiling. In fact, for any form of vaporization, whether evaporation, sublimation, boiling or the spheroidal state, heat must be supplied. If heat is not supplied from without, the necessary quantity of heat will be abstracted from the body itself and the temperature of the body will fall. By allowing liquid helium to evaporate at a pressure of 1.2 mm. of mercury, a temperature of $-271.3°$ C., has been reached.

When no change of temperature occurs, the ratio of the heat required, to the mass of substance vaporized is called the *heat equivalent of vaporization.**

Thus, if H units of heat are required to vaporize m units of mass of a given substance, the value of the heat equivalent of vaporization L_v, is

$$L_v = \frac{H}{m}.$$

Heat equivalent of vaporization is expressed in calories per gram, B.t.u. per pound, etc.

The heat equivalent of vaporization of liquids is usually referred to the boiling point under a pressure of 76 cm. of mercury. The heat equivalent of vaporization of water boiling under a pressure of 76 cm. of mercury is 538.7 calories per gram. The heat equivalent of vaporization increases when the pressure decreases. For problem work, we shall take the heat equivalent of vaporization of water at a pressure of 76 cm. of mercury to be 539 calories per gram or 970 B.t.u. per lb. These values should be memorized.

The heat equivalent of vaporization of a substance can be determined by the Method of Mixtures in the same manner as already applied to the determination of the heat equivalent of fusion. If the vapor is passed into a liquid contained in a calorim-

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* Formerly the term "latent heat of vaporization" was used instead of heat equivalent of vaporization, but now the term "latent heat" is out of date.

THE BOILING POINTS, AND VALUES OF THE HEAT EQUIVALENT OF VAPORIZATION OF WATER

Pressure in mm. of mercury	Temperature in ° C.	Heat Equivalent of Vap. in calories per gram	Pressure in mm. of mercury	Temperature in ° C.	Heat Equivalent of Vap. in calories per gram
4.579	0	595.4	355.1	80	551.1
9.205	10	590.2	525.8	90	544.9
17.51	20	584.9	760.0	100	538.7
31.71	30	579.6	1074.5	110	532.3
55.13	40	574.2	1488.9	120	525.6
92.30	50	568.4	2025.6	130	518.6
149.19	60	562.8	2709.5	140	511.5
233.53	70	556.9	3568.7	150	504.1

eter, the quantity of heat gained by the calorimeter and contained liquid equals the sum of the quantity of heat lost by the vapor during condensation and the quantity of heat lost by the condensed vapor in falling to the temperature of the mixture. The mass of vapor condensed is obtained by weighing the calorimeter with its contents before and also after the experiment.

A dry thermometer indicates the same temperature whether exposed to a breeze or protected from it. But a person feels cooler when a breeze is blowing because the current of air, by dissipating the vapor near the body, facilitates evaporation.

The degree of humidity of the air is measured by the difference in the indications of two thermometers—one with a dry bulb and the other with a bulb kept moist by a wick dipping into water. If the air be saturated there will be no evaporation and the two thermometers will indicate the same temperature. If the air be unsaturated, the indication of the thermometer with the wet bulb will be lower than the other by an amount depending upon the degree of humidity of the air.

In the same way, on a warm day, when the air contains much moisture, we feel much warmer than on a dry day of the same temperature.

Rheumatism is commonly treated by applying heat to the parts of the body affected. It is found that the skin will not bear a wet cloth at a temperature much above 135° F., whereas it will bear dry air as high as 350° F. In the latter case, the dry air induces such rapid evaporation that the temperature of the skin does not attain a higher value than when the wet cloth was applied.

On removing the cover from a pot of boiling water some steam escapes. If an amount of steam equivalent to 10 drops of water (about 0.5 cc.) should escape, the pot of water would lose sufficient heat to raise the temperature of one pound of water more than 1° F. "A watched pot never boils."

If steam at 100° C. be passed into a saturated aqueous solution at near the same temperature, some of the steam will be condensed. For every gram of steam that is condensed, the solution will receive nearly 539 calories of heat. This heat will raise the temperature of the solution till the vapor pressure of the solution becomes equal to that of water at 100° C. If the vapor pressure of the solution at 100° C. be less than that of water at 100° C., steam will be condensed till the boiling point of the solution is reached. That is, steam at 100° C. will raise the temperature of the aqueous solution to a higher temperature. In this manner, steam at 100° C. will raise the temperature of an aqueous solution of calcium chloride to about 112° C.

SOLVED PROBLEM

PROBLEM.—In the manufacture of catsup, tomato pulp at 80° F., enters an open kettle and is evaporated by the heat given up by high pressure steam traversing a coil of pipe within the kettle. Suppose that the mean boiling point of the pulp is 219° F., that the saturated steam enters the coil at 328° F., that this steam condenses at 219° F., and the condensed steam escapes at 216° F.

Assuming that the heat equivalents of vaporization of water at 219° F., and at 216° F., are 961 and 963 B.t.u. per lb., respectively; and that the specific heat of tomato pulp and that of steam are 0.9 and 0.48, respectively, find the number of pounds of water that must be evaporated by the boiler in order to reduce 1000 lb. of tomato pulp to 500 lb.

SOLUTION.—

Heat gained by the pulp = heat lost by the steam.

$$0.9\times 1000(216-80)+500\times 963=0.48m(328-219)+m961+1m(219-216)$$

where m represents the number of pounds of water that must be evaporated by the boiler.

224. Mechanical Refrigeration.—The fact that a vaporizing liquid absorbs considerable heat is utilized for producing low temperatures. As ammonia gas is cheap and can be liquefied by pressure at ordinary temperatures, this substance can be advantageously employed for mechanical refrigeration. One process used for making ice artificially and for cooling cold storage rooms is represented diagrammatically in Fig. 217.

In its simplest form the apparatus consists of a gas compressor *P* connected to two coils of pipes *A* and *D* through valves *B*, *C* and *E*, as shown. On the upstroke of the piston, ammonia gas is drawn into the compressor from the coil of pipes *A*. On the downstroke this gas is largely liquefied and forced into the coil *D*. The large amount of heat developed in the substance by the compression is absorbed by cold water circulating about the coil *D*. The cooled liquefied ammonia is allowed to escape through the regulating valve *E* and expand into the gaseous form in the coil *A*. This vaporization and expansion entails such a great absorption of heat that the coil *A* and its surroundings are cooled considerably below the freezing point of water.

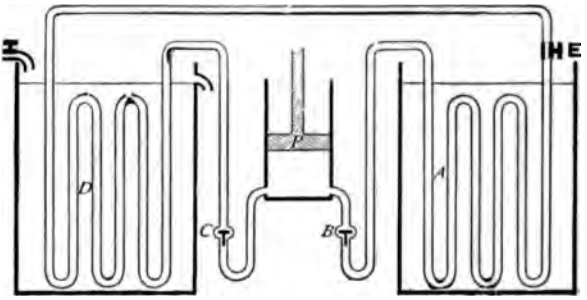

FIG. 217.

The coil *A* is usually immersed in a tank of brine. If a cold storage room is to be cooled, this brine is pumped through coils of pipes suspended on the walls and ceiling of the room. If articifial ice is to be made, cans filled with the water to be frozen are placed in the tank of cold brine.

The lowest temperatures that have been attained have been produced through the cooling developed by a suddenly expanding gas. Linde has developed a device in which by expanding one portion of a compressed gas at atmospheric temperature, the remainder of this compressed gas is considerably lowered in temperature. By allowing a portion of this cooled compressed gas to expand, the remainder is still further cooled. By continuing this process, the following low temperatures have been attained.

Boiling point of liquid air	−185 ° C.
Boiling point of liquid hydrogen	−252.5° C.
Boiling point of liquid helium	−271.3° C.

225. Heat of Reaction.—For any particular substance, the number of grams numerically equal to the molecular weight of the substance is called the *gram molecule* or *mol* of the substance. For example a gram molecule of $ZnSO_4$ is $[64.9+31.8+4\times15.9]=$ 160.3 gm. The gram molecule divided by the number of atoms of hydrogen equivalent in combining power to the metal of the molecule is called the *gram equivalent* of the substance. For instance, since zinc has twice the combining power of hydrogen, the gram equivalent of $ZnSO_4$ is $\frac{1}{2}(160.3)$ gm.

In the case of a chemical reaction, the number of calories of heat developed by molecular quantities of the reacting substances is called the *heat of reaction.* The reaction of one gram molecule of zinc and of one gram molecule of sulphuric acid in dilute aqueous solution produces an evolution of 37,730 calories. The combination of molecular quantities of copper and sulphuric acid in dilute aqueous solution is accompanied by an absorption of 12,400 calories. Contrariwise, the decomposition of one gram molecule of $CuSO_4$ is accompanied by an evolution of 12,400 calories.

The total evolution of heat by a chemical system in passing from one state to another is independent of the intermediate states. This is called the law of Hess.

226. Heat Value of Fuels.—The number of units of heat developed by the complete oxidation of unit mass (or unit volume, for gases) of a substance is called the *heat value,* thermal value or calorific value of the substance. The heat values of domestic fuels such as coal, wood and illuminating gas vary through a wide range. Good anthracite coals vary from 12,000 to 14,000 B.t.u. per lb.; semi-bituminous (e.g., Pocahontas) coals from 13,500 to 15,000, and bituminous coals from 10,000 to 14,000. The heat value of seasoned hard wood varies from 8300 to 8600 B.t.u. per lb. For resinous woods like pine and fir the heat value may be as high as 9150 B.t.u. per lb. Crude petroleum varies from 17,000 to 21,300 B.t.u. per lb.

Coal gas, produced by distilling bituminous coal, has a heat value of about 600 B.t.u. per cu. ft. Water gas, produced by blowing steam through a bed of incandescent coke, has a value of about 300 B.t.u. per cu. ft. Most commercial gas works produce a mixture of coal gas and water gas. Both the light and heat values of water gas can be raised by introducing oil into the white hot retort. The resulting carburetted water gas will be increased in heat value through a considerable range by an amount proportional to the amount of oil added. One gallon of oil will increase the volume about 75 cu. ft., and the thermal value of the gas about 85 B.t.u. per 1000 cu. ft. As usually supplied, illuminating gas has a heat value of about 600 B.t.u. per cu. ft.

COST OF HEAT PRODUCED BY CERTAIN DOMESTIC FUELS

Fuel	Cost	Heat Value	B.t.u. for one cent
Anthracite coal...	$10.00 per ton	14,000 B.t.u. per lb.	28,000
Bituminous coal..	6.00 per ton	12,000 " "	40,000
Seasoned oak....	12.00 per cord of 3700 lb.	8,300 " "	25,591
Kerosene........	0.22 per gal. of 6.7 lb.	20,000 " "	6,091
Illuminating gas...	1.00 per 1000 cu. ft.	600 " per cu. ft.	6,000
Electricity.......	0.10 per kilowatt hour	....	381

Questions

1. What property must a metal possess in order that it shall give a good impression of the mold when cast? Name such a metal. Name one that will not give good castings.

2. What effect has the pressure on the melting point of substances that expand on solidifying? Those that contract on solidifying? Explain.

3. A piece of solid type metal floats on the surface of liquid type metal. Does this substance expand or contract on solidifying? Explain. What effect does this quality have on the process of type casting?

4. In order to keep the contents cool, farmers often cover their water jugs with a piece of wet carpet. Explain the action.

5. Steam at 140° C. will not burn the flesh as severely as steam at 100° C. Explain.

6. The hot water from a steam radiator may be as hot as the steam which entered it. How then has the room been warmed?

7. Open vessels filled with water are frequently placed in cellars to prevent freezing of vegetables in cold weather. Explain fully.

8. A little alcohol sprinkled on a thermometer bulb causes an immediate lowering of the mercury, even though the thermometer shows no change in temperature when the bulb is immersed in the alcohol. Explain.

9. Why does steam produce so much more severe burns than the same mass of hot water at the same temperature?

10. The sweat secreted from the skin holds the temperature of the body at its normal value even though the surrounding air is warmer than the body. Explain fully. Why is summer heat often oppressive before a shower?

11. A vertical cylinder is one-third filled with water, and immediately above the water is fitted an air-tight piston. The piston is then drawn out nearly to the end of the cylinder. If no heat passes in or out through the piston or the walls of the cylinder, how does the temperature of the water change? Explain.

12. Explain how the height of a mountain may be measured by means of thermometer readings.

13. The boiling point of a salt solution is higher than that of pure water. If steam at 100° C. is passed into such a solution it will cause it to boil. Explain.

14. It is impossible to cook beans by boiling them in an open kettle on the top of a high mountain. Explain.

15. How would you expect a liquid to behave when heated to the boiling point, provided that the heat equivalent of vaporization were zero?

16. In a closed vessel is contained some warm water which has recently ceased boiling. How may the water be made to boil again without applying heat to the vessel?

17. What is the change of state which boiling water is undergoing? Is the boiling water receiving heat? If so what becomes of it? Is the water getting warmer?

18. What limits the elevation of the temperature inside of a loaf of bread or a piece of meat that is being baked?

19. Why will water extinguish fire?

20. Why is a pan of water often put into an oven in which custards are baking?

§ 3. *Expansion*

227. Linear Expansion.—With few exceptions bodies expand while increasing in temperature. It is found that for small temperature ranges the increase in length of a body under constant pressure varies directly as the original length, nearly as the change in temperature, and that it is independent of all other quantities. That is, if a body of length l_0 at $o°$ be raised in temperature to $t°$, the increase in length

$$(l_t - l_0) \propto l_0,$$

and

$$(l_t - l_0) \propto (t - o).$$

Whence,

$$l_t - l_0 = a l_0 t. \qquad (133)$$

where a is a constant of proportionality. This constant is called the mean coefficient of linear expansion of the given substance from o° to t°.

Solving for a,

$$a = \frac{l_t - l_0}{l_0 t}. \qquad (134)$$

Therefore, the *mean coefficient of linear expansion* of any substance between o° and t° may be defined as the ratio of the average change in length of a bar of that substance for one degree change in temperature, to the length of the same bar at o°. Hence, the temperature coefficient of linear expansion is numerically equal to the change in length per degree change in temperature per unit length at o° C.

For any given substance, a is slightly different for different temperature ranges. However, for most substances, the change with temperature is very small and in this book will be neglected.

If, then, a rod which has a length l_0 at o° be heated successively to temperatures t°_1 and t°_2, its length at these two temperatures will be, (133),

$$l_1 = l_0 + a l_0 t_1,$$

and

$$l_2 = l_0 + a l_0 t_2.$$

Consequently, on being heated from t°_2 to t°_1, the increase in length of the rod is

$$l_1 - l_2 = l_0 a(t_1 - t_2). \qquad (135)$$

In the case of solids, the coefficient of expansion is so small that for ordinary temperatures l_2 differs so little from l_0 that it is usually accurate enough to write (135) in the form

$$l_1 - l_2 = l_2 a(t_1 - t_2).$$

Whence

$$l_1 \doteq l_2[1 + a(t_1 - t_2)]. \qquad (136)$$

The mean coefficients of linear expansion of several substances from 0° C. to 100° C. are given below:

Brass	0.000019	per ° C.
Copper	0.000017	"
Iron	0.000012	"
Nickel	0.000013	"
Platinum	0.000009	"
Ordinary glass	0.000009	"
Invar*(0.36 Nickel, 0.64 Iron)	0.0000091	"
Silver Iodide (Amorphous)	−0.000001	"
Iceland Spar, parallel to axis	0.000025	"
Iceland Spar, perpendicular to axis	−0.000006	"
Quartz (fused)	0.0000005	"

* The alloy called invar not only has a very low coefficient of expansion, but it changes in length so sluggishly that two months are required for the complete change. For these reasons invar is useful for constructing standard meter bars.

A small space must be left between railway rails in order that each rail may change in length with changes in temperature without distorting the track.

Steel bridge girders are mounted on rollers so that the change in length produced by changes of temperature shall not disturb the piers. As the pavement has a smaller coefficient of expansion than the supporting steel girders, provision must be made to avoid buckling and cracking of the pavement when the bridge is subjected to considerable temperature changes.

In riveting boilers and the steel members of buildings and bridges, the rivets are placed in the holes while red hot and then a head hammered onto the end of the shank. On cooling, the contracting shank draws together the two pieces of steel with great force.

If a thick piece of glass be quickly changed in temperature it will crack on account of unequal expansion. Due to the same phenomenon a sheet of glass or a glass vessel may be divided along any predetermined line by "leading" a crack along the line by means of a piece of hot metal.

The coefficients of expansion of fused quartz and of "pyrex" glass are so small that objects of these materials may be plunged while red hot into cold water without breaking.

In designing an object to be made of cast iron, care must be exercised that the shape and thickness of the different parts are such that all parts will cool at about the same rate. Otherwise, on cooling, unequal contractions will occur which will produce internal strains. Such internal strains will greatly reduce the strength of the object.

In order that a clock pendulum may have a constant period of vibration, the equivalent length must be constant. Two devices are in common use for maintaining the length constant while the temperature changes. In one device, the bob consists of a reservoir of mercury, Fig. 218. If the temperture rises, the pendulum rod lengthens, thereby lowering the center of mass. At the same time, the upper surface of the mercury rises, thereby raising the center of mass of the pendulum. With the proper relation between the lengths of the pendulum and the mercury column, the center of mass of the pendulum will be at a constant distance from the supporting knife-edge, whatever the temperature.

Another compensation pendulum consists in a bob attached to a frame composed of alternate rods of brass and steel. With the rods arranged as in Fig. 219, the bob will be lowered by an increase in the length of the unshaded rods, and raised by an increase in the length of the shaded rods. By making the lengths of the two sorts of rods inversely proportional to the coefficients of linear expansion of the materials of which they are composed, the equivalent length of the pendulum will remain constant however the temperature may change.

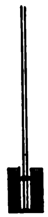
FIG. 218.

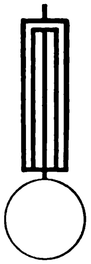
FIG. 219.

Large guns are made of a series of concentric rings or tubes, each shrunk on the next inner one like the tire of a wagon wheel. After a limited number of shots have been fired, the inner tube becomes so deformed or roughened that it must be replaced by another. To remove this inner tube, the entire gun is heated, and cold water squirted into the bore. The sudden contraction of the inner tube permits its easy withdrawal.

228. Cubical Expansion.—If a body of volume V_0 at $o°$ be raised in temperature to $t°$ while the pressure is kept constant, it will increase in volume by the amount

$$\gamma V_0 t,$$

where γ is the mean coefficient of cubical expansion at constant pressure—or the coefficient of dilation— from $o°$ to $t°$.

Consequently, the volume of the body at $t°$ is

$$V_t = V_0 + \gamma V_0 t = V_0(1+\gamma t). \quad . \quad . \quad . \quad . \quad (137)$$

From this equation,

$$\gamma = \frac{V_t - V_0}{V_0 t}.$$

Therefore, the mean coefficient of cubical expansion at constant pressure of any substance between $o°$ and $t°$ may be defined as the ratio of the average change in volume for one degree change in temperature, to the volume at $o°$. Hence, the temperature coefficient of cubical expansion is numerically equal to the change in volume per degree change in temperature per unit volume at $o°$ C.

For any given substance γ is slightly different for different temperature ranges. However, for most substances the change with temperature is small, and in this book it will be neglected. If, then, a body which has a volume V_0 at $o°$ is heated successively to temperatures t_1 and t_2, its volumes at these temperatures will be

$$V_1 = V_0 + \gamma V_0 t_1,$$

and

$$V_2 = V_0 + \gamma V_0 t_2.$$

Consequently, when heated from $t°_2$ to $t°_1$, the change in volume is

$$V_1 - V_2 = V_0 \gamma (t_1 - t_2). \quad . \quad . \quad . \quad . \quad . \qquad (138)$$

In the case of solids and liquids, the coefficient of cubical expansion is so small that for ordinary temperatures V_2 differs so little from V_0 that we are usually sufficiently accurate if in place of the preceding equation we write

$$V_1 - V_2 = V_2 \gamma (t_1 - t_2). \quad . \quad . \quad . \quad . \quad . \qquad (139)$$

In the case of gases, the coefficient of cubical expansion is so great that we are not accurate enough in writing V_2 in place of V_0. Consequently for gases the approximate equation (139) is inadmissible.

For considerable temperature ranges, the coefficient of expansion of any given gas at constant pressure is very nearly constant, and for all gases it has very nearly the same value. When temperatures are expressed in centigrade degrees, the coefficient of expansion of gases is about 1/273, and when expressed in Fahrenheit degrees it is about 1/459. Consequently the volume of a gas at $t°$ C. is, (137),

$$V_t [= V_0(1 + \gamma t)] = V_0\left(1 + \frac{t}{273}\right), \quad . \quad . \quad . \quad . \qquad (140)$$

and at $t'°$ F. the volume is

$$V_t' = V_0\left(1 + \frac{t'}{459}\right). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (141)$$

The fact expressed by these two equations is called Charles' Law.

229. Relation between the Coefficient of Linear and of Cubical Expansion.—Consider a solid cube, each edge of which at o° has a length l_0. Its volume at o° is $V_0 = l_0^3$. If the temperature be increased 1°, each edge will attain a length, (133), $l_1 = l_0(1+a)$, and the volume at this temperature will be $V_1 = V_0(1+\gamma)$.

Thus the volume at 1° is

$$V_1[=l_1^3=(l_0+l_0a)^3]=l_0^3+3l_0^2(l_0a)+3l_0(l_0a)^2+(l_0a)^3. \quad . \quad (142)$$

The volume at 1° is made up of the original cube, Fig. 220, together with three square slabs, each having an area of base l_0^2 and thickness l_0a, three rectangular strips of length l_0 and area of cross section $(l_0a)^2$, and a cube of edge (l_0a), Fig. 221.

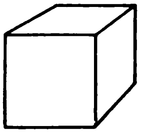

FIG. 220.

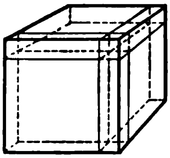

FIG. 221.

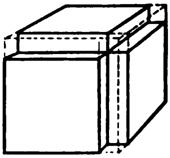

FIG. 222.

If a be sufficiently small compared with l_0, the terms of (142) containing a^2 and a^3 are negligible. That is, the volume of the cube in Fig. 221 and expressed by (142) approximately equals the volume shown in Fig. 222 and expressed by the equation

$$V_1 \doteq l_0^3+3l_0^3a.$$

But $$V_1[=V_0(1+\gamma)]=l_0^3+l_0^3\gamma.$$

Whence, $$\gamma \doteq 3a.$$

That is, the coefficient of cubical expansion is approximately equal to three times the coefficient of linear expansion. For this reason, tables of physical constants seldom give coefficients of cubical expansion of solids.

230. Uniform and Ununiform Expansion.—By arbitrary convention the ratio between any two changes of temperature is taken to be the ratio between the changes in pressure of a fixed mass of hydrogen, kept at constant volume, when subjected to the given temperatures (Art. 197). A body that changes in volume in proportion to the change of temperature as measured by a normal thermometer, is said to expand uniformly, or to have a constant coefficient of expansion. This is equivalent to saying that a body has a constant coefficient of expansion when the ratio between any two changes in volume equals the ratio between the changes in pressure of a fixed mass of hydrogen kept at constant volume and subjected to the same temperature changes.

For all temperatures between the freezing point and the boiling point of water, mercury changes in volume in very nearly the same proportion that hydrogen at constant volume changes in pressure when subjected to the same temperature changes. At temperatures much higher than the boiling point of water, the ratio of the changes in the volume of mercury when the temperature is changed, does not very nearly equal the ratio of the changes in the pressure of hydrogen kept at constant volume, when subjected to the same temperature changes. Consequently, mercury is said to expand nearly uniformly from 0° to 100° C., but not at temperatures much above 100° C. The same is true for glass. For this reason, a mercury-in-glass thermometer with a uniform bore may have one hundred equal spaces between the freezing point and boiling point of water, and at all temperatures between 0° and 100° C., the indications will be nearly correct.

On the other hand, water expands so ununiformly that it can scarcely be said to have a coefficient of expansion even for a limited range of temperature. In fact, if the temperature of water be raised from the freezing point, it will first contract. This contraction continues till the temperature is about 4° C. Above that temperature water expands when the temperature is increased, but not in the same ratio as the pressure of hydrogen increases for the same change in temperature.

231. The Fundamental Law of Perfect Gases.—If the temperature of a gas be kept constant, then through a considerable range of pressure, the volume varies inversely with the pressure. It also varies directly with the mass. That is,

$$V \propto \frac{m}{P} \text{(when temp. is constant).}$$

This is one form of Boyle's Law.

If the pressure of a fixed mass of gas be kept constant, then through a considerable range of temperature, we have, from Charles' Law, (140), when temperatures are expressed according to the centigrade scale,

$$V \propto \left[1+\frac{t}{273}\right] \text{(when pressure is constant).}$$

A gas that would obey Boyle's law at all pressures and temperatures is called an *ideal* or *perfect* gas. For an ideal gas, the above variations give the equation,*

$$V=C\frac{m}{P}\left[1+\frac{t}{273}\right]$$

where C is a constant of proportionality. From this equation we obtain

$$PV=\frac{C}{273}m(273+t).$$

Denoting the constant quantity $\frac{C}{273}$ by the symbol R, we have

$$PV=Rm(273+t).$$

If we were to measure temperature from a zero which is 273° C. below the centigrade zero, and denote these temperatures by T, the preceding equation would become

$$PV=RmT. \quad . \quad . \quad . \quad . \quad . \quad . \quad (143)$$

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* "If A varies as B when C is constant, and A varies as C when B is constant, then will A vary as BC when both B and C vary."—Hall and Knight, Higher Algebra, p. 23.

This is called the Fundamental Law of Perfect Gases. All gases obey this law for a certain range of temperature and pressure which is different for different gases. No actual gas, however, obeys this law for all temperatures and pressures.

If we take as the unit of mass the number of grams numerically equal to the molecular weight of the gas, then for any gas the *gas constant* R has the value 83.15(10^7) ergs per ° C. In all of our problems this constant cancels out and so its value need not be memorized.

Unsaturated vapors not near the temperature of condensation very nearly obey the law of perfect gases. Saturated vapors do not obey this law or the law of Charles or that of Boyle.

232. The Ideal Gas Temperature Scale.—The simplicity of the fundamental law of ideal gases is utilized in constructing a temperature scale in which the zero point is 273 centigrade degrees below the centigrade zero (or 459 Fahrenheit degrees below the Fahrenheit zero), and in which temperatures are expressed in terms of the change in pressure of a fixed mass of ideal gas kept at constant volume.

According to the Ideal Gas Temperature Scale, *the ratio between two temperatures equals the ratio between the pressures of a fixed mass of ideal gas at constant volume when at the given temperatures.*

Only at temperatures above 1000° C. is the departure of the Normal Hydrogen Scale so much as one degree centigrade from the Ideal Gas Scale. Since the departure of hydrogen from being an ideal gas has been determined, it is possible to reduce an observed "normal temperature" to the corresponding "ideal gas temperature."

Temperatures reckoned from the absolute zero, (−273° C., or −459° F.), are called *absolute temperatures.* Thus, 20° C. equals 293° C. Absolute, and 20° F. equals 479° F. Absolute. It should be kept in mind that T in (143) represents the temperature reckoned from the absolute zero.

SOLVED PROBLEM

PROBLEM.—A town is supplied with gas at a pressure of 3 inches of water above the atmospheric pressure. When the barometric pressure is 31 inches, and the gas temperature is 40° F., the town uses 10,000,000 cubic feet per week, and the company neither gains nor loses. Assuming that the same mass of gas is used during a week when the average barometric pressure is 30 inches, and the average gas temperature is 50° F., find the profit or loss of the company for the week.

SOLUTION.—Representing the pressure, volume, mass and absolute temperature of the gas when the company neither gains nor loses by P_1, V_1, m and T_1, respectively; and the corresponding quantities in the second case by P_2, V_2, m and T_2, respectively, we may write for the two cases,

$$P_1V_1 = RmT_1$$

and $$P_2V_2 = RmT_2.$$

Dividing each member of the latter equation by the corresponding member of the former, we obtain

$$\frac{P_2V_2}{P_1V_1} = \frac{T_2}{T_1}.$$

Hence the volume of the gas in the second case is

$$V_2 = \frac{P_1V_1T_2}{P_2T_1}.$$

Now, $P_1[=3+31\times 13.6]=424.6$ inches of water.

$P_2[=3+30\times 13.6]=411.0$ " "

$V_1 = 10{,}000{,}000$ cu. ft.

$T_1[=40+459]=499$°F. Absolute.

$T_2[=50+459]=509$°F. "

$$\therefore \quad V_2\left[=\frac{P_1V_1T_2}{P_2T_1}\right]=\frac{424.6\times 10{,}000{,}000\times 509}{411\times 499}=10{,}537{,}903 \text{ cu. ft.}$$

Consequently, for the same mass of gas there is a gain in volume of 537,903 cu. ft. There is, then, a profit of $537.90.

QUESTIONS

1. Why do iron workers sometimes use red-hot rivets? Why do water pipes sometimes burst in cold weather? Explain.

2. If air is heated and not allowed to expand, what change occurs? State the law which shows the relation between this change and the change of temperature.

3. A bicycle tire when filled with air and left standing in the hot sunshine will sometimes burst. State the law of heat which accounts for this.

4 What property of matter must be guarded against in the manufacture of clock pendulums and balance wheels for watches? How is it guarded against?

5. Why is a steel wagon tire put on the wheel while hot? When a glass stopper is fast in a bottle it can sometimes be loosened by heating the neck of the bottle. Explain.

6. An ordinary cheap alarm clock may be regulated to keep time fairly well if kept at ordinary room temperatures. If in winter the windows are thrown open so that the room cools, the clock gains. Explain.

7. There is a nickel-steel having a coefficient of elasticity that increases with rise of temperature. Show that a watch having a hair spring made of this material in connection with a brass balance wheel can be constructed which will not be affected by temperature changes.

8. If the bulb of a thermometer be plunged into hot water, the mercury at first falls. Why?

9. Illuminating gas is bought by volume. Will a customer obtain a greater mass of gas per thousand cubic feet when (*a*) the barometric pressure is high or when it is low, (*b*) when the temperature is high or when it is low?

10. Why will a cake "fall" if the oven door be opened before the loaf is done?

11. What causes the enamel to flake off of granite ware cooking utensils?

CHAPTER XV

PROPAGATION OF HEAT

§ 1. *Convection and Conduction*

233. Three Modes of Propagation.—There are three means by which one body may receive heat from another. They are called convection, conduction and radiation.

If a dish of cold water be set on a hot stove, the lower layers of the liquid will become heated, will expand and be pushed upward, thereby carrying heat to the upper layers. The transfer of heat effected by heated matter moving from one place to another is called *thermal convection*. Convection occurs in fluids in which there exist temperature differences.

If one end of a metal rod is at a higher temperature than the other, heat will travel from the hotter to the colder end. In this case the body itself does not move, but the molecules composing the body, perhaps by impact with neighboring molecules, hand on the energy from the hotter end to the colder end. The flow of heat effected by heat being handed on successively by one set of molecules to the next is called *thermal conduction*.

A thermometer placed a few feet from a red hot coal will indicate a rise of temperature even though the intervening space be occupied by a glass jar from which the air has been pumped. Since much of the space between the coal and the thermometer is devoid of matter we conclude that matter is not necessary for the transportation of whatever passes from the hot coal to the thermometer. And since we conceive heat to be the kinetic energy of particles of matter, we conclude that it is not heat that passes from the hot body through the vacuum to the thermometer. It is found that the energy emitted by hot bodies can produce interference effects. Consequently we believe that this energy is transmitted by waves. These waves are propagated by the ether and not by matter. The process by which a heated body sets up waves in the surrounding ether is called *radiation*. The energy of these waves is called *radiant energy* or *radiance*. Radiant energy is absorbed readily by some sorts of matter and transformed into the disorderly molecular motion which we call heat. Thus it appears that in the above case, the hot coal set up waves in the surrounding ether; these waves moved outward in all directions; and some of the energy of the wave motion was absorbed by the thermometer and transformed back into heat.

At noon during midsummer, on a clear day and at the latitude of New York, the earth receives from the sun an amount of radiance equal to about two calories per minute per square centimeter. If all the radiance received from the sun at midday on a clear day by a ship at the equator could be turned into work and applied to the propeller, the ship would be propelled at a speed of about ten knots.

In convection and conduction there is an actual transfer of heat, but in radiation such is not the case. In radiation, there is a transfer of energy which may be absorbed by matter and by it be transformed into heat. By radiation one body may lose heat and by absorption another body gain heat, but heat does not pass from one body to the other. Another case where heat energy disappears at one place and appears at another is that of a system consisting of a boiler, engine, belt and machine with a dry bearing. With a fire under the boiler, the dry bearing will become hot although no heat goes from the boiler to the dry shaft.

234. Convection.—The circulation of water in the hot water apparatus used for supplying the hot water taps in private houses is a familiar example of convection. The hollow casting *B* forms one side of the fire box of the kitchen range. The tank *A*, " water back" *B*, and all the pipes are kept full of water by being connected to the service pipe at *C*. After being heated, the water in the water back is less dense than that in the bottom of the tank and so is pushed upward. In this manner cold water from the bottom of the tank flows into the water back, and after being heated flows into the upper part of the tank. On opening a tap connected to the pipe above D, hot water will be drawn from the water back and from the upper part of the tank.

The heating of a house by hot water or by hot air is due to convection. The arrangement of the hot-water system is shown in Fig. 224. The entire apparatus consisting of boiler B, " radiators " R, and piping, is kept filled with water. The hot water in the boiler rises to the top and passes out through the pipe H while the more dense cold water flows into the bottom of the boiler through the pipe C. After the hot water has been cooled in the " radiators " it falls through the pipes C' and its place is taken by hot water rising through the pipe H'.

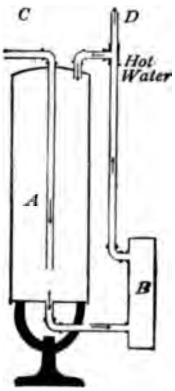

Fig. 223.

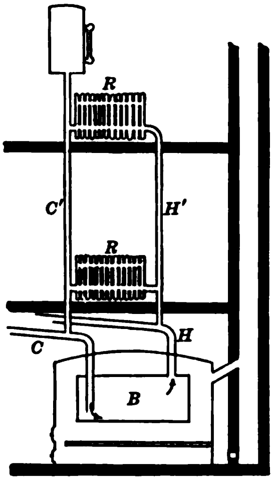

Fig. 224.

In order that the system may be kept full of water under practically constant pressure, a small tank is joined to the piping, at a point higher than the "radiators." When the water in the system expands or contracts with change of temperature, the level of the water in the expansion tank rises or falls.

Winds and ocean currents are convection currents on an enormous scale. If one portion of the earth's surface be heated to a higher temperature than the surrounding region, the air in contact with the earth will expand and form a region of low barometric pressure. The flow of air from the surrounding region into this low pressure area constitutes wind.

235. Thermal Conduction.—In thermal conduction heat is handed on from molecule to molecule in the direction of decrease of temperature. Metals are the best conductors whereas the other mineral substances are poor conductors. With the exception of liquid metals, fluids are very poor conductors. Air is such a poor conductor that it is very effective in preventing a rapid change in the temperature of any body which it envelops. But in order that the air may not convect heat, it must be prevented from circulating. Charcoal, sawdust, mineral and animal wool, and other porous bodies, owe their efficacy as nonconductors largely to the air entangled within them. Clothing and steam pipe covering, as well as the " fireless cooker " and the ordinary refrigerator are examples of the application of this fact.

The fact that metals are good conductors of heat is utilized in the Davy Safety Lamp employed to warn miners of the presence of inflammable gases. Davy's device consists in surrounding the flame with a cage made of fine wire gauze supported by massive metal rods. If such a lamp is taken into an explosive mixture, the gas that gets inside the gauze cage will ignite, but on account of the rapidity with which the heat is conducted away by the metal gauze and frame, the temperature outside the cage will not quickly rise to the point necessary to ignite the gas outside. When a "working" is found to be dangerous it is ventilated before the workmen with their ordinary lamps are allowed to enter.

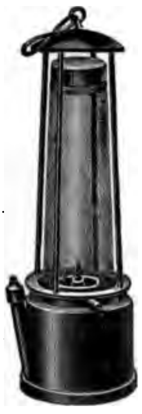
FIG. 225.

A piece of metal feels colder to the hand than does a piece of wood at the same temperature, if this temperature is below that of the body. The reason is that the metal conducts heat away from the hand more rapidly than does the wood. If the temperature of the metal and wood were higher than that of the hand, then the metal would feel hotter than the wood.

That a metal conducts heat much better than does wood may also be illustrated by passing through a flame a piece of paper stretched over a cylinder made of alternate disks of metal and of wood. It will be found that the portions of paper backed by metal remain unscorched, whereas the portions of paper backed by wood are scorched. This is due to the fact that the metal disks conducted heat away from the paper in contact with them so rapidly that these portions did not rise to a high temperature, whereas the wood disks did not conduct heat away from the paper in contact with them with sufficient rapidity to prevent a considerable rise in temperature of these portions.

If an unbaked cake in a metal dish be placed in a hot oven, heat will be so quickly conducted through the dish that the outside of the cake will harden long before the center. The subsequent expansion of the air bubbles within the dough will cause an eruption through the center of the upper crust and the consequent volcano-like form sometimes seen. If the dish be of glass or earthen ware, and the oven less hot, the upper crust will be nearly level.

To cook rice with only a little more water than is necessary to cover it requires about half an hour. But if the rice be sprinkled into a large kettle of violently agitated boiling water, only about fifteen minutes is required. In the first case the heat is slowly conducted through the mass. In the second case the heat of the boiling water passes directly into each separate grain.

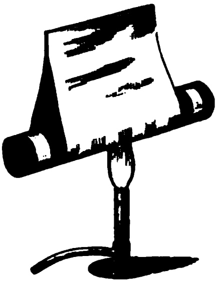

Fig. 226.

236. Coefficient of Thermal Conduction.—Let us now consider the flow of heat along a rod. Let AG, Fig. 227, represent the rod in which are imbedded the bulbs of a number of thermometers. If the end A be placed in the fire it will be observed that all the thermometers begin to rise and continue to rise for some time. After an hour or more, depending upon the material and dimensions of the rod, B will stop rising; a little later C will stop rising, and finally G will stop rising. If the temperature of the source at A does not change, the temperature of each element of the rod will be constant. The rod is said to be now in the "stationary state." Before the rod attains the stationary state, each element of the rod receives heat by conduction from the adjacent element on the left, absorbs part of this heat and rises in temperature, emits a part laterally, and hands on the remainder to the adjacent element on the right. After the stationary state has been attained no element absorbs any heat.

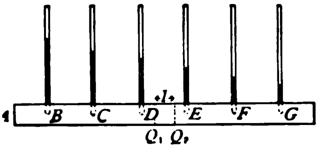

Fig. 227.

If, while in the stationary state, there were no loss of heat into the air through the sides of the rod, the heat flow would be uniform throughout the length of the rod. It is found by experiment that under these conditions, if the opposite faces of an element of length x and area of cross-section A are maintained at a temperature difference $(\Theta_1-\Theta_2)$, then the quantity of heat H conducted through the element during time t varies directly with A; varies directly with $(\Theta_1-\Theta_2)$; varies directly with t; varies inversely with x; and for a given material is independent of every other quantity. Consequently we may write

$$H=\frac{kA(\Theta_1-\Theta_2)t}{x}, \quad . \quad . \quad . \quad . \quad . \quad (144)$$

the quantity k being constant for any given material but having different values for different materials. It is called the "coefficient of thermal conduction" of the given substance. The value of the coefficient of thermal conduction is

$$k=\frac{Hx}{A(\Theta_1-\Theta_2)t}.$$

The thermal conductivity or coefficient of thermal conduction is the time rate of transfer of heat by conduction through unit thickness, across unit area for unit difference of temperature. In the C.G.S. system of units it is measured in calories per second per square centimeter for a thickness of one centimeter and a temperature difference of 1° C. In the F.P.S. system of units, it is usually measured in B.t.u. per second per square foot for a thickness of one inch and a temperature difference of 1° F. Sometimes, however, the time may be for a minute, an hour, or a day.

For many substances the thermal conductivity is very different at different temperatures. Since there are no molecules in a vacuum, no heat is conducted through a vacuum. And as in a gas under ordinary pressures the molecules are separated by considerable spaces, gases are poor conductors. If the gas occupies a large space, however, there will be considerable heat propagated by convection. Most of the available heat insulators or non-conductors are porous materials containing a large number of small air spaces. Woolen clothing, bed quilts filled with cotton and asbestos steam pipe covering, are examples.

In the following table are given representative values of coefficients of thermal conduction of several poor conductors, expressed in B.t.u. per hour conducted through a layer one square foot in area and one inch thick when the opposite faces are at a difference of temperature of 1° F.*

Substance	% air	k
Sheep's wool	95.7	0.36
Cotton	98.	0.44
Hair felt	91.5	0.56
Poplar sawdust	83.8	0.75
Mineral wool	94.3	0.50
Carbonate of magnesium	94.	0.50
Asbestos	97.	0.56

If two substances of low conductivity and widely different temperatures are separated by a plate of highly conducting material, the temperatures of the layers adjacent to the plate will be considerably different than the temperature of the surface of the plate. The difference between the temperatures of the two surfaces of a boiler plate is small compared with the differences between the temperatures of the water on one side and the fire on the other.

SOLVED PROBLEMS

PROBLEM.—Steam at 227° F. is conveyed through 1000 ft. of bare iron pipe 3 in. outside diameter laid in ground at a mean temperature of 50° F. Assuming that steam costs \$0.003 per lb., that at the given temperature the heat equivalent of vaporization of water is 960.5 B.t.u. per lb., and that bare iron pipe conducts 51 B.t.u. per sq. ft of outside surface per ° F. difference of temperature in one day, find the number of dollars lost per day due to condensation.

SOLUTION.—Heat lost in one day

$$= kA(\Theta_1 - \Theta_2) = \left[(51)1000\,\frac{2\pi 1.5}{12}(227-50)\right] = 7{,}089{,}000 \text{ B.t.u.}$$

Steam condensed in one day $\left[=\dfrac{7{,}089{,}000}{960.5}\right] = 7380$ lb.

Loss due to condensation $[=7380\times 0.003] = \$22.14$ per day.

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* Ordway—Ice and Refrigeration, 1891, p. 216.

Problem.—Find the economy effected per day by covering the pipe in the preceding problem with a layer one inch thick of Nonpareil covering which has a thermal conductivity of 7.4 B.t.u. per sq. ft., per inch thickness, per ° F., per day.

Solution.—Heat lost in one day $=7.4A(\Theta_1-\Theta_2)$. In the present case A represents the product of the length of the pipe and the mean circumference of the covering.

Heat lost in one day $\left[=(7.4)1000\frac{2\pi 2}{12}(227-50)\right]=1{,}371{,}000$ B.t.u.

Steam condensed in one day $\left[=\frac{1{,}371{,}000}{960.5}\right]=1427$ lb.

Loss due to condensation $[1427\times 0.003]=\$4.28$ per day.
Economy effected by use of pipe covering $[=\$22.14-\$4.28]=\$17.86$ per day, or 86%.

Problem.—A room 10 ft. by 10 ft. and 8 ft. high has floor, ceiling and walls of concrete 6 in. thick. It is desired to line this room with cork board of such thickness that the temperature of the room will be maintained at 50° F. lower than the outside temperature by the melting of 1000 lb. of ice per day. Assuming that the conductivity of concrete and of cork board are respectively 103 and 8.6 B.t.u. per sq. ft., per inch thickness, per ° F., per day, find the thickness of cork board required.

Solution.—For the amount of heat conducted through the concrete, we may write,

$$H=\frac{k_1A(\Theta_1-\Theta_2)t}{x_1}\quad\text{or}\quad\frac{H}{At}\left(\frac{x_1}{k_1}\right)=\Theta_1-\Theta_2.$$

Since the same heat passes through the cork board, we may write,

$$H=\frac{k_2A(\Theta_2-\Theta_3)t}{x_2}\quad\text{or}\quad\frac{H}{At}\left(\frac{x_2}{k_2}\right)=\Theta_2-\Theta_3.$$

Adding to each member of the former equation, the corresponding member of the latter,

$$\frac{H}{At}\left(\frac{x_1}{k_1}+\frac{x_2}{k_2}\right)=\Theta_1-\Theta_3.$$

On substituting in this equation the data of the problem, and remembering from the solved problem on p. 266 that 144 B.t.u. are required to melt 1 lb. of ice at its melting point, we have, if the melted ice escapes at the melting point,

$$\frac{1000\times 144}{520\times 1}\left(\frac{6}{103}+\frac{x_2}{8.6}\right)=50.$$

Whence $x_2=1$ inch (very nearly).

§ 2. *Radiation*

237. Absorbing Power and Radiating Power.—In general, when radiance is incident upon any body, part of the energy will be reflected, part absorbed and transformed into the disorderly molecular motions we call heat, and the remainder will be transmitted. Polished silver is the best reflector; it reflects about 97 per cent. of the radiant energy incident upon it. Lampblack is the best absorber; it absorbs about 98 per cent of all the energy incident upon it. Clear rock salt is the best known transmitter; a plate 0.25 cm. thick will transmit about 90 per cent. of the total radiant energy incident upon it.

If radiant energy be incident on a body consisting of molecules that have natural periods of vibration equal to the periods of the incident waves, the molecules of matter will be set into sympathetic vibration. This entails a loss of energy by the incident wave and a gain of internal energy by the absorbing body. The energy gained by the absorbing body is exhibited by a rise in the temperature of the body. That is, the radiant energy is transformed into heat energy. The heated body will now emit radiance of longer wave-length than that of the incident radiance.

Some bodies absorb the energy of waves of all periods, but most bodies absorb energy of a certain range of periods only. Lampblack transmits no radiant energy and reflects only about 2 per cent. of the energy incident upon it. A body that absorbs only waves of certain periods is said to have the property of *selective absorption.* Clear glass and pure water absorb a large part of the energy of infra red and ultra violet vibrations, but absorb very little of the energy of visible vibrations. A solution of iodine in carbon bisulphide absorbs the energy of visible vibrations, but does not absorb infra red vibrations.

After radiant energy has traversed a plate of any given material, it can traverse another plate of the same material with very little additional absorption. Some materials are such strong absorbers that for them absorption is practically a surface effect. The metals are examples.

Since the vibrations of the molecules of a radiating body set up in the ether vibrations of the same periods, and since the natural periods of the molecules of an absorbing body must be the same as the periods of the incident waves, it follows that any material that is a good radiator is a good absorber, and any material that is a good absorber is a good radiator.

Lampblack is our best absorber and our best radiator. The gases are poor absorbers and poor radiators. A burning gas that contains no solid particles emits very little energy in either luminous or nonluminous waves. But if the flame contains particles of carbon there will be considerable radiant energy emitted in both luminous and nonluminous waves.

The ratio of the radiance absorbed by a body to the radiance incident on the body during the same time is called the *absorption* of the body at the given temperature. A body that absorbs all the radiance incident upon it has an absorption of unity and is called a *black-body* or *perfect absorber.* The amount of radiance which a body will emit per second, due to heat alone, is called the *emission* of the body at the given temperature. Kirchhoff and Balfour Stewart have proved that (*a*), the ratio of the emission to the absorption of any body depends upon the temperature only; (*b*), this ratio numerically equals the emission of a perfectly black body at the same temperature.

Again, the radiance emitted by a body is limited to the same range of wave-lengths that the given body absorbs. This fact is expressed in Stokes' Law—" substances that are good absorbers of any specified kind of radiance are also good emitters of that same kind of radiance." Kirchhoff enunciated a similar law in the form, " A substance which emits waves of definite periods when heated, will selectively absorb waves of the same periods when cool." For example, clear glass absorbs very little of the radiant energy of luminous waves, and if a piece of glass be heated even to the melting point it will emit very little radiant energy in luminous waves.

The land being a better absorber of radiance than water, the temperature of the land will rise during the day time more rapidly than will that of lakes and seas. Also, on a clear night the surface of the land will cool more rapidly than the surface of a large body of water. Large bodies of water tend to keep the temperature of the surrounding region more uniform.

238. Transmission.—Of the radiant energy that enters a body, all that is not absorbed by being transformed into heat is transmitted as radiant energy and emerges as radiant energy. The property of a body for transmitting radiance is called *diathermancy;* the absence of this property is called *athermancy.* The property of a body for transmitting radiance capable of affecting the eye is called *transparency;* the absence of this property is called *opacity.*

Gases not near the point of condensation are highly diathermanous. When the atmosphere contains little moisture, a thermometer exposed to the sun's radiance may rise as high as 30° C., whereas if placed in the shade it may indicate 0° C. The cause of this difference is not far to seek. A thermometer indicates its own temperature. This will be the temperature of the surrounding body only when the thermometer is shielded from all outside thermal influences. But if the thermometer be placed within a diathermanous body exposed to radiance emitted by a body at a higher temperature than the thermometer, the thermometer will absorb radiant energy and rise in temperature, whereas the surrounding diathermanous body will not absorb any energy and will not rise in temperature. The thermometer will rise in temperature until it emits energy at the same rate it absorbs energy. If the body beyond the diathermanous body be at a lower temperature than the thermometer, the thermometer will radiate more energy than it absorbs from the colder body, and so it will drop to a temperature below that of the surrounding diathermanous body.

Water is athermanous, that is, absorbs a large portion of the radiance incident upon it. Moist air is warmer than dry air exposed to the same solar radiation. Frosts are less likely to occur when the air is moist than when dry for two reasons: the earth loses less energy by radiation when covered by moist air, and heat is liberated when any of the moisture of the air freezes. Garden plants may be protected from a light frost by a liberal sprinkling of water. On dry cold nights smoke clouds are frequently produced over orchards to prevent radiation from the earth and the consequent lowering of temperature.

Glass is diathermanous to the waves which effect vision, but is athermanous to many of the longer waves which constitute the greater part of solar radiance. This fact is applied in the "cold frame" used to protect plants from frost. The cold frame consists of a bottomless box with a glass top and resting on the ground. Most of the solar radiance transmitted by the glass is absorbed by the ground and transformed into heat. Though the warmed ground radiates energy, this energy is of such long wave-lengths that very little is transmitted by the glass. Consequently the temperature of the enclosure is higher than that of the air outside.

In regions where the winter atmosphere is very clear and not too cold, enough energy may be trapped by this "cold frame effect" to heat water within black pipes to a temperature sufficiently high for a comfortable bath.

239. Reflection.—Of the radiance incident upon a body, that which is neither absorbed nor transmitted is reflected. Polished metals are our best reflectors. Silver can be polished so that it will reflect 97 per cent. of the total radiance incident upon it. Lampblack reflects only about 2 per cent. of the incident radiance. Radiance, visible or invisible, obeys the same laws with respect to the equality of the angles of reflection and incidence that apply to other forms of wave motion (Art. 158).

The facts that vacuum is our most effective non-conductor of heat, and polished silver is our best reflector of radiance, are utilized in designing the ordinary "thermos bottle." This consists of a double-walled bottle with the space between the walls exhausted of air, and the outer side of the inner bottle and the inner side of the outer bottle coated with a thin layer of polished silver. A liquid within such a bottle will change in temperature very slowly even though the temperature outside is very different from the temperature of the liquid.

240. Kirchhoff's Black-body or Perfect Radiator.—Though no known substance fulfills the definition of perfect radiator, Kirchhoff has shown that black-body radiation can be experimentally realized. Imagine an ideal black-body within a uniformly heated athermanous enclosure and in thermal equilibrium with it. Being in thermal equilibrium with its surroundings, it radiates to the walls of the enclosure the same amount of energy it receives from them. That is, every element of area of a uniformly heated enclosure radiates as the black-body. If a small aperture be made into the enclosure, the aperture will emit radiance as from a black-body. A black-body is experimentally realized in various forms of electrically heated furnaces.

241. Temperature of, and Energy emitted by, a Black-body.—The radiance emitted by any body consists of waves of different lengths which may be dispersed by a rock salt prism and their energy determined by means of a sensitive thermometer. Imagine a radiating body to be placed in front of the slit of a spectroscope provided with rock salt prism and lenses. By replacing the eye-piece by a sensitive thermometer the radiance of different wave-lengths can be measured. Curves indicating the different amounts of energy associated with waves of different lengths emitted by a perfect radiator at various temperatures are given in Fig. 228. In this figure wave-lengths are plotted as abscissas and are expressed in thousandths of millimeters. A thousandth of a millimeter is called a *micron* and is represented by the symbol μ. Energy is plotted on the axis of ordinates according to an arbitrary scale that need not here be explained.

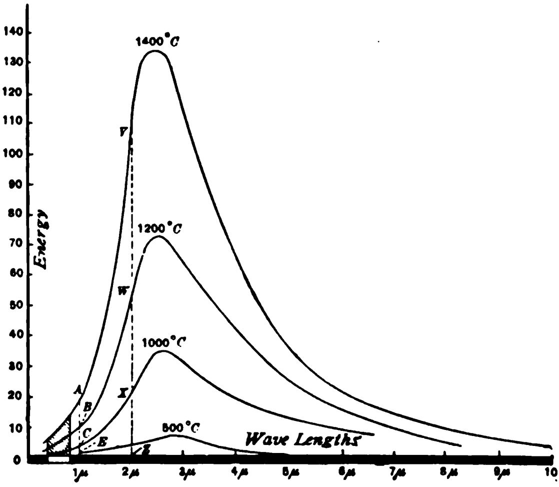

FIG. 228.

The total energy emitted by a black-body at any given temperature is proportional to the area between the corresponding energy curve and the axis of abscissas, Fig. 228. The part of the energy carried by waves of lengths between given limits is proportional to the area bounded by ordinates drawn from the assigned wave-lengths. For instance, the energy of waves of lengths extending from 1 μ to 2 μ emitted by the given black body at 1000° C. is represented by the area *ECXZ;* at 1200° C. it is represented by *EBWZ;* at 1400° C it is represented by *EAVZ.*

When the temperature of a body is raised, there is a greater increase in the energy carried by the shorter waves than in the energy carried by the longer waves. In Fig. 228 it will be observed that when the temperature of a body is raised the wave-length which corresponds to the highest point of the energy curve is displaced toward the shorter wave-lengths. Experiment indicates that through wide temperature ranges the wave-length for which the energy emitted by a black-body is maximum, varies inversely with the absolute temperature of the body. This is called " Wien's Displacement Law."

242. The Black-body Temperature Scale.—Any body at a temperature above the absolute zero radiates energy at a rate which depends upon the temperature of the body and upon the nature of the surface. If the nature of the surface be constant, the temperature of bodies can be compared in terms of their radiance. When at the same temperature, all black-bodies radiate at the same rate. It follows that the temperature of black-bodies can be compared by means of their radiance.

It was found by Stefan and Boltzman that the rate with which energy due to thermal causes is radiated by a black-body is proportional to the fourth power of the absolute temperature. This is the basis of an important temperature scale. *According to the Black-body Temperature Scale, when the energy radiated from two surfaces is due to purely thermal causes, the ratio between the absolute temperatures of those surfaces equals the fourth root of the ratio between the rates of radiation of the surfaces per unit of area.*

Two bodies will be at the same black-body temperature when the rate of their thermal radiation per unit surface is the same. It will be recalled that two bodies are at the same ideal gas temperature if, when placed in contact, they are in thermal equilibrium.

A piece of retort carbon absorbs almost all of the radiance of whatever frequency, incident upon it. Consequently retort carbon is nearly black. A piece of polished platinum absorbs partially, but to practically the same extent, radiance of all frequencies. Consequently polished platinum is gray. A lump of gold absorbs nearly all the radiance incident upon it with the exception of the waves that produce the visual sensation we call yellow. This selective absorption of gold is described by the statement that gold is yellow. If pieces of retort carbon, polished platinum and gold be placed together within a uniformly heated enclosure until they are in thermal equilibrium and be then withdrawn, it will be found that the carbon will radiate at a greater rate than the platinum or gold. That is, although all three bodies are at the same temperature according to the ideal gas scale, they are at different black-body temperatures.

In the case of a black-body the same number that expresses its ideal gas temperature is used to express its black-body temperature. But since a nonblack-body at a given ideal gas temperature radiates less than a black-body at the same ideal gas temperature, the number which expresses the black-body temperature of a nonblack-body is less than the number which expresses its ideal gas temperature.

Black-body temperatures can be determined from either the intensity of the radiance of all wave-lengths emitted by the body, or the intensity of the radiance of a single wave-length emitted by the body. The radiance of a particular wave-length can be obtained by isolating one part of the spectrum of the body and measuring its luminous intensity. The method of determining black-body temperatures from total radiance is called radiation pyrometry and the method of determining black-body temperatures from radiance of a single wave-length is called optical pyrometry. These methods are of importance when the body is either inaccessible or at such a high temperature as to destroy a thermometric device placed in contact with it.

One instrument for determining black-body temperatures by means of the radiance emitted by the body is the Féry Spiral Pyrometer, Fig. 229. This instrument consists of a concave mirror which reflects the incident radiance on a small blackened spiral, Fig. 230, which will coil or uncoil as the temperature of the spiral is increased or decreased. This sensitive spiral consists of a double ribbon of two metals of different thermal expansion coefficients. The two ribbons being fastened together throughout their length, an increase in temperature, by causing the outer ribbon to expand more than the other, will result in the spiral coiling up more closely. The scale of the instrument is empirically graduated to indicate black-body temperatures.

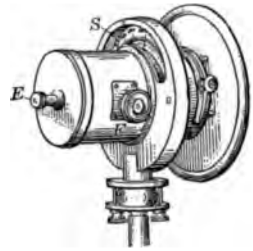

FIG. 229.

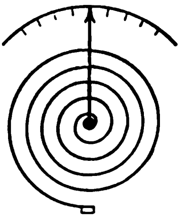

FIG. 230.

243. Relation between Light and Radiant Energy.—Radiant energy produces interference effects. Consequently it is propagated by waves. It can be polarized. Consequently these waves are transverse. The medium by which it is propagated is the ether. Radiant energy of wave-lengths between about 0.000033 cm. and 0.000081 cm. (0.33μ and 0.81μ) is capable of exciting the sensation of sight. Consequently light is that radiant energy having waves of lengths between those limits.

If a black-body be raised in temperature it will be found that below 400° C. all the energy emitted is in waves longer than those which affect the eye. At about 400° C. waves of length about 0.81μ are emitted along with the longer ones previously emitted, and the body appears dull red. As the temperature is increased shorter waves are added to those previously emitted and the body changes in color from red to yellow, and finally white. When white, the body is emitting waves of all lengths from the long ones below the red that do not affect the eye to the very short ones beyond the blue that do not affect the eye. The waves longer than 0.81μ are called *infra red* waves and those shorter than 0.33μ are called *ultra violet* waves.

The ratio of the energy of the visible portion of the radiance to the total energy emitted, is called the *luminous efficiency* of the body at the given temperature. In Fig. 228 the ratio of the shaded area to the entire area included between the 1400° curve and the axis of abscissas is the luminous efficiency of the given body at 1400° C. The curves in the figure show the great increase in luminous efficiency produced by increasing the temperature of the source.

The luminous efficiency of all artificial illuminants is very small. The luminous efficiency of arc lamps is about 10 per cent., of carbon filament incandescent lamps from 3 to 5 per cent., and of gas flames from 2 to 3 per cent. In all these cases the luminous body is carbon. The difference in the luminous efficiency is due to the different temperatures of the emitting bodies.

It is interesting to note that the fire-fly and the glow-worm have a luminous efficiency of more than 90 per cent. and that this high value is not due to a high temperature.

QUESTIONS

1. (*a*) The skin is cooled much more rapidly when placed in cold water than when placed in air of the same temperature. Why? (*b*) Men have remained for some time in rooms where the temperature was as high as 126° C. without the temperature of their bodies rising much above the normal, 37°. How was the temperature kept down? (*c*) A piece of iron at 5° feels colder than a piece of cloth at 5°. Why? (*d*) A piece of iron at 40° feels warmer than a piece of cloth at 40°. Why?

2. Describe how you would construct a vessel which will retain its contents as nearly as possible at a definite temperature when the surroundings are at variable temperature.

3. In cold weather we keep ourselves warm by a woolen blanket. In summer we may to some extent keep a piece of ice from melting by wrapping it in the same blanket. Explain.

4. Why will a moistened finger freeze instantly to a piece of metal on a cold day, but not to a piece of wood?

5. Many advertisements and references are seen to-day of the fireless cookers. Explain what device is used and state the principle which is applied in its construction and use.

6. A thermometer placed in contact with the different objects in a room shows no variation in temperature, although some of the objects feel colder than others to the hand. Explain.

7. Explain clearly why a tall chimney gives a better draft than a short one. What connection is there between Archimedes' principle and the answer to this question?

8. A thermometer placed in the sunshine on a snow-clad mountain in summer will indicate a higher temperature than when placed in the valley. Why does not the snow on the mountain melt when in the shade?

9. If a black body absorbs more radiance than a white body and both are placed on non-conducting insulated stands in a vacuum and are exposed to the same radiance, will the black body become hotter than the white? Give reason for your answer.

10. If a cake in a metal pan be baked in a hot oven, the center of the top will rise much higher than the edges. But if the pan be lined with thick paper, this bump will be less. Explain.

CHAPTER XVI

THERMODYNAMICS

244. First Law of Thermodynamics.—That branch of the theory of heat which treats of the relations between heat and mechanical work is called *thermodynamics.* The experiments of Rumford, Davy, and Joule prove, (*a*) that heat is an aspect of energy, (*b*) that energy in the mechanical form can be transformed into energy in the thermal form, (*c*) that energy in the thermal form can be transformed into energy in the mechanical form. From these facts, together with the principle of the conservation of energy—" in any self-contained system of bodies the quantity of energy remains constant during any reaction or transformation between its parts"—it follows that *when thermal energy is transformed into mechanical energy, or when mechanical energy is transformed into thermal energy, the amount of thermal energy equals the amount of mechanical energy.* This corollary of the principle of the conservation of energy is called the first Law of Thermodynamics.

It may also be expressed in the form: *when work is transformed into heat, or heat into work, the quantity of work is mechanically equivalent to the quantity of heat.*

If H represent the amount of heat transformed into mechanical work, and W the quantity of work thereby produced, the first law of thermodynamics can be expressed in the form

$$H = W.$$

In this expression heat and work must be measured in the same unit—for instance, both must be measured in ergs, in calories, in foot-pounds, in British thermal units, etc. If both forms of energy are not measured in the same unit, the above equation must be put into the form

$$JH = W,$$

in which J represents the number of units of mechanical energy in the unit of thermal energy. The number of work units in one heat unit is called the *mechanical equivalent of heat.*

The results of careful determinations by many observers using quite different methods of experiment show that the value of the mechanical equivalent of heat is very nearly

$$J = 4.19\ (10^7) \text{ ergs per calorie.}$$

To maintain the vital processes of an animal, food having a certain heat value is required. This subject has been studied by means of men performing various tasks while living for several days at a time within a calorimeter consisting of a small jacketed room. The heat value was determined of the air, food and drink supplied as well as that of the waste products. The man was weighed on entering and on leaving the calorimeter. The heat given to the calorimeter while the man was performing various tasks was measured. From such data obtained from several men during a long series of experiments it has been concluded that the number of calories required for a man, per pound of weight, per hour, according to different kinds of physical activity is about as follows—sleeping, 500; sitting quietly, 600; standing, 750; light exercise, 1000; moderate exercise, 1250–1500; active exercise 1750–2000; severe exercise, 3000 or more. Mental effort also requires a supply of energy.

245. Second Law of Thermodynamics.—It is a matter of common observation that heat can be transferred from a body at any temperature to a body at a lower temperature. Such a transfer can be effected by conduction, convection, or radiation.

Under certain conditions heat can also be transferred from one body to another at a higher temperature. For example, consider two cylinders A and B provided with air-tight frictionless pistons connected as shown in Fig. 231. Imagine that the pistons and the walls of the cylinders are impervious to heat. In A is a mass of gas at high pressure and low temperature, and in B is a mass of gas at low pressure and high temperature. If the double piston be allowed to move, the gas in A will expand while that in

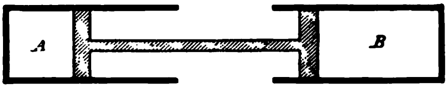

Fig. 231.

B will be compressed. During this operation work was done by the gas in A, and work was done on the gas in B. The loss of energy suffered by the gas in A manifests itself by a fall in temperature and the gain of energy by the gas in B by a rise in temperature. Consequently, the gas in A has lost heat, and the gas in B has gained heat. It thus appears that heat has been transferred from one body to another at a higher temperature.

It is to be noted that not only the temperature, but also the potential energy of A, is less after the expansion than before. The operation above described is an illustration of the law *unless energy is supplied from some outside source, no mechanism can convey heat from one body to another at a higher temperature and be in the same condition after the transfer as before.*

As the operation above described cannot be repeated unless energy is supplied to A from some outside source, this law is also expressed in the form, *it is impossible for a self-acting machine, unaided by external agency, to convey heat continuously from a body at one temperature to another body at a higher temperature.*

The fact enunciated in these two forms is called the Second Law of Thermodynamics.

If the attempt be made to transfer heat from one body to another at a higher temperature by means of any mechanism, it must be clearly understood that the second law of thermodynamics applies only if the internal energy of the mechanism at the end of the operation is the same as it was before the operation. In other words, the mechanism must not supply energy—it must only transfer energy.

246. The Indicator Diagram.—Imagine a gas inclosed in a cylinder by a piston of area A. Suppose that due to the pressure of the gas the piston is pushed forward through a distance x, thus increasing the volume of the gas by an amount ΔV. If during this expansion the pressure of the gas has changed uniformly from an initial value P_1 to a final value P_2, the average pressure during the expansion is $\frac{1}{2}(P_1+P_2)$. Since the total force acting on the piston due to the expanding gas is $F=\frac{1}{2}(P_1+P_2)A$, the work done by the gas is

$$W(=Fx)=\tfrac{1}{2}(P_1+P_2)Ax=\tfrac{1}{2}(P_1+P_2)\,\Delta V. \quad . \quad . \quad (145)$$

It will now be shown that since the magnitude of the work done equals the product of the mean pressure and the change of volume of the gas, and since the magnitude of an area equals the product of two distances, it follows that if the pressures and volumes of a gas at successive instants are plotted on coordinate axes, the work done by the gas is represented by an area on the diagram thus formed. In Fig. 232 let the pressure of the gas at successive instants be plotted along the axis of ordinates, and the corresponding volumes be plotted along the axis of abscissas. For example, if at a given instant the pressure and volume are P_1 and V_1, the condition of the gas with respect to pressure and volume is completely represented by the position of the point A on the diagram. If at a later instant the pressure and volume are P_2 and V_2, then the condition of the gas with respect to pressure and volume is represented by the position of the point B. If during this interval of time the pressure changed at a uniform rate, the condition of the gas at the various instants during this interval is represented by the straight line AB. If the gas does not expand uniformly, the line which shows the relation between the pressure and the volume at successive instants of time is not straight. The line BC represents an ununiform expansion. A line which shows the relation between the pressure and the volume of a gas at successive instants of time is called an *indicator diagram.*

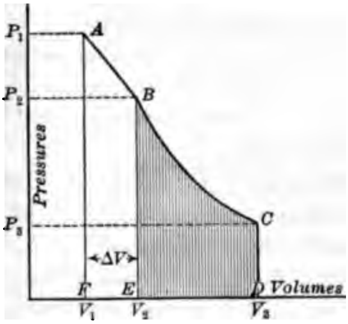

Fig. 232.

In Fig. 232, the area of the trapezoid $ABEF$ represents $\frac{1}{2}(P_1+P_2)\Delta V$. Consequently the work done by the gas in expanding uniformly from the volume V_1 to the volume V_2 is represented by the area $ABEF$.

During the interval while the gas changes its volume from the value V_2 to the value V_3 the pressure does not change uniformly. But by dividing this interval into portions sufficiently narrow, the change of pressure during one of those small changes of volume may be made as nearly uniform as we desire. Thus by dividing the area *BCDE* into strips sufficiently narrow, each strip will be a trapezoid and its area will represent the work done during the corresponding change of volume. And since the total work done equals the sum of the amounts of work done during all these small changes of volume, it follows that the total work done while the gas expands from V_2 to V_3 is represented by the area *BCDE.*

It has now been shown that if a gas expands either uniformly or ununiformly from some volume V_1 to some volume V_2, and if the pressures and volumes of the gas at successive instants be plotted on an indicator diagram, *AB*, then work is done by the gas of a magnitude represented by the area *ABEF.*

On the other hand, if the gas is compressed from a volume V_2 to a volume V_1, work is done *on* the gas of a magnitude represented by an area constructed as above described.

247. Isothermal Processes.—Any change of the condition of a body by which the temperature remains constant is called an *isothermal process.* An indicator diagram of an isothermal process is called an *isothermal line.*

In the case of a perfect gas

$$PV = RmT.$$

If the temperature is constant,

$$PV = \text{const}.$$

Since this is the equation of an equilateral hyperbola, it is seen that isothermal curves of a perfect gas are equilateral hyperbolas. A number of isothermal curves for a perfect gas at various fixed temperatures is shown in Fig. 233.

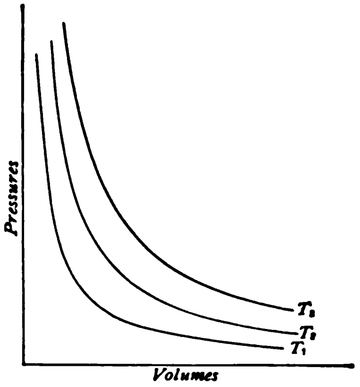

FIG. 233.

If a substance expands it does external work and loses an equivalent amount of thermal energy. If its temperature remains constant, thermal energy must be supplied from outside.

Conversely, if a substance is compressed isothermally thermal energy must be emitted by the body.

248. Adiabatic Processes.—Any change of the condition of a body by which no heat enters or leaves the body is called an *adiabatic process.* A line expressing the relation between the pressures and volumes of a body undergoing any change during which no heat is either gained or lost is called an *adiabatic line.* An adiabatic line is an indicator diagram of an adiabatic process.

If a substance expands against an opposing force, without heat being supplied from outside, its temperature will fall. If the temperature of a body falls, the pressure corresponding to any given volume will be less than if the temperature did not fall. Therefore the adiabatic line is steeper than the isothermal line. In Fig. 234 the lines *BC* and *AD* are isothermals of a perfect gas, and the lines *BA* and *CD* are adiabatics of a perfect gas.

If a substance expands adiabatically its temperature will fall. If a substance is compressed adiabatically, energy is put into it, and the temperature will rise.

If one suddenly rarefies a mass of water vapor contained in a clean flask, by means of the mouth for instance, a faint fog will be observed to fill the flask. Here the exhaustion occurred so quickly that the expansion of the vapor was adiabatic. A sufficient lowering of temperature was thereby produced to cause the vapor to condense. Precipitation of moisture in the form of rain is usually due to the cooling of moist air produced by a local adiabatic expansion of the atmosphere.

When steam escapes rapidly from a boiler three distinct divisions of the jet are often observed. At the nozzle is a clear space occupied by a column of invisible vapor at a temperature above the condensation point. This merges into a cloud of minute water particles produced by the adiabatic expansion of the vapor and the consequent cooling to the condensation point. Beyond this zone there will usually be observed a second clear space where the kinetic energy of the moving particles has been absorbed by friction through the air and heat thereby developed in sufficient amount to revaporize the condensed particles. The temperature of the visible cloud is about 100° C. The temperatures of the clear spaces are higher. Although the middle region is cooler than the others, one is more liable to be scalded in this region than in the others. The reason is that in this region water will condense on the hand and in so doing will give out the heat due to condensation: whereas, in the other regions the temperature is too high for the vapor to condense and the only heat given to the skin is that due to the fall in temperature of the small mass of vapor in contact with the skin. Since this mass is small, and the thermal capacity of water vapor is not great, the heat imparted to the skin will not be sufficient to produce a scald. Naturally, if the hand is held there for a considerable length of time, then the mass of vapor moving past the skin will not be small and a scald will be produced.

249. Reversible Cycles.—If a piece of ice at 0° C. be compressed, an amount of work W will be done on the ice, the freezing point will be lowered (Art. 208), and the ice will liquefy. If the pressure be now removed thereby allowing the undercooled liquid to expand, an amount of work W will be done by the expanding substance and ice will be reformed at 0° C. The substance is again in the initial condition. The above series of operations could start with the substance in any one of the specified conditions and could proceed in either direction.

A series of operations by which a substance after passing through various conditions is brought to the initial condition is called a *cycle*. A cycle of operations that can be traversed in both directions is said to be *reversible*.

As another example of a reversible cycle consider the following operations on a saturated vapor. Suppose a quantity of heat H to be supplied to a liquid at the boiling point contained in a cylinder having a conducting end, non-conducting sides and a non-conducting frictionless piston. If a quantity of heat H enter the liquid through the conducting end of the cylinder, a mass m of vapor will be formed. Let the vapor be maintained under constant pressure, and therefore at constant temperature. In expanding, the vapor will do an amount of work W. If now the heat source be removed, the pressure of the piston will produce a diminution of volume and a consequent condensation of the vapor (Art. 216). When the constant pressure has done work on the vapor equal to W, the mass m of vapor will be condensed and a quantity of heat H will have escaped through the conducting end of the cylinder. The substance is now in the initial condition.

In estimating the amount of work done during a cycle, no account need be taken of any changes in the internal energy of the substance, because, at the end of a cycle a substance is in the same condition as at the beginning.

250. The Carnot Cycle.—The simplest reversible thermodynamic cycle is that devised by Carnot, which consists of four operations. In two of these operations there is a work change without any temperature change, while in the other two there is a work change without the passage of any heat either into or out of the working substance. That is, two of the operations are isothermal and two are adiabatic.

Let us now consider a substance, the volume of which changes with temperature, to go through Carnot's cycle of operations. Imagine the substance to be in a cylinder one end of which is made of a perfect heat conductor, while the remainder of the cylinder and also the piston are made of non-conducting materials. In addition, imagine that we have a non-conducting stand and two large tanks of water. Let the water in one tank be at the temperature T_1, and the water in the other at a lower temperature T_2.

Let the condition of the working substance with respect to pressure, volume and temperature be that represented by the point A, Fig. 234. Most substances expand when their temperature is raised. These substances rise in temperature when their volume is adiabatically decreased. A few substances behave in the opposite manner. We will imagine that the working substance under consideration belongs to the former class.

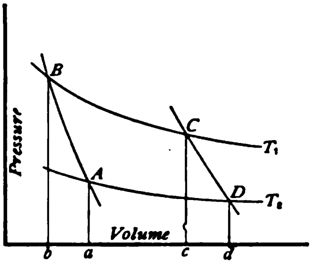

FIG. 234.

Let the working substance go through the following four processes:

First. With the working substance at the temperature, pressure and volume represented by the point A, place the cylinder on the non-conducting stand and apply pressure to the piston until the temperature of the working substance rises to T_1. The pressure, volume and temperature of the working substance is now that corresponding to the point B of the diagram. During this adiabatic process the work done *on* the working substance is represented by the area *bBAa;* and as no heat has either entered or left the substance, the heat change is zero.

Second. Place the cylinder in the tank of water of temperature T_1 and allow the substance to expand. After this expansion the condition of the substance is represented by the point C, Fig. 234. Being in the large tank of water, the temperature of the working substance remains constant. But this fact requires heat to enter the substance. Denote this amount of heat by the symbol H_1. During this isothermal process the work done *by* the working substance is represented by the area $BCcb$ and the heat absorbed equals H_1.

Third. Transfer the cylinder to the insulating stand and allow the working substance to expand adiabatically until its temperature falls to T_2. After this expansion the condition of the substance is represented by the point D, Fig. 234. During this adiabatic process the work done *by* the working substance is represented by the area $CDdc$, and the heat change is zero.

Fourth. Place the cylinder in the tank of water at the temperature T_2 and apply pressure to the piston until the substance attains its original condition represented by the point A in the diagram. During this isothermal process, the work done *on* the substance is represented by the area $DAad$ and an amount of heat that may be denoted by the symbol H_2 has left the substance.

The results of these four operations can be summarized as follows:

1st operation—Work done by substance $= -bBAa$; heat absorbed by substance $= 0$
2d operation—Work done by substance $= BCcb$; heat absorbed by substance $= H_1$
3d operation—Work done by substance $= CDdc$; heat absorbed by substance $= 0$
4th operation—Work done by substance $= -DAad$; heat absorbed by substance $= -H_2$.

Therefore the total work done by the substance is represented by the sum of the areas

$$-bBAa+BCcb+CDdc-DAad;$$

and from the diagram this sum is seen to be equal to the area $ABCD$. The total heat absorbed by the substance is H_1-H_2.

Consequently, since the final condition of the working substance is the same as the original condition, the result of this cycle of four operations is that an amount of work represented by the area $ABCD$, has been produced at the expense of an amount of heat (H_1-H_2).

Obviously the cycle could start at any point and proceed in either direction. If the direction be reversed, H_2 will be absorbed from the cold body, and a larger quantity H_1 lost to the hotter body at the expense of mechanical work supplied from outside represented by the area *ABCD*. This cycle is consequently perfectly reversible.

251. The Reversible Thermodynamic Engine.—Any arrangement capable of transforming heat into work is a *thermodynamic engine*. An engine in which the working substance traverses a reversible cycle is called a reversible engine.

When heat is transformed into work or mechanical work is transformed into heat, the quantity of work is equivalent to the quantity of heat. But it must be noted that under the conditions of temperature possible on the earth's surface, it is impossible to transform all of the heat taken from a body into mechanical work.

The discussion of the transformation of heat into work will be much simplified by considering the engine to go through a cycle of operations such that at the end of the cycle the working substance is in the same condition that it was in the beginning. By this device the internal energy of the working substance will be the same at the end as at the beginning, and any work done by the engine will be due solely to the heat supplied to it from outside.

If an engine possesses friction it is irreversible because heat is developed at the expense of mechanical energy in whichever direction the cycle is traversed. In an actual engine there are always unavoidable irreversible thermal losses due to conduction and radiation. Consequently no actual engine is perfectly reversible and no actual engine can do the amount of work corresponding to a perfectly reversible cycle.

252. The Thermodynamic Efficiency of a Reversible Engine is Greater than that of any other Engine.—The thermodynamic efficiency of an engine which is transforming thermal energy into mechanical energy is the ratio of the work done by it to the mechanical equivalent of the heat absorbed from the hot body. Or, in symbols,

$$\text{Thermodynamic efficiency} = \frac{W}{JH_1}, \quad . \quad . \quad . \quad (146)$$

where W represents the number of units of work done by the engine, H_1 is the number of units of heat absorbed from the hot body, and J is the number of work units in one heat unit.

In the case of an engine which is transforming work into heat, the efficiency for transforming work into heat is the ratio of the mechanical equivalent of the heat emitted to the work absorbed, while the efficiency for transforming heat into work is the ratio of the work absorbed by the engine, to the mechanical equivalent of the heat imparted to the hot body.

It will now be shown that a reversible engine working between any two temperatures will transform into mechanical work a greater fraction of the heat absorbed than any other engine working between the same temperatures. In other words, it will be shown that the reversible engine has the highest possible efficiency.

Thus, suppose a certain irreversible engine X, Fig. 235, be conceived to have a higher efficiency than the reversible engine Y. Imagine the two engines coupled together so that the irreversible engine X drives the reversible engine Y in the reverse direction. By this process, at every stroke X is putting into Y a certain amount of mechanical work W, and by the expenditure of this work Y will absorb from the cold body an amount of heat H_3 and give to the hot body a greater amount H_4. The engine X will absorb from the hot body an amount of heat H_1, deliver a part of it H_2 to the cold body and transform the remainder into an amount W of mechanical work. By the assumption that X is more efficient than Y, that is, that

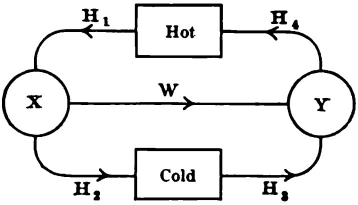

Fig. 235.

$$\frac{W}{JH_1} \text{ is greater than } \frac{W}{JH_4},$$

it would follow that H_4 is greater than H_1; in other words, that the engine Y imparts more heat to the hot body than the engine

X takes from it. Consequently, if the original assumption be correct, the combined self-contained system enables heat to pass continuously from a cold to a hot body until the entire quantity of heat in the cold body is exhausted. Since this result is contrary to the second law of thermodynamics, it proves that the original assumption is false. Therefore no engine can be more efficient than a reversible engine.

By letting both X and Y be reversible engines and proceeding as above, it can be shown that all reversible engines working between the same temperatures, and using the same working substance, have the same efficiency.

By means of the same method employed above, it can be shown that if X uses a different working substance than Y, the efficiency of X will be the same as that of Y. Whence, the efficiency of any thermodynamic engine is independent of the working substance.

Since no irreversible engine can do the amount of work corresponding to a perfectly reversible cycle (Art. 251), a perfectly reversible engine is more efficient than any other.

It has now been shown that *the mechanical energy developed by any thermodynamic engine, working between any two temperatures, depends only upon the quantity of heat transformed; and that a reversible engine will transform into mechanical work a greater fraction of the heat absorbed than any other engine working between the same temperatures.*

It is not self-evident that the amount of work done by an engine depends only on the amount of heat transformed, and is independent of the working substance. For example, ethyl ether boils at 35° C., whereas water boils at 100° C.; the heat equivalent of vaporization of ether is 90 calories per gram, whereas that of water is 539 calories per gram; the specific heat of both ether vapor and of water vapor is about 0.45. Consequently to produce a given mass of ether vapor at 150° C. requires about one-fifth as much heat as is required to produce the same mass of water vapor at the same temperature. Again, the vapor pressure of ether at 150° C. is about four times as great as the vapor pressure of water at the same temperature. Consequently by the expenditure of a given amount of heat on ether there could be produced twenty times as great a force on a piston as if the same amount of heat were given to water. After the ether vapor has expanded it could be readily recovered for use again, and so the original greater cost of ether over water would not be a serious objection to its use.

These facts have led many untrained practical men to imagine that the efficiency of an engine could be increased by substituting for water, ether or some other volatile liquid. The plausibility of the scheme is so great that several companies have been formed to build such engines.

The fact lost sight of is that for work to be developed the vapor must expand. This expansion causes cooling. If a small amount of heat was required to vaporize the liquid, the loss of a small amount of heat will cause the vapor to condense. When the vapor condenses, the pressure which it exerts on the piston becomes zero.

253. The Value of the Thermodynamic Efficiency of a Reversible Engine.—If a thermodynamic engine absorbs from a hot body an amount of thermal energy equal to JH_1 mechanical units and develops ΔW units of work, we have from definition,

$$\text{Thermodynamic efficiency} = \frac{\Delta W}{JH_1}.$$

A value of the efficiency of a reversible engine, expressed in terms of the temperature at which heat is received from the hot body and the temperature at which heat is rejected to the cold body will now be deduced.

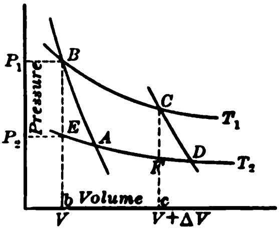

FIG. 236.

Since the efficiency of all reversible engines, working between the same two temperatures, is the same, and is independent of the working substance, the efficiency can be determined from the consideration of a reversible engine using whatever working substance is most convenient. It will be simplest to consider a perfect gas. Let a mass of perfect gas go through the cycle of operations represented by $ABCD$, Fig. 236, between the temperatures T_1 and T_2. Draw the lines Bb and Cc parallel to the pressure axis. The mechanical work done during the cycle is represented by the area $ABCD$ = area $EBCF$ when the isothermals and adiabatics are drawn so close together that the figure $ABCD$ may be considered to be a parallelogram. The work done

$$\Delta W = (BE)\ (bc).$$

Let V denote the volume of gas when in the condition represented by the point B and let ΔV denote the small change of volume represented by the line bc. When at the constant volume V, let P_1, and P_2 be the pressures of the gas at the temperatures T_1 and T_2, respectively. Then

$$(BE) = P_1 - P_2.$$

Whence the work done during the cycle, when the change in volume is small—

$$\Delta W[=(BE)\ (bc)] = (P_1 - P_2)\Delta V. \quad . \quad . \quad . \quad (147)$$

Now the heat received from the source, expressed in dynamical units, is JH_1; and when the change in volume is small, this equals $P_1\Delta V$. Whence, for a reversible engine,

$$\text{Thermo. eff.}\left[=\frac{\Delta W}{JH_1}\right] = \frac{(P_1 - P_2)\Delta V}{P_1\Delta V} = \frac{P_1 - P_2}{P_1}. \quad . \quad . \quad (148)$$

We shall now find an expression for $\frac{P_1 - P_2}{P_1}$. For the condition represented by the point B, Fig. 236, we have, (143),

$$P_1V = RmT_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (149)$$

and for the condition represented by the point E,

$$P_2V = RmT_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (150)$$

in which the temperatures T_1 and T_2 are reckoned from the absolute zero which is 273° C., below 0° C., or 459° F., below 0° F.

Dividing each member of (150) by the corresponding member of (149),

$$\frac{P_2}{P_1} = \frac{T_2}{T_1},$$

or
$$\frac{P_2}{P_1} - 1 = \frac{T_2}{T_1} - 1,$$

whence,
$$\frac{P_2 - P_1}{P_1} = \frac{T_2 - T_1}{T_1}.$$

Substituting this value in (148), we have for a reversible engine

$$\text{Thermo. eff.}\left[=\frac{\Delta W}{JH_1}\right]=\frac{T_1-T_2}{T_1}. \quad . \quad . \quad . \quad . \qquad (151)$$

And since a perfectly reversible cycle is the most efficient method for the conversion of heat into work, it follows that the maximum amount of work that can be produced by the transfer of the quantity of heat H_1 from the absolute temperature T_1 to the absolute temperature T_2 is

$$\Delta W=JH_1\frac{T_1-T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \qquad (152)$$

It should be noticed that if the changes of pressure and volume are small this result applies to any reversible cycle in which heat is transformed into any other form of energy. The result is independent of the working substance employed and of the sort of energy into which the heat is transformed. In deriving this result, the sole reason for considering Carnot's cycle is the great simplicity of this particular series of operations.

By more elaborate mathematical methods these same results may be obtained for a cycle in which the changes of pressure and volume are as great as we please.

Another expression for the efficiency of a reversible engine may be derived as follows. Let the quantity of heat not transformed into work be denoted by H_2. Then, from the first law of thermodynamics the work produced is

$$W=J(H_1-H_2).$$

Substituting this value in (146),

$$\text{Thermo. eff.}\left[=\frac{\Delta W}{JH_1}\right]=\frac{J(H_1-H_2)}{JH_1}=\frac{H_1-H_2}{H_1}. \quad . \quad . \qquad (153)$$

254. Conditions which Limit the Efficiency of an Actual Engine.—We have seen that the thermodynamic efficiency of a reversible engine is greater than that of any other; that the efficiency is independent of the working substance; and that when all of the heat entering the working substance is absorbed at an absolute temperature T_1, and that the heat which is rejected is at T_2, the magnitude of the efficiency is directly proportional to $T_1 - T_2$. If any of the heat received is at a lower temperature than T_1, or if any of the heat rejected is above T_2, the mechanical work developed will be less than if all of the heat absorbed had been at T_1 and that rejected had been at T_2.

In order that the cycle of operations may be reversible, (*a*) the temperature of the working substance when receiving heat must be the same as that of the source, and when rejecting heat must be the same as the temperature of the condenser; (*b*) no energy must be absorbed in friction or in setting the piston into motion; (*c*) no heat must be emitted except to the condenser.

For a steam engine the temperature range is limited by the freezing point of water and the greatest practicable boiler pressure. Up to the present time boiler pressures greater than 300 lb. per sq. in. have not been in successful use. At this pressure, water boils at 417° F. Remembering that the temperatures T_1 and T_2 are reckoned from a zero point 459° F. below the Fahrenheit zero, the value of the thermodynamic efficiency of a reversible engine working between 417° F. and 32° F. is seen to have the value

$$\text{Thermo. eff.} \left[= \frac{T_1 - T_2}{T_1} \right] = \frac{(417+459)-(32+459)}{417+459} \doteq 0.44$$

For the following reasons the efficiency of any actual engine working between these temperatures will be much less.

1. Some of the heat will be received at a temperature below 417° F.

2. Some of the heat will be rejected at a temperature above 32° F.

3. Mechanical energy will be lost by friction.

4. Heat energy will be lost on account of conduction through the cylinder walls and condensation of steam within the cylinder.

Losses from these causes are made as small as possible by using (*a*) pipes of short length and large diameter to connect the engine to the boiler and to the condenser, (*b*) quickly operating valves having large ports, (*c*) thorough piston lubrication, (*d*) superheated steam, (*e*) steam jackets about the cylinders.

The greatest efficiency actually obtained with a reciprocating engine is about 0.25. This degree of efficiency is only possible when the steam is supplied at high temperature (above 400° F.), and the condenser is operated at a low temperature.

255. Internal Combustion Engines.—There are many types of engine in which liquid or gaseous fuel is burned within the cylinder. If the piston could be driven by the expansion of the products of combustion starting at the temperature of combustion, such an engine would be much more efficient than any possible steam engine. Unfortunately, however, such high temperatures would destroy any lubricant as well as the surface of the cylinder walls. But the cylinder can be cooled to a practical operating temperature which is still higher than that which can be economically supplied by steam. Internal combustion engines using crude oil as fuel have been constructed that yield 2500 H.P. per cylinder at an efficiency exceeding 30 per cent.

SOLVED PROBLEMS

PROBLEM.—A locomotive burning 863 lb. coal per hour and running 30 mi. per hr. exerts a draw-bar pull of 3467 lb. wt. The coal has a thermal value of 14500 B.t.u. per lb. Find the H.P. developed by the locomotive and also the actual efficiency of the combined boiler and engines.

SOLUTION.—

$$\text{H.P.}\left[=\frac{\text{ft. lb. per sec.}}{550}\right]=\frac{3467(30\times 5280)}{550(60\times 60)}=277.$$

$$\text{Actual eff.}=\frac{\text{Work performed}}{\text{Mechanical equiv. of heat supplied}}$$

$$=\frac{3467(30\times 5280)}{863\times 14500\times 778}\doteqdot 0.06.$$

PROBLEM.—A 450 H.P. condensing engine is supplied with steam from a boiler that has an efficiency of 47%. Each hour there are consumed 1170 lb. of coal having a heat value of 12000 B.t.u. per lb. The temperature of the steam supplied to the engine is 370° F., and the temperature of the condenser is 132° F. Find the thermodynamic efficiency, and the actual efficiency of the engine.

SOLUTION.—

Remembering that the temperatures T_1 and T_2 are reckoned from a zero point 459° F. below the Fahrenheit zero, we have

$$\text{Thermo. eff.}\left[=\frac{T_1-T_2}{T_1}\right]=\frac{(370+459)-(132+459)}{(370+459)}=0.29,$$

$$\text{Actual eff.}=\frac{\text{Work performed}}{\text{Mechanical equivalent of heat supplied}}$$

$$=\frac{450\times550\times60\times60}{1170\times12000\times0.47\times778}=0.17.$$

PROBLEM.—A 125 H.P. non-condensing engine is supplied with steam from a boiler that has an efficiency of 55%. Each hour there are consumed 732 lb. of coal of a thermal value of 12000 B.t.u. per lb. The steam entering the engine is at 324° F., and the exhaust is at 226° F. Find the relation between the actual and the thermodynamic efficiency of the engine.

SOLUTION.—

$$\text{Actual eff.}=\frac{\text{Work performed}}{\text{Mechanical equivalent of heat supplied}}$$

$$=\frac{125\times550\times60\times60}{732\times12000\times0.55\times778}=0.066.$$

$$\text{Thermo. eff.}\left[=\frac{T_1-T_2}{T_1}\right]=\frac{(324+459)-(226+459)}{(324+459)}=0.13.$$

$$\therefore\ \frac{\text{Actual eff.}}{\text{Thermo. eff.}}=\frac{0.066}{0.13}=0.5.$$

256. The Utility of the Principle of the Reversible Thermodynamic Engine.—The principle of the reversible engine is of wide application in both science and engineering. For example, it shows that the quantity of work produced by any continuously operating engine is independent of the working substance and depends only on the amount of heat transformed. It shows that the efficiency of an engine is increased when the temperature of the source of heat is raised and when the temperature of the exhaust is lowered. For this reason large engines now seldom exhaust into the air, but use condensers, and it is now becoming more and more the practice to supply engines with high pressure (i.e., high temperature) steam.

If all the heat absorbed by an actual engine enters at the same high temperature, and all the heat emitted escapes at the same lower temperature, then the efficiency of the given engine will approximate to the efficiency of the ideal reversible engine, working between the given temperatures, in proportion to the degree of reversibility of the cycle traversed by the working substance.

This principle has also been used to determine the amount of elevation of the boiling point, and the depression of the freezing point of liquids by pressure. The elevation of the boiling point, and the depression of the freezing point of solutions produced by the addition of a solute can also be computed. The standard methods of determining the molecular weights of chemical compounds depend upon this principle. If the chemical actions that occur in a galvanic cell are known, this principle affords a method of computing the electromotive force that will be developed.

257. Kelvin's Thermodynamic Temperature Scale.—The temperature scales heretofore considered depend upon some property of a given substance. But since the work done by a reversible engine depends only upon the temperatures between which it operates, and not upon the nature of the working substance, it follows that the difference between the temperatures can be expressed in terms of the work done by a reversible engine operating between those temperatures. On this basis Lord Kelvin has devised the "thermodynamic" or "absolute" scale of temperature.

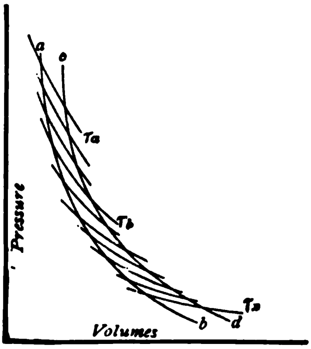

FIG. 237.

In Fig. 237 suppose that τ_a is the isothermal of any substance at the temperature of boiling water, and that τ_b is the isothermal at the temperature of freezing water, while *ab* and *cd* are two adiabatics of water. Let the area included between these four lines be divided into *n* equal parts by a series of isothermals. Then the same amount of work will be developed by a Carnot cycle operating between any two consecutive isothermals. Lord Kelvin's suggestion is that the temperature difference of those isothermals be called equal between which equal amounts of work are developed by the operation of a Carnot cycle. Some of the properties of a temperature scale constructed on this basis will now be considered.

The area of the figure included between the isothermals τ_b and τ_a represents the work done by a Carnot cycle operating between these temperatures. If this area be denoted by A, then the area of each element into which it is divided is $\frac{A}{n}$, and the area of the figure included between any two isothermals equals the product of $\frac{A}{n}$ and the difference in temperature of the given isothermals. If a reversible engine operating between the temperature τ_1 and τ_2 absorbs from the hotter body an amount of heat H_1, and emits to the colder body an amount H_2, then, (Art. 244), the work done by the working substance is $J(H_1-H_2)$. Since the area of the figure included between the two isothermals τ_1 and τ_2 is $\frac{A}{n}(\tau_1-\tau_2)$, it follows that

$$J(H_1-H_2)=\frac{A}{n}(\tau_1-\tau_2). \quad . \quad . \quad . \quad . \quad (154)$$

This equation is true whatever the amount of heat received from the hot body, and the amount emitted to the cold body. Consider the particular case where all the heat received from the hot body is transformed into work; that is, in which the quantity of heat emitted to the cold body equals zero. In order that the engine may transform into work the entire quantity of heat received from the hot body, the adiabatic expansion of the working substance must continue until the working substance is entirely devoid of heat. The temperature which the exhaust of a reversible engine would need to have in order that the engine may convert into work all of the heat supplied to it is taken as the thermodynamic zero of temperature. At this temperature a substance would be entirely devoid of heat.

In this case $H_2 = o$ and $\tau_2 = o$. Therefore, for this particular case (154) becomes

$$JH_1 = \frac{A\tau_1}{n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (155)$$

It is to be noted that (154) apples to a reversible engine working between τ_1 and τ_2, and which receives a quantity of heat H_1 from the hot body; while (155) applies to a reversible engine working between τ_1 and the thermodynamic zero of temperature and which receives from the hot body a quantity of heat H_1. On dividing each member of (154) by the corresponding member of (155) we obtain

$$\frac{H_1 - H_2}{H_1} = \frac{\tau_1 - \tau_2}{\tau_1}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (156)$$

This is the value of the efficiency of a reversible engine operating between the temperatures τ_1 and τ_2 measured on the thermodynamic temperature scale. And since the efficiency of such an engine is independent of the working substance, it follows that the relation between any two temperatures measured on the thermodynamic scale is independent of the working substance employed.

On putting (156) into the form

$$\frac{H_1}{H_2} = \frac{\tau_1}{\tau_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (157)$$

it is seen that the ratio of the quantity of heat absorbed by the working substance to the quantity emitted, equals the ratio of the temperatures between which the reversible engine is operating. Hence, *the ratio between any two temperatures measured on the thermodynamic scale equals the ratio between the quantity of heat absorbed to the quantity emitted by a reversible engine operating between those temperatures.*

From (153) and (151) we have

$$\frac{H_1 - H_2}{H_1} = \frac{T_1 - T_2}{T_1},$$

or

$$\frac{H_1}{H_2} = \frac{T_1}{T_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (158)$$

where temperatures are measured on the ideal gas scale.

From (157) and (158) it follows that

$$\frac{\tau_1}{\tau_2}=\frac{T_1}{T_2}. \qquad (159)$$

Whence, the ratio between the numbers expressing two temperatures according to the thermodynamic scale equals the ratio between the numbers expressing the same temperatures according to the ideal gas scale.

258. The Experimental Realization of the Thermodynamic Scale of Temperatures.—No thermometer has been made the action of which depends upon the reversible engine. And no actual gas obeys the laws of perfect gases except through a limited range of temperature. Consequently, before the above equality can be used, it will be necessary to investigate the departure of actual gases from perfection.

This subject was investigated by Joule and Kelvin by measuring the temperature and pressure of gases before and after traversing a porous plug of cotton wool. In order that a gas may be perfect, it must obey Boyle's Law. This implies that there is no force between the molecules. If such a gas traverse a porous plug, the potential energy of the molecules before entering the plug will be the same as their potential energy on emergence. The total energy as well as the potential energy being unchanged, there will be no alteration in the kinetic energy of their vibration and consequently no change of temperature. If, however, there be a force of repulsion between the molecules of a certain gas, the potential energy of the molecules on emergence will be less than their potential energy before entering the plug. The total energy being constant, their kinetic energy will be greater on emergence than on entrance, that is, the temperature on emergence will exceed the temperature on entrance. Contrariwise, if it be found that a given gas be cooled by passing through a porous plug, one would know that the molecules of this gas attract one another.

From these considerations and certain thermodynamical principles which we have not developed, it is possible to construct an equation coordinating the indicated temperature of an actual gas within the bulb of a constant volume thermometer, and the thermodynamic temperature of the gas. Thus if T and T' represent the constant-volume-thermometer temperatures of the specimen on entering and after traversing the porous plug, P and P' the pressures on entering and after leaving the plug, c the specific heat of the gas under constant pressure, J the mechanical equivalent of heat, and τ the thermodynamic temperature of the gas on entering the plug, it can be shown that

$$\tau = T + \frac{Jc(T-T')}{k \log_e \frac{P}{P'}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (160)$$

where k is a constant of the particular gas that can be determined by experiment.

The difference between the values obtained for a given temperature on the thermodynamic scale and on the constant volume hydrogen thermometer is very small. For instance, between −50° C., and 150° C., the difference is less than 0.001° C. At 1000° C., the hydrogen thermometer gives a reading about 0.044° C. too low. In work of precision observed mercury or gas thermometer indications are corrected so as to express absolute thermodynamic temperatures.

Temperatures expressed according to Kelvin's thermodynamic temperature scale are indicated by the symbol K. Thus, "1000 K" represents a temperature of 1000 centigrade degrees above the absolute zero on the thermodynamic scale.

QUESTIONS

1. A cylinder of compressed air bursts. The gas is cooled below the coldest surrounding objects and external work is done. Is this in violation of the second law of thermodynamics? Give reasons for your answer.

2. The earth possesses a large amount of heat energy. State the principle which prevents us from accumulating all of this energy at a definite place, thus creating a means of using this energy in doing work.

3. If the gage pressure of a locomotive boiler be maintained constant, will the power of the engine be different when the locomotive is at a high altitude than when at low levels?

4. Does the economy effected by the steam condenser of a power plant depend upon the altitude of the place where the plant is situated?

ELECTRICITY

CHAPTER XVII

MAGNETISM

259. Natural and Artificial Magnets.—Certain iron ores strongly attract small pieces of iron. As the first known bodies possessing this property came from Magnesia in Asia Minor, the Greeks called them magnets. By sprinkling iron filings over a magnet it is found that the property of attracting iron is not uniform over the surface of the magnet. On each specimen there are two or more spots at which the force is much greater than over the remainder of the surface. These small areas are termed *magnetic poles.*

If a magnet of two poles be delicately suspended so that it can easily turn about a vertical axis, it will usually turn till the line joining its poles lies approximately north and south. The pole directed toward the north is called the *north-seeking pole,* and the pole which turns toward the south is called the *south-seeking pole.* These names are commonly abbreviated as *north pole* and *south pole.*

A magnet mounted so that it can easily turn about a vertical axis is called a magnetic compass. The magnetic poles of the earth toward which a compass points do not coincide with the geographical poles. There are places at which a compass points east, west or even south. At one of the magnetic poles of the earth, a freely suspended magnet will stand vertical.

Artificial magnets can be produced by bringing pieces of iron or steel into contact with a magnet. In the case of soft iron the magnetic effect is soon lost, especially if the specimen be dropped or struck. But the magnetism imparted to hardened steel persists even though the specimen be roughly handled. A magnetized piece of soft iron is called a *temporary magnet.* A magnetized piece of hardened steel is called a *permanent magnet.*

If a magnet be heated to about 785° C. it will cease to be magnetic. Consequently, a natural magnet cannot be forged into shape. But steel, while hot, can be forged into any desired shape and then hardened and magnetized. For this reason artificial magnets are commonly used instead of natural magnets.

The tendency of a magnetized steel needle to place itself north and south was utilized by Italian navigators as early as the tenth century. The early compass consisted of a bowl containing liquid on which floated a cork, to which was attached a horizontal circular card and a magnetized steel needle. The edge of the card was divided into "the points of the compass." On the edge of the containing bowl was a line parallel to the keel of the ship. The angle between the keel of the ship and the meridian was indicated by the mark on the card coinciding with the mark on the bowl.

260. Magnet Poles.—If a magnet pole be drawn from one end of a thin steel rod to the other end, a magnetic north pole will be developed at one end of the rod and a south pole at the other. If a magnet pole be drawn from one end of a thin steel rod to the middle, there raised, and then drawn from the other end to the middle, four poles will be developed on the rod—two of the same sort at the ends, and two of the other sort at the middle of the rod. The double pole at the middle of the rod is called a consequent pole.

The straight line joining the two poles of a magnet is called the *magnetic axis* of the magnet. Although a floating magnet will turn until its axis is parallel to the earth's magnetic meridian, there is no tendency for the entire magnet to move either toward the north or toward the south. Whence we conclude that the force acting on the north pole of the magnet equals the force acting on the south pole of the magnet.

The position of a magnet pole can be determined by the aid of a delicately pivoted compass needle. When such a magnetic needle is brought close to one end of a bar magnet it will point toward the pole. The lines along which the needle points when it is placed near the end of a magnet intersect in a region which may be of some size. The magnet pole may be thought of as occupying this region. A magnet pole may be almost a point or it may have considerable dimensions.

261. The Constitution of Magnets.—If a magnet of two poles be broken, each piece will have two poles. Even though the process of breaking the magnet be continued till each fragment is of minute dimensions, each fragment will have two poles. Consequently, we conclude that magnetism is a property of the entire mass of a magnet and is not limited to the poles. This suggests that iron consists of small particles each of which is a magnet of two poles, and that the difference between an unmagnetized rod and a magnetized rod is due to the arrangement of these minute magnets.

If the north poles of all these minute magnets point toward one end of the rod, then this end face will consist of north poles and the opposite end face of the rod will consist of south poles. At any cross-section of the rod between the ends there will be both north and south poles which neutralize one another's effect. If, however, the minute magnets constituting the rod point in all directions, then at every point of the rod the north and south poles of neighboring magnets neutralize the effect of one another, and no magnetic poles are developed on the rod.

This conception of the constitution of magnets can be illustrated as follows: By means of a file reduce a steel magnet to small particles. Each of these filings is a magnet of two poles. Place these filings in a glass tube stoppered at each end. If the filings be now shaken and the tube suspended by a fine thread, there will be no tendency for the tube to point in the north and south direction. But if the tube be stroked from end to end with a strong magnet, many of the little steel filings will be turned so that their north poles point in the same direction. The tube of filings now has a north pole at one end and a south pole at the other, and if suspended will set itself in the north and south direction.

262. Force Acting upon Magnet Poles.—It is found that two north magnet poles repel one another, also two south magnet poles repel one another, but that a north and a south pole attract one another. That is, similar magnet poles repel, dissimilar poles attract.

Let the force acting upon magnet poles A and X be f_1. Let A be replaced by another magnet pole B, and let the force acting upon B and X be f_2. Then the poles A and B are said to have pole strengths in the ratio of f_1 to f_2. Expressed in another manner, the stress between any given pole and a pole X varies directly with the pole strength of the given pole. Likewise, the force must vary directly with the pole strength of X. Thus, if the pole strengths of two isolated poles are m_1 and m_2, then for a given distance between them, the force

$$F \propto m_1 m_2.$$

For any two isolated poles it is found that the stress between them varies inversely with the square of their distance apart. That is,

$$F \propto \frac{1}{r^2},$$

where r is the distance between the two poles.

So long as the medium surrounding the magnet poles remains the same—whether this medium be air, hydrogen, kerosene, or any other—the stress between magnetic poles depends only upon their pole strength and upon the distance between them. Consequently, the above experimental facts are expressed by the equation

$$F = \frac{m_1 m_2}{\mu r^2}, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (161)$$

where μ is a factor which depends upon the medium surrounding the poles. The law expressed by this equation is called Coulomb's law for magnet poles.

It should be noted that the above equation is true only when the two poles are of point dimensions. No actual magnetic pole is of point dimensions. Therefore, in using Coulomb's law, r is taken as the distance between the points of application of the resultant magnetic forces which the two poles exert upon any test pole placed near them. Thus, if in Fig. 238, *R* applied at *P* represents the resultant of all the forces acting between an isolated test pole *n* and the magnet pole *N*, and *R′* applied at *P′* represents the resultant of the magnetic forces between *n* and the various parts of *S*, then the distance between *P* and *P′* is the length *r* used in Coulomb's law.

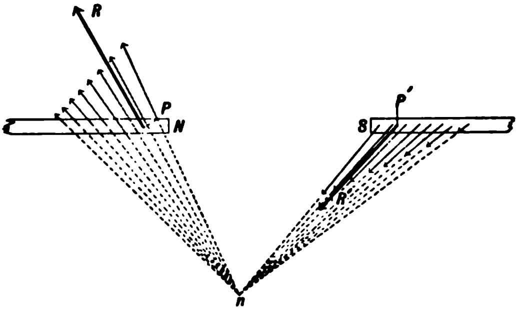

FIG. 238.

263. Unit Pole Strength.—In (161), μ is a factor characteristic of the medium surrounding the magnet poles. For any medium its value will depend upon the magnitudes of the units adopted for force and pole strength. Since most magnetic experiments are conducted in air we define unit pole strength, in terms of the above equation, to be the strength of a magnet pole such that if placed at a distance of one centimeter *in air* from another equal pole, the force acting upon them will be one dyne.

From this definition of unit pole strength it follows that in the C. G. S. system of units, the value of μ for air is unity. Consequently, when surrounded by air, the force acting upon two isolated poles of pole strengths m_1 and m_2, separated by a distance r, is

$$F = \frac{m_1 m_2}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (162)$$

264. Intensity of Magnetic Field.—A region in which a force would act upon a magnet pole if placed there is called a *magnetic field of force.* The region about a magnet is an example of a magnetic field of force. It will be shown later that the region about a conductor carrying an electric current is also a magnetic field of force.

A magnetic field of force has direction and magnitude. The positive direction is taken as that in which a north pole would tend to move if placed there. A line which at every point is in the direction of the magnetic field at that point is a *magnetic line of force.* In the region about a magnet the lines of force extend from the north pole to the south pole. The ratio of the force F, that would act upon a magnetic pole placed at any given point in a magnetic field to the strength of that pole, m_1, is called the intensity of the magnetic field at the given point. Thus the intensity of the magnetic field is given by

$$H=\frac{F}{m_1}. \qquad \qquad (163)$$

Hence, the magnetic intensity, or field strength, is numerically equal to the force that would act upon a unit pole placed at the given point. Intensity of magnetic field is expressed in dynes per unit pole. An intensity of magnetic field of one dyne per unit pole is called a *gauss.*

A magnet pole of strength m_1 at a point where the intensity of the magnetic field is H gausses is acted upon by a force (163), $F=m_1H$ dynes.

A magnetic field is said to be *uniform* in a given region, if throughout that region the lines of force are parallel and the intensity is the same.

265. Magnetic Flux.—The product of the mean intensity of a magnetic field and an area normal to the direction of the field is called the *magnetic flux* across the area. Thus, representing by the symbol Φ the magnetic flux across an area of A square centimeters which at every point is normal to a magnetic field of a mean intensity of H gausses, we have

$$\Phi=A\,H. \qquad \qquad (164)$$

The unit of magnetic flux is the flux through 1 square centimeter perpendicular to a magnetic field of one gauss and is called the *maxwell.*

The total magnetic flux emanating from an isolated magnet pole situated in air will now be determined. Let the pole be of strength m and let a sphere of radius r centimeters be drawn with the given isolated magnet pole as center. Then, since the area of this sphere is $A=4\pi r^2$ square centimeters, and the magnetic intensity at the surface of the sphere is

$$H\left[=\frac{F}{m_1}\right]=\frac{\frac{mm_1}{r^2}}{m_1}=\frac{m}{r^2} \text{ gausses,}$$

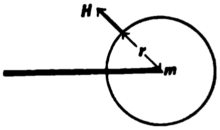

FIG. 239.

the total magnetic flux emanating from the surface of the sphere, in air, is

$$\Phi[=A\,H]=\frac{4\pi r^2 m}{r^2}=4\pi m \text{ maxwells.} \quad . \quad . \quad (165)$$

266. Representation of Magnetic Flux by Lines of Force.—It is often convenient to represent graphically lines of force in a magnetic field. In order that such a representation may express the flux as well as the direction of the magnetic field, electrical engineers use the following convention.

Imagine a surface drawn across the magnetic field at every point normal to the direction of the field. For example, if the lines of force of the field are parallel and straight, this surface would be plane: if the lines of force are straight and radiate from a point in all directions, this surface would be spherical. Now suppose that across each element of area of this surface there are drawn a number of lines of force numerically equal to the mean intensity of the magnetic field over the given element of area. For example, if at a certain part of the surface the mean magnetic intensity is x dynes per unit pole, there would be drawn through that part of the surface x lines of force per square centimeter.

From the fact that a unit magnetic field can be arbitrarily defined as a certain number of lines of force per square centimeter, it is evident that a line of force is a purely mathematical conception and has no actual physical existence.

In terms of the convention of lines of force, a flux of x maxwells is expressed as a flux of x " lines."

267. Magnetic Induction.—When an unmagnetized piece of iron or steel is placed in a magnetic field of force it will become a magnet. The phenomenon of a body becoming a magnet when placed in a magnetic field of force is called *magnetic induction.*

The magnetic field of force about the magnet is the resultant of the field that existed in the given region before the introduction of the iron and the field of force due to the poles developed on the newly formed magnet. The direction and magnitude of the resultant magnetic field at any point can be obtained by compounding the two component magnetic fields at the given point by the ordinary parallelogram law.

By cutting the piece of iron into two pieces by a section normal to the resultant external magnetic field, separating the two pieces slightly, and exploring the space between the two pieces, it is found that a magnetic field of force exists within the magnet.

The magnetic flux per unit area of cross-section at any point within a piece of iron is much greater than at the same point in space before the iron was introduced. The magnetic flux per unit area at any point within a body in a magnetic field is called the *induction density* in the body at the given point. Induction density is measured in gausses and is usually denoted by the symbol B. The magnetic flux per unit area, that is, the field strength, at the same point in air before the specimen is introduced, is called the *magnetizing field.* The magnetizing field is measured in gausses and is usually denoted by the symbol H.

268. The Induction Density within an Iron Rod in a Magnetizing Field.—The relations between the magnetizing field H and the induction density B for three specimens are represented in Fig. 240. As the magnetizing field increases from zero, the induction density of silicon steel and of wrought iron is increased very much more rapidly than the magnetizing field until H becomes about 15 gausses, after which further increases in H produce only equal increases in B. When an increase in H produces an increase in B equal to the increase in H, the specimen is said to be *saturated.*

The figure does not extend far enough to the right to show the magnetizing field necessary to saturate the mild steel.

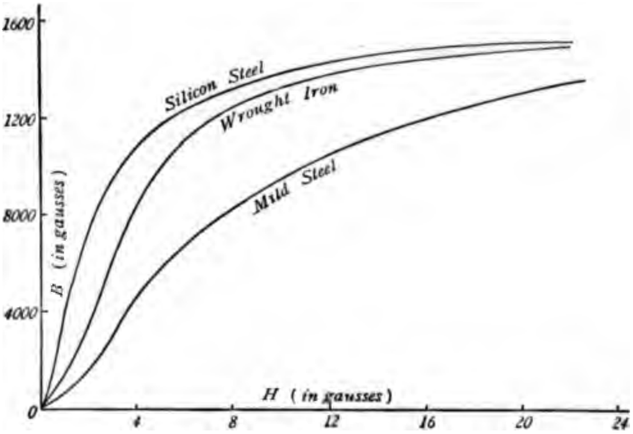

FIG. 240.

Invisible cracks and lumps of slag in a specimen of iron can be detected by magnetizing the specimen and then observing the degree of uniformity of the induction density in different parts of the specimen.

269. Orientation of a Piece of Iron in a Magnetic Field.—If a compass needle be placed in a magnetic field of force, the north pole will be drawn in the direction of the magnetic field, and the south pole in the opposite direction, thereby causing the compass needle to set itself parallel to the magnetic field at the place where it is situated. It is also found that when an unmagnetized rod of steel or iron is placed in a magnetic field of force it will become a magnet, and that if this induced magnet is free to move in every direction it will turn till its long axis is in the direction of the magnetic field at the place where it is situated.

These facts are in accord with the supposition (Art. 261) that unmagnetized iron consists of minute magnets, the axes of which point in diverse directions, and that when a piece of unmagnetized iron is placed in a magnetic field of force these minute magnets are turned into approximately the direction of the field, thereby transforming the unmagnetized iron into a magnet. In terms of this idea, if a particle of iron consisting of two minute magnets be placed with its longer dimension at right angles to a magnetic field of force, each minute magnet will turn in the direction of the field. The forces between the poles of the minute magnets will now be as represented in Fig. 241. These forces balance and there is no tendency for the axes of the minute magnets to be turned out of the direction of the magnetic field, and consequently, no tendency for the specimen to turn out of the original position at right angles to the field of force.

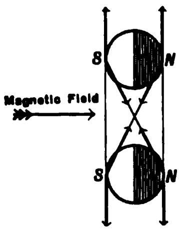

FIG. 241.

Suppose the particle of iron to be placed with its longer dimension inclined to the direction of the field. If the minute magnets turn till their magnetic axes are in the direction of the external field, Fig. 242, the forces of repulsion between the two pairs of similar poles will be equal, but the forces of attraction between the two pairs of dissimilar poles will be unequal. This will cause the two minute magnets to rotate into some such position as that shown in Fig. 243. When in this position, the adjacent poles of the two minute magnets are so close together that the external magnetic field will exert but slight effect upon either of them. Upon the two poles at the end of the particle, however, the external field will develop two forces constituting a couple which tends to cause the particle to turn till its long axis becomes parallel to the external field.

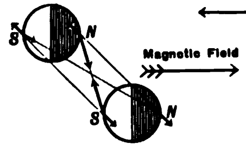

FIG. 242.

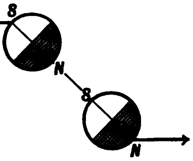

FIG. 243.

Similar reasoning shows that if an iron rod of any size be placed in a magnetic field of force, with the long axis of the rod inclined to the direction of the field, then, due to the poles of the tiny magnets, each tiny magnet will be turned and held in position with reference to the others. Free magnet poles are developed toward the ends of the rod. Between these poles and the external magnetic field there exist forces which tend to turn the rod till its long axis is parallel to the direction of the magnetic field.

270. Mapping Magnetic Lines of Force with Iron Filings.—In the preceding article it has been shown that a small specimen of iron placed anywhere in a magnetic field tends to set itself in the direction of the line of force at the given point. This fact furnishes a very convenient method for mapping out the lines of force in a magnetic field. Fig. 244 was obtained by sprinkling iron filings on a sheet of paper placed over a bar magnet. While falling through the air, the filings became magnets and were then easily turned into the direction of the magnetic field. Dissimilar poles attract one another, thereby causing the filings to arrange themselves in strings along the lines of force.

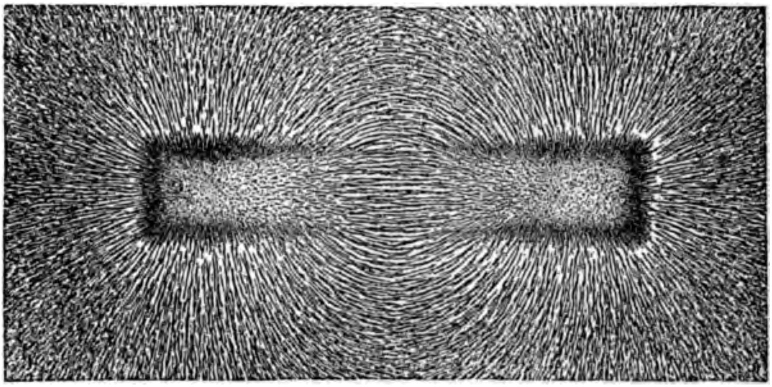

Fig. 244.

Fig. 245 represents the lines of force between dissimilar poles of two bar magnets placed in the same straight line.

Fig. 246 represents the lines of force between similar poles of two bar magnets placed in the same straight line.

Figs. 247 and 248 represent the lines of force about a piece of soft iron placed in a magnetic field that was uniform before the advent of the piece of iron.

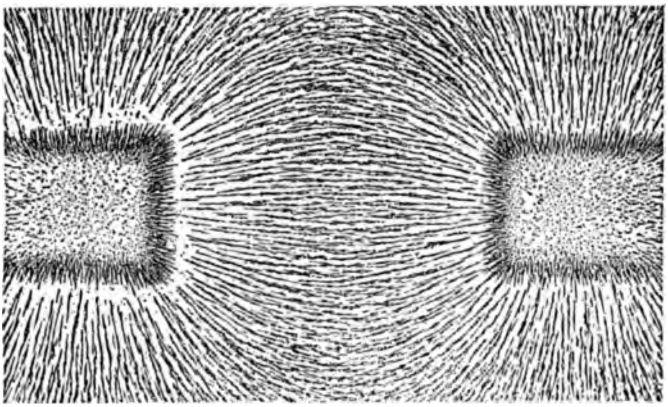

Fig. 245.

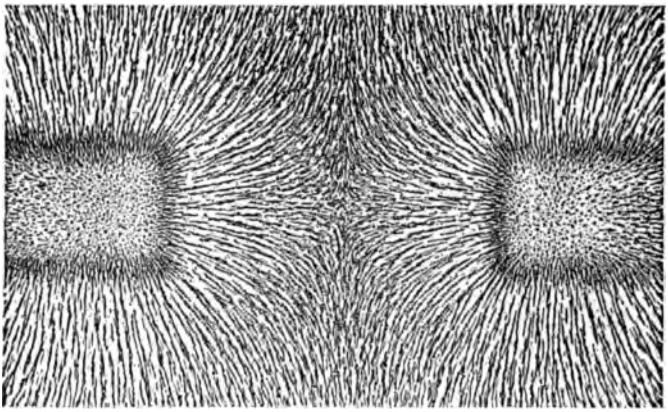

Fig. 246.

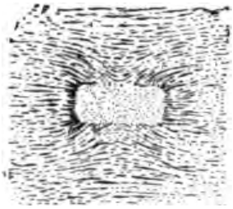

Fig. 247.

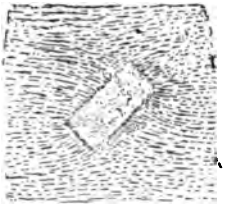

Fig. 248.

271. Magnetic Screening.—A comparison of Fig. 245 with Figs. 247 and 248 shows that when a piece of soft iron is introduced into a magnetic field, there is produced a change of the magnetic intensity of the field in the neighborhood of the iron. The region where the magnetic intensity is diminished by the presence of a piece of soft iron is said to be *magnetically screened.*

If a given space be entirely surrounded with a thick layer of soft iron it will be almost completely screened from external magnetic influences. Fig. 249 is an iron filing map of the magnetic field in the neighborhood of a hollow iron cylinder situated in a strong magnetic field. In this map it will be observed that the iron filings within the cylinder exhibit no tendency to arrange themselves in lines. This shows that within the cylinder the directive force is very small: that is, the magnetic intensity of the field is there negligible.

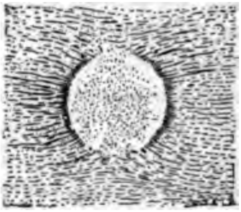

FIG. 249.

272. Magnetic Permeability.—That property of matter to which the phenomenon of magnetic induction is due is called *magnetic permeability.* The magnetic permeability of a body is measured by the ratio of the magnitude of the magnetic induction density developed in the body, to the intensity of the magnetic field producing it.

Thus, if when a given substance is placed in a magnetic field of intensity H, there is developed in the specimen a magnetic induction density B, then the magnetic permeability of the given material is

$$\mu=\frac{B}{H}. \qquad (166)$$

It should be kept in mind that H represents the intensity of the magnetic field at a given point in air, while B represents the magnetic induction density at the same point in space when occupied by some given substance.

It can be shown, though the proof will not be here given, that the factor μ in (161) is numerically equal to the magnetic permeability of the medium in the magnetic field of force.

From the above equation, together with the definition of H, it follows that the magnetic permeability of air is unity. The magnetic permeability of iron is much greater than that of any other substance. It depends upon the intensity of the magnetizing field, and upon the purity, the temperature and the previous heat treatment of the specimen. For example, under a magnetizing field of 5 gausses a certain specimen of iron at 21° C. had a permeability of 2600; at 500° C. it had a permeability of 2100; at 775° C. a permeability of 1700; and at 785° C. a permeability of unity. High-carbon steel is unmagnetic at about 680° C.—that is, at this temperature the permeability is unity.

At the same temperature and under the influence of magnetizing fields of the same intensity, the approximate values of the permeability of certain substances are given below. These values vary when the magnetizing field changes.

Substance	Permeability
Soft iron	2500
Mild steel	2000
Hard steel	250
Cobalt	225
Nickel	200
Heusler's Alloy	33
Air	1
Bismuth	0.99

Almost all substances have a magnetic permeability of nearly unity. But iron, steel, cobalt, nickel, manganese, chromium, platinum and oxygen have a permeability greater than unity, while antimony, lead, zinc and bismuth have permeabilities slightly less than unity. No substance is known that has greater magnetic permeability than iron, nor one that has smaller permeability than bismuth.

If the magnetic permeability is different at two neighboring points in a magnetic field, a magnetic pole will be developed. For example, if a homogeneous steel rod be placed in a magnetic field a pole will be developed where the magnetic lines of force enter the rod, and another where they leave the rod. Again, if there is a high-carbon spot in the rod, there will be a pole at each end of the spot.

Substances of magnetic permeability greater than unity are attracted by a magnet pole and are said to be *paramagnetic.* Substances of high permeability, such as iron, cobalt and nickel, are strongly attracted by a magnet pole and are said to be *ferromagnetic.* Substances of permeability less than unity are slightly repelled by a magnet pole and are said to be *diamagnetic.* Bismuth, phosphorus, antimony, water and many gases belong to this class.

Substances of widely different magnetic permeabilities can be separated by means of magnets. In this manner magnetic iron ores are commercially separated from rock. A thin sheet of the pulverized ore falls in front of a number of magnets *mm*, Fig. 250. The materials of small permeability fall vertically, while the substances of high permeability are drawn to one side and fall into a separate receptacle.

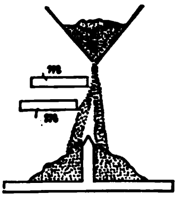

FIG. 250.

If steel be gradually heated the size of grain will become minimum when the decalescent point is reached (Art. 205). If now the temperature be either increased or maintained constant, the size of the grain will increase and the strength of the material will diminish. But if the specimen be quenched by plunging it into cold oil or water as soon as the decalescent point is attained, the hardness, the fineness of grain and the strength of the specimen will be rendered permanent.

Steels containing from about 0.4 per cent to 0.9 per cent carbon cease being ferromagnetic and become paramagnetic at a temperature called the Curie point, which practically coincides with the decalescent point. Consequently, for these steels the correct moment for quenching may be readily determined by bringing a pivoted magnet near the pieces being heated, Fig. 251. Until the correct temperature is attained, the magnet pole will be attracted toward the pieces of steel; but when the proper temperature for quenching is reached, the magnet will be horizontal. If the steel be now quickly quenched, it will have the finest grained structure.

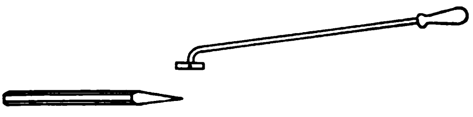

FIG. 251.

273. Properties of Paramagnetic and Diamagnetic Substances.—If a rod of iron be placed in a magnetic field, magnet poles will be developed in the iron as indicated in Fig. 252. If instead of iron, a piece of bismuth be used, poles will be developed in the bismuth as indicated in Fig. 253.

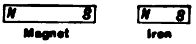

Fig. 252.

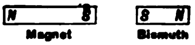

Fig. 253.

If the magnetic field be uniform, neither the iron nor the bismuth will either approach or recede from the magnet. But if the magnetic field be ununiform as in the above figures, then the iron will move toward the stronger part of the field while the bismuth will move toward the weaker part of the field.

If a rod of iron, free to turn in all directions, be placed in an ununiform magnetic field, it will set itself parallel to the direction of the field at the place where it is situated. On the other hand, if a freely suspended rod of bismuth be placed in an ununiform magnetic field, it will set itself perpendicular to the direction of the field at the place where it is situated.

A rod of any paramagnetic substance in a magnetic field, (*a*) tends to set itself parallel to the direction of the field, (*b*) develops within itself an induced field in the direction of the magnetizing field, and (*c*) if the magnetic field in which it is situated be ununiform the rod tends to move in the direction in which the strength of the field increases most rapidly.

A rod of any diamagnetic substance in a magnetic field, (*a*) tends to set itself perpendicular to the direction of the field, (*b*) develops within itself an induced field in the direction opposite to the magnetizing field, and (*c*) if the magnetic field in which it is situated be ununiform the rod tends to move in the direction in which the strength of the field diminishes most rapidly.

The behavior of a substance in a magnetic field depends not only upon the permeability of the substance, but also upon the permeability of the surrounding medium. A magnetic substance sets itself across the magnetic lines of force when surrounded by a medium of greater permeability, and parallel to the lines of force when surrounded by a medium of less magnetic permeability. For example, a thin-walled glass tube filled with a dilute solution of ferric chloride suspended in a concentrated solution of the same substance between the poles of a strong magnet will set itself perpendicular to the lines of force; but when suspended in air or water, the tube of ferric chloride will set itself parallel to the lines of force.

274. Heusler's Magnetic Alloy.—Although manganese and aluminium are slightly paramagnetic while copper is slightly diamagnetic, an alloy consisting of 25 parts manganese, 14 aluminium and 61 copper, has a magnetic permeability of 33. This suggests that magnetic properties may not depend upon the atoms of the substance, but depend upon the constitution of the molecules or groups of molecules composing the substance.

275. Remanence, Coercive Force and Hysteresis.—If a specimen of iron be placed in a magnetic field that can be altered in intensity, and if for each value of this magnetizing field the induction density set up in the iron be determined, it will be found that the induction density is not uniformly proportional to the magnetizing field. Consequently, magnetic permeability is not a constant quantity, but it depends upon the magnetizing field.

For instance, starting with a specimen of unmagnetized iron, suppose we gradually increase the magnetizing field and determine the induction density in the iron corresponding to various magnetizing fields. By plotting magnetizing fields as abscissas and induction densities as ordinates, we will obtain a curve similar to *ab*, Fig. 254. This curve shows that when the magnetizing

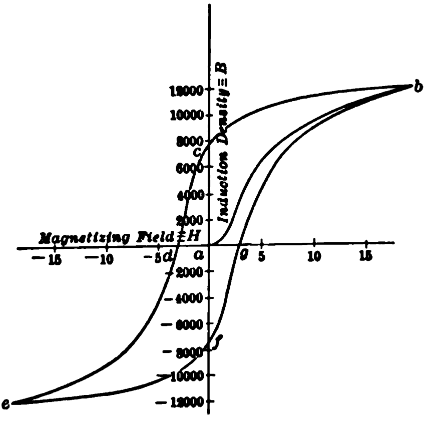

Fig. 254.

field is sufficiently small, a slight change in the value of the magnetizing field produces a great change in the induction density; whereas, when the magnetizing field is large, a change in the magnetizing field produces a smaller change in the induction density.

If after the condition represented by the point *b* has been attained the magnetizing field be gradually reduced to zero, it will be found that the magnetic condition of the iron does not retrace the previous series of values, but that the curve co-ordinating *B* and *H* has the form *bc*. Thus, while under a zero magnetizing field this particular specimen had an induction density of 8000 gausses. The value of the residual induction density when the magnetizing field has been reduced to zero is called *residual magnetism* or *remanence*. In the figure, the remanence corresponds to the distance *ac*.

If now the specimen be subjected to a gradually increasing magnetizing field in the opposite direction, the induction density will be reduced to zero when the magnetizing field has a certain value which in the case of the actual specimen here considered amounted to 3 gausses. The value of the magnetizing field required to reduce the induction density to zero is called the *coercive force*. In the figure, the coercive force is given by the distance *ad*. The magnitude of the remanence and of the coercive force depend upon the quality of the iron and also upon the degree to which the specimen has been magnetized.

The fact that when the magnetizing field is zero the induction density has not become zero, and that not till the magnetizing field has been reversed in direction and has a certain finite value will the induction density become zero, is described by the statement that when iron is subjected to a changing magnetic field the change of the induction density lags behind the change of the magnetizing field. The lagging of the change of induction density in a magnetic substance behind the change of magnetizing field is called *magnetic hysteresis*.

If, after the magnetizing field has reached the value represented by *d* it be still farther increased in the negative direction, the curve co-ordinating magnetizing field and induction density will be of the form *de*. If now, the magnetizing field be decreased to zero, the induction density will not become zero, but will have the value represented by *af*. In fact, the induction density will not become zero till the magnetizing field has increased in the positive direction to the value represented by *ag*. By gradually increasing the magnetizing field, the magnetic condition of the specimen will go through the changes represented by the curve *gb*.

The magnetic condition of the specimen has now gone through a complete cycle of changes that may be represented by the curve *bcdefgb*. If the magnetizing field be caused to repeat the above series of changes, the induction density will go through a cycle of values very nearly the same as before.

To increase the induction density of a magnetic substance, energy must be supplied. When the induction density diminishes, energy is liberated. It can be shown, though the proof will not be here given, that to carry a specimen through a complete cycle of magnetization, there must be supplied an amount of energy that is proportional to the area of the *hysteresis loop bcdefgb*. As the iron parts of many kinds of electric machinery go through many complete cycles of magnetization every minute, the knowledge of the amount of energy absorbed due to hysteresis is of great importance. The hysteresis loss per cycle depends upon the quality of the iron and upon the range of the magnetizing field to which it is subjected.

QUESTIONS

1. Steel drills while being used in drilling gas or water wells become magnetized. What is the cause of this magnetism? What is the supposed change in the structure of the drills?

2. It is required to make a magnet with both ends of the same polarity. Show how this is to be accomplished.

3. After a certain steel rod has been magnetized, it is suspended by a delicate fiber, but the rod exhibits no tendency to point in a north and south direction. Explain. If the rod were broken at the middle and each half separately suspended, would it be expected that then the two pieces would have a directive tendency?

4. Show (*a*) that when two similar poles are brought near one another each pole is weakened, and (*b*) that when two opposite poles are brought near one another each pole is strengthened.

5. Two bar magnets are placed in line with one another, (*a*) with opposite poles toward one another, (*b*) with similar poles toward one another. If the distance between the magnets is the same in the two cases, will the attraction in the first case be greater or less than the repulsion in the second case? Why?

6. Two equal bar magnets are fastened together so as to make a right-angled cross, and the system is suspended so that it can turn freely about a vertical axis. Find the direction it will set in the magnetic field of the earth.

7. Two circular iron rings are magnetized, the first by being placed between the poles of a strong horseshoe magnet so that the line joining the poles of the magnet is a diameter of the ring; the second, by having one pole of a bar magnet drawn round it several times. How do the magnetic states of the rings differ?

8. Explain how it is that although a compass needle is rotated by the earth's magnetic field, there is no motion of translation.

9. It is suspected that a magnetized bar of steel has consequent poles. How would you ascertain whether this is so or not?

10. If a bar magnet be placed inside a thick iron tube slightly longer than the magnet, it does not affect compass needles placed outside the tube, but if placed in a similar brass tube it does affect them. Explain.

11. What fact proves that the two poles of a magnet are of equal and opposite strength?

12. If the north pole of a magnet be brought slowly from a distance toward the north pole of a weakly magnetized compass needle, the compass needle will be at first slightly repelled, but later when the magnet is closer, it will be strongly attracted. Explain.

13. How would the reading of a compass be affected by placing it in an iron box? Explain.

14. Supposing an iron ship to behave like a permanent magnet with a weak north pole at the bow and a weak south pole at the stern, explain how a compass on board would be affected as the ship swings through 360°.

15. A long horizontal soft iron bar is placed in the axis of a ship so that one end is close to the compass, the bar lying wholly in front of the compass needle. What effect does this have on the compass? Does it make any difference whether the ship is in the northern or in the southern hemisphere? Why?

CHAPTER XVIII

ELECTRICITY AT REST

276. Electrification.—As early as 600 B.C. it was known by the Greeks that if after being in contact with woolen cloth a piece of amber be separated from it, the amber will attract chaff or other light bodies. Later, it was found that the "amber effect" is possessed by any two bodies of different material after having been separated from intimate contact. The Greek name for amber being *elektron,* the property that a body acquires on being separated from intimate contact with a dissimilar body is now called *electrification.* Electrification is conceived to be due to something called electricity. Of the ultimate nature of electricity we know very little, but with its effects we are more conversant. A body that has been electrified is said to have acquired a *charge* of electricity, or, to have become *charged* with electricity.

A convenient light object to exhibit the force between electrified bodies is a small ball made of the pith of a corn stalk or sunflower stalk. The pith ball is usually suspended by a silk fiber.

If a pith ball be allowed to touch an electrified body it will also become electrified, and will in turn attract any unelectrified body, e.g., the hand. After being touched with the hand the pith ball exhibits a weaker attraction than before. This is described by saying that the electrification of the pith ball has been partly lost, or that the pith ball has been partly *discharged.*

If two unelectrified pith balls, suspended as in *A*, Fig. 255, be allowed to touch the same electrified body, they will afterward be repelled by the charging body, as shown in *B*, Fig. 255. If the charging body be now removed, the two charged pith balls will be seen to repel one another, as shown in *C*, Fig. 255. This shows that two bodies charged in the same way repel one another.

Let two pith balls be suspended as in Fig. 256. Let the one marked *a* be charged by contact with an ebonite rod that has been separated from fur, and let the one marked *r* be charged with a glass rod that has been separated from silk. If now, a glass rod that has been separated from silk be brought near the two charged pith balls, *a* will be attracted and *r* will be repelled. It thus appears that there are two kinds of electrification, one which is attracted by the charged glass rod and one which is repelled by the same charged glass rod.

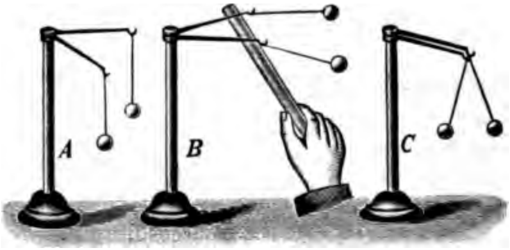

FIG. 255.

If the charged glass rod in Fig. 256 be replaced by the silk with which the glass rod has been rubbed, it will be found that the pith ball *a* will be repelled from the silk and the pith ball *r* attracted to the silk. This shows that both the glass rod and the silk from which it has been separated are electrified but that their electrifications produce opposite effects.

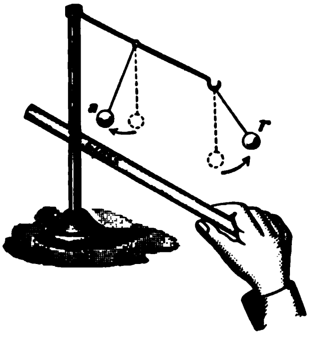

FIG. 256.

If after contact and separation, the glass rod and the silk cloth be brought together near an uncharged pith ball, it will be found that there is neither attraction nor repulsion. Consequently the charge on the silk cloth is equal in magnitude to the charge on the glass rod, but opposite in kind. From this fact it is logical to call the charge on one body positive and that on the other negative. Benjamin Franklin called the charge on glass after separation from silk, *positive.* And since his time physicists have continued to use this arbitrary convention.

In the following list, any substance becomes positively charged when brought into contact with and then separated from any substance lower down in the series.

+
1. Fur
2. Wool
3. Ivory
4. Quartz
5. Glass
6. Cotton
7. Silk
8. The hand
9. Wood
10. Metals
11. Ebonite
12. Sealing wax
13. Rosin
14. Sulphur
15. Guttapercha

It is seen that any substance except two may be made either positive or negative by properly selecting the substance with which it is brought into contact and from which it is then separated. For example, glass is positive if rubbed with silk, but negative if rubbed with fur or wool. It should be noted that the only purpose in rubbing is to produce close contact over a large area.

The previous experiments illustrate the following facts:

1. Any body can be electrified by separating it from contact with any dissimilar body.
2. The charges developed when two dissimilar bodies are brought together and then separated are opposite in kind and equal in magnitude.
3. A body can be electrified by contact with an electrified body at the expense of the charge on the latter.
4. An electrified body becomes discharged by intimate contact with the hand.
5. There is a force of attraction between an electrified and an unelectrified body.
6. Two similarly electrified bodies repel one another and two dissimilarly electrified bodies attract one another.
7. The charge on glass when brought into contact with silk and then separated, is arbitrarily called positive.

277. Coulomb's Law.—Suppose that we have two insulated metal spheres of the same size, one of them charged and the other not charged. If we cause one sphere to touch the other we shall find that both are then charged, and that the charges are equal. Each one has, therefore, half the charge that was at first on one of them. Proceeding in this manner we can obtain charges in the ratios 1 : 2, 1 : 4, 1 : 8, etc.

By measuring the forces of attraction or repulsion acting upon charged spheres of diameters small compared with their distances apart, Coulomb found that the force varies directly with the charge on each sphere, and inversely as the square of the distance between their centers. And for any given medium surrounding the charged bodies, the force is independent of all other quantities. If the medium surrounding the charged bodies be changed, the force will be altered. Denoting the charges on the two spheres by q_1 and q_2, and the distance between centers by r, these results may be expressed in the form

$$\begin{aligned} F &\propto q_1, \\ &\propto q_2, \\ &\propto \frac{1}{r^2}. \end{aligned}$$

Whence

$$F = \frac{q_1 q_2}{k r^2}, \qquad \qquad (167)$$

where k is a constant depending upon the medium surrounding the bodies. It should not be forgotten that this equation is accurate only when the charged bodies are very small compared with their distance apart.

The quantity k is different for different media. It is called the " dielectric constant " of the medium surrounding the charged bodies. The *dielectric constant* of a given medium is the measure of that quality of the medium which determines the magnitude of the force acting upon two charged bodies at a given distance apart when surrounded by the medium. The force acting upon two charged bodies in a medium of high dielectric constant is smaller than when the same charged bodies are in a medium of small dielectric constant.

If we assign a numerical value to the dielectric constant of some given medium, the magnitude of unit charge, that is, unit quantity of electricity, can be obtained from (167). The dielectric constant of vacuum is taken to be unity. Remembering this convention, an inspection of (167) shows that we may take as the unit quantity of electricity that quantity which repels an equal quantity at a distance of one centimeter, in empty space, with a force of one dyne. This is called the C. G. S. electrostatic unit of charge.

Taking the dielectric constant of vacuum as unit charge per dyne per centimeter, the dielectric constant of air at 76 cm. pressure is 1.0006; that of sulphuric ether is 4.8; ethyl alcohol, 26.5; ebonite, 3.15; glass from 4 to 9; mica, 6.6; paraffin, 2.3; shellac, 3.1; sulphur, 3.8; water, 81; metals, infinity.

Since for air the dielectric constant is so nearly equal to that for empty space, it is customary to call the dielectric constant for air unity.

278. Electric Field of Force between Charged Bodies.—The forces between charged bodies indicate the existence of potential energy in the medium between the bodies. This potential energy may be attributed to a state of electric strain developed in the medium by the charges. Any medium capable of being electrically strained is called a *dielectric.*

A region where an electric charge experiences a force is called an *electric field of force.* An electric field of force is bounded partly by positively and partly by negatively charged bodies. The property of an electric field of force which gives rise to a mechanical force when an electrified body is brought into it is called the *intensity* of the electric field. The intensity e of an electric field at any given point is measured by the ratio of the mechanical force F acting upon an electrified body situated at the point, to the charge q on the body. Thus

$$e=\frac{F}{q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (168)$$

Consequently, intensity of electric field is expressed in *dynes per unit charge.*

The direction of the electric field at any point is taken to be the direction in which a small positively charged body would tend to move if placed there. A line drawn through an electric field of force so as to be at each point in the direction of the electric field at that point is called an *electric line of force* or a *line of electric strain.*

279. Electric Potential.—If an electric charge q be placed in the neighborhood of a charged body Q, the system consisting of the two charges and the intervening medium will possess potential energy. The magnitude of the potential energy will depend upon the point at which the charge q is placed relative to Q. The point a, Fig. 257, differs from b and c in that the potential energy of the system would have different magnitudes if a given charge were placed at a, b and c. Thus, each point in the region of a charged body possesses a certain quality which is measured by the change in the potential energy of the electric system produced by placing a given charge at the point in question. That characteristic of a point in the region of a charged body or system of bodies by virtue of which potential energy is given to the system when a unit positive charge is placed there is called the *electric potential* at the given point.

Fig. 257.

If a small body electrified with a unit positive charge be brought into the neighborhood of an isolated positively charged body there will be a force of repulsion acting upon the two bodies; or, if brought into the neighborhood of an isolated negatively charged body there will be a force of attraction acting upon them. In either case the force acting upon the two charged bodies will be zero when they are separated by an infinite distance. When in this position the potential energy of the system equals zero. Consequently, at a point at an infinite distance from a charged body, the electric potential may conveniently be taken as zero, Hence, the magnitude of the electric potential at a given point equals the amount of work which must be done by an external agent in carrying a unit positive charge from infinity to the given point. If positive work must be done by an external agent in moving the unit positive charge from an infinite distance to the given point, the potential at this point is said to be *high;* if, however, positive work must be done by the external agent in moving the unit positive charge from the given point to an infinite distance, the potential at the given point is said to be *low.*

The difference of electric potential between two points is measured by the work required to move a unit positive charge from one point to the other.

For example, if 100 ergs of work are required to move a unit positive charge from a point B to a point A in opposition to an electric field directed from A toward B, the difference of potential between A and B is 100 ergs per unit charge, and the potential of A is higher than that of B. If 10 units of charge had been moved, the work required would have been 10 times as great, but the potential difference between A and B would, as before, be 100 ergs per unit charge.

In general, if W units of work are required to move a charge q from a place of potential V_B to a place of potential V_A in opposition to an electric field directed from A toward B,

$$W = q(V_A - V_B), \quad \ldots\ldots \quad (169)$$

or

$$V_A - V_B = \frac{W}{q}. \quad \ldots\ldots\ldots \quad (170)$$

The electrostatic unit of electric potential is the *erg per unit charge.*

280. The Practical Zero of Electric Potential.—Although in mathematical work it is very convenient to use as the zero of electric potential, the potential of a point in space at an infinite distance from the charged body or system under consideration, in experimental work it is quite otherwise. In experimental work it is customary to take the electric potential of the earth as zero. This is allowable for the following reasons. As the earth is a rather good conductor, the potentials at various points on it will never be very different. And since the size of the earth is so great, and since with every positive charge we always produce an equal negative charge, any change in the potential of the earth due to any charge we can impart to it is almost exactly balanced by the presence of the neighboring charge of opposite sign.

The use of two zero points from which to reckon electric potential in different cases is similar to the use in heat of the absolute zero and the centigrade zero of temperature. Since in experimental work only differences of potential are usually required, it is seldom necessary to know the difference between the potential of the earth and that at infinity. As a matter of fact the difference is great.

Taking the electric potential of the earth as zero, a body is said to be at *high potential* when on being connected to the earth by a conductor, positive electricity will pass from the body to the earth. Similarly, a body is said to be at *low potential* when on being connected to the earth by a conductor, positive electricity will pass from the earth to the body. Since the earth is so large that its potential cannot be sensibly altered by any charge that we can impart to it, any body after being connected to the earth will be at zero potential if energy is not supplied to it, and any uncharged body which is close to the earth and far from any charges except that of the earth will also be at zero potential.

281. Potential in Space due to Charged Bodies.—Since a positive charge tends to move away from an isolated positively charged body, points near an isolated positively charged body are at higher potentials than points at greater distances. And since a positive charge tends to approach an isolated negatively charged body, points near an isolated negatively charged body are at lower potentials than points at greater distances. The entire region about an *isolated* positively charged body is at high potential, and the entire region about an *isolated* negatively charged body is at low potential.

The word *isolated* is here used to mean, far from any other charge and far from any other conductor.

The presence of an electrified body affects the entire region round about it. A high potential body raises the potential of the whole region round about it, raising to a higher potential those points near the body than those points farther away. Similarly, the entire region in which a low potential body is situated is at a lower potential than it would be if the body were not present, and those points near the low potential body are at a lower potential than those points farther away from it. If a positively charged body be moved from one point to another it raises the potential of all points that it approaches and lowers the potential of all points from which it recedes. And if a negatively charged body be moved from one point to another it lowers the potential of all points that it approaches and raises the potential of all points from which it recedes.

If there are a number of charged bodies in a given region, each produces its own effect upon the electric potential at any given point independently of the other bodies. For example, when alone, the positively charged body A, Fig. 258, is at high potential, but on bringing a low potential body L into the neighborhood of A, the potential of A will be lowered and the potential of L will be raised. If the low potential body L be brought nearer and nearer to A, the potential of the latter will be reduced more and more. It may become zero or even "low." Unless L be brought close enough to A for a spark to jump across, there will be no change in the magnitude of the charge on either body.

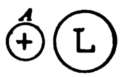

FIG. 258.

282. Flow of Electricity.—The potential energy of any system tends to diminish to a minimum. For example, a mass tends to move from a place of higher to a place of lower gravitational potential. Similarly, a positive charge tends to move from a place of higher to a place of lower electric potential, and a negative charge tends to move from a place of lower to a place of higher electric potential.

Let a narrow strip of aluminium or gold leaf BD, Fig. 259, be attached to a metal rod AC as shown in the figure. If a charged body be brought into contact with A, the leaf BD will be deflected away from the vertical position. This means that electricity has been transmitted along the rod till both BC and BD are charged. These being charged similarly, the movable vane BD is repelled and deflected away from the original position.

If the metal rod AC be replaced by one made of sulphur, glass or hard rubber, and the upper end be charged as before, there will be no deflection of the movable vane BD. This means that electricity is not transmitted by sulphur, glass or hard rubber.

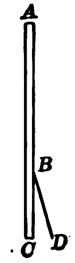

Fig. 259.

These experiments show that some substances transmit electricity, and other substances do not. Substances that transmit electricity readily are called conductors, and substances that do not are called non-conductors or insulators. Metals are good conductors. Air, glass, silk, porcelain, rubber, sulphur, amber and shellac are good insulators. A body that is not in contact with any conductor is said to be insulated.

If two points at a difference of electric potential are connected by a conductor and no energy enters the system from outside, a positive charge will move along the conductor from the place at higher potential toward the place at lower potential. This flow will continue until the entire conductor is at a uniform potential. A *conductor* may be defined as a body in which a difference of potential cannot persist unless energy be supplied from outside. A flow of electricity which occurs in a small fraction of a second is called a *discharge.*

If two points maintained at a constant potential difference be connected by a conductor, the flow of electricity will be steady. The rate of flow of electricity is called electric *current strength.* The strength of a current is measured by the quantity of electricity which passes any section of the conductor in a given time. Thus, representing strength of current by the symbol I,

$$I=\frac{q}{t}. \qquad \ldots\ldots\ldots \qquad (171)$$

This equation shows that the *C. G. S. electrostatic unit of current strength* is a flow of one C. G. S. electrostatic unit quantity of electricity per second.

283. Piezo-Electrification.—Some crystals become electrically charged when subjected to stress. The phenomenon called piezo-electrification (pressure-electrification), is exhibited by boracite, quartz, tourmaline, and especially strongly by Rochelle salt. If a crystal of Rochelle salt be compressed perpendicularly to the end faces, Fig. 260, the two end faces will be charged positively and the equatorial zone negatively. By applying a torque about the principal axis of the crystal, charges will also be developed.

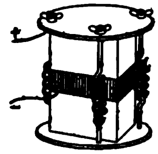

Fig. 260.

By applying compression in the direction of the principal axis of a crystal of Rochelle salt, A. M. Nicolson has obtained charges of 200 electrostatic units, and potentials of 1.66 electrostatic units per kilogram weight.

By causing a needle attached to the spring compressor, Fig. 260, to rest in the groove of a graphophone disk in such a manner that the transverse motion of the needle caused by the rotation of the disk will cause slight torsions of the crystal, charges will be developed which vary with the form of the groove, that is, with the sound which produced the groove. If a telephone receiver be joined to the electric terminals of the crystal, the sound recorded on the graphophone disk can be reproduced at a distant point.

284. Electrostatic Induction.—Consider the effect of an electrified body upon an insulated uncharged conductor. In Fig. 261, *A* and *B* are two metal bodies supported on glass rods. Let *A* be discharged by touching it with the finger. Let *B* be negatively charged by stroking it with a piece of fur. Since *B* is at a distance from all other charged bodies it is at low potential. And since *B* is supported by a non-conducting rod, its charge remains.

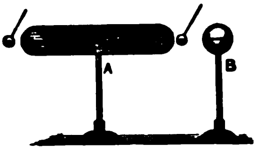

Fig. 261.

On bringing the charged body *B* near to the uncharged body *A*, but not touching it, it is found (by means of a charged pith ball, for example) that the body *A* has become positively charged on the end nearer the body *B* and negatively charged on the end distant from *B*. About an intermediate zone the charge is zero. It is found that the longer the body *A*, and the nearer the body *A* is to *B*, the greater is the magnitude of the charges on the two ends.

Since the body A is quite insulated, no charge has been either added to it or taken from it during these operations. Consequently the positive charge on one end is equal in magnitude to the negative charge on the other end. The production of electrification on a body by the mere proximity of an electrified body is called *electrostatic induction* or *influence*.

In terms of the idea of electric potential, the cause of the production of the charges on A is as follows: The presence of the low potential body B lowers the potential of the whole surrounding region—points nearer the body being at a lower potential than points farther away. Thus, the potential of a point on the end of A nearer B is made lower than the potential of a point on the end of A farther from B. These two points being connected by a conductor, a positive charge will flow from the end at higher potential to the end at lower potential. The charge on the end farther from B will be thereby reduced from its original zero value to a negative value, while the charge on the end nearer B will be raised from its original zero value to a positive value. It should be noted, (*a*) that although one end of A is charged negatively and the other positively, the total charge on A is still zero; (*b*) that after the momentary flow of positive charge along A, the electric potential of A is uniformly low throughout.

The entire body A being at one potential, and this potential being lower than that of the earth, if the earth be connected to A, a positive charge will flow from the earth to A and the magnitude of this charge will be the same whatever point of A be joined to the earth. If the connection between the body A and the earth be broken, and the body B then removed to a distance, the body A will be left positively charged.

The charged body B and the uncharged body A tend to approach one another because the force of attraction between B and the nearer end of A exceeds the force of repulsion between B and the distant end of A. This is the cause of the attraction between a charged body and any uncharged body. As there is a greater separation of charges on a good conductor than on a poor conductor, good conductors are more strongly attracted than poor conductors.

The fact that good conductors are more strongly attracted by a charged body than are poor conductors is the basis of the electrostatic method of separating metallic particles from particles of rock. Crushed ore drops from a hopper onto a rotating drum and thence falls in a thin sheet in front of a highly charged body Q, Fig. 262. The poorly conducting particles are but slightly deflected by the charged body whereas the conducting particles are drawn to one side and fall into a different chute. To recover any metallic particles which may be still with the poorly conducting material, the tailings are subjected to several repetitions of the above action.

285. The Gold Leaf Electroscope.—This sensitive instrument for the detection of charges and the determination of the sign of a charge consists of a narrow strip of gold or aluminium foil attached by one end to a metal rod, Fig. 263. The gold leaf should be protected by a metal case provided with a window. The rod is separated from the case by an insulating plug of sulphur or amber. On charging the upper end of the rod, either positively or negatively, the rod and the leaf will become similarly charged and the gold leaf will recede from the rod. If the charge be increased, the deflection will become greater; if the charge be decreased the deflection will become smaller.

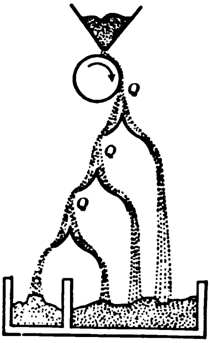

Fig. 262.

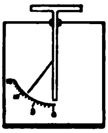

Fig. 263.

The gold leaf is so frail that it would be broken if the instrument were strongly charged. For this reason it is never charged by bringing it into contact with a charged glass or hard rubber rod.

To charge an electroscope negatively, a glass rod that has been separated from silk is brought toward the instrument. The whole region about the glass rod being at high potential (Art. 281), the electroscope is now at a higher potential than the earth. On touching the instrument with the finger, the instrument and the earth will be joined by a conductor. Since a difference of potential cannot persist on a conductor unless energy be supplied, a positive charge will flow from the electroscope to the earth until the potential of the electroscope is that of the earth. Since a positive charge has left the previously uncharged electroscope, the instrument is now negatively charged. If the finger be removed from the electroscope without moving the glass rod, the electroscope will remain at zero potential and will be charged negatively. But the potential of the electroscope is higher than it would be if the high potential glass rod were not present. If the glass rod now be removed, the potential of the electroscope will be less high than when the glass rod was present. Hence the electroscope is now at low potential. And as positive charge left the previously uncharged electroscope while connected to the earth, the instrument is now negatively charged.

By using a hard rubber rod that has been separated from fur and proceeding as above, an electroscope can be charged positively.

286. The Electrophorus.—Electrostatic induction furnishes a convenient means for producing charges. The electrophorus consists of a plate of ebonite, sealing wax or other substance low down in the electrostatic series (Art. 276), together with a sheet metal cover provided with an insulating handle. On stroking this plate *A*, Fig. 264, with a piece of fur, the plate will be charged negatively. On removing the fur the plate will be at low potential.

Fig. 264.

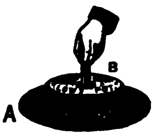

Fig. 265.

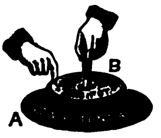

Fig. 266.

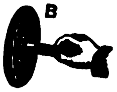

Fig. 267.

If we bring the uncharged cover *B* near the plate *A*, Fig. 265, the cover will become of low potential. And if the plate and cover do not touch, the charge of the cover will remain zero.

If while the cover is near the plate, it is touched by a body connected to the earth, Fig. 266, charge will flow from the zero potential earth to the low potential cover till the cover becomes of zero potential. The cover is now positively charged.

If the connection with the earth be now broken and the cover be removed to a distance, the potential of the cover will not be so low as when the cover was in the neighborhood of the low potential plate. Therefore the potential of the cover is now high. The charge is the same as it was before it was removed from the neighborhood of the plate.

If the cover be now brought into contact with the earth, electricity will flow from the high potential cover to the zero potential earth till the cover is at zero potential. The cover is now uncharged.

The charge and the potential of the plate *A* are the same now as they were after the plate was stroked with the fur. The above series of operations can be repeated indefinitely. The energy of the high-potential charged cover in Fig. 267 is due to the work expended in separating the positively charged cover from the negatively charged plate in Fig. 266.

287. Electric Capacity or Capacitance.—Let a positive charge be given to a body *A*, and an equal negative charge to a body *B*.

This is equivalent to taking a positive charge from B and giving it to A. This process develops a potential difference between the bodies with A at higher potential than B. If the charge taken from B and given to A is doubled, the potential difference is also doubled; and in general, the potential difference is proportional to the charge taken from one body and given to the other.

Since the carrying of a positive charge q from B to A produces a potential difference $V_A - V_B$, proportional to q, we may write

$$C = \frac{q}{V_A - V_B}, \quad \ldots\ldots\ldots \quad (172)$$

where C is a constant which depends upon the size of the bodies, the distance between them and the nature of the surrounding medium. This constant is numerically equal to the charge that must be carried from one body to the other in order to develop unit potential difference between the bodies. It is called the *electric capacity* or *capacitance* of the system.

Sometimes one speaks of the capacity of a single body. In this case it is understood that the other body is the wall of the room, the earth or something else at zero potential.

288. Condensers.—Let a plate, A, be charged positively, and let a plate, B, be charged negatively. If the two plates are far apart the charge on either of them produces little effect on the potential of the other. But if they are brought close together, the negative charge on B lowers the potential of A and the positive charge on A raises the potential of B. Hence the difference of potential between the plates is much smaller than when the plates are far apart. It follows (172), that the capacitance of the system is much greater when the plates are close together than when they are far apart. In other words, when the plates are close together a much larger charge can be carried from one to the other without producing a great potential difference between them. The term *electric condenser* is applied to an apparatus consisting of two conductors insulated from one another and so arranged that a large charge can be carried from one to the other without the potential difference between the conductors being greatly changed.

The electric capacitance of a condenser is numerically equal to the number of units of charge that must be communicated to one conductor in order that the difference in potential between the two conductors may change by unit amount. The capacitance of an electric condenser depends upon the area and shape of the conducting surfaces, their distance apart, and upon the nature of the intervening dielectric. In order that a condenser may have great capacitance, the area of the conducting surfaces should be large, and the distance between these surfaces should be small.

By the electric capacitance of a condenser one does not mean the greatest quantity of electricity it can hold. The greatest quantity of electricity that can be imparted to a condenser depends not only upon its electric capacitance, but also upon the ability of the substance between the conductors to resist being punctured by an electric discharge. The instrument is called an electric condenser because a considerable charge can be imparted to it with an expenditure of but a small amount of work.

Condensers usually consist of a pile of thin metal sheets separated by a layer of sheet mica, paraffined paper or glass. The alternate metal sheets joined together constitute one conductor, while the other metal sheets joined together constitute the other conductor. Large commercial condensers are also made by enclosing the metal sheets separated by thin spaces in a strong tank filled with air under a pressure of 200 or more pounds weight per square inch. The earliest condensers were made in Leyden and consisted of glass jars coated with tinfoil inside and outside to within a few inches from the mouth. Such condensers are still used and are called "Leyden jars."

One purpose for which condensers are used is to accumulate a gradually increasing charge until when a certain potential difference is attained all the accumulated charge will spark across a gap left in the circuit

289. Specific Inductive Capacity.—Consider two conducting plates, one charged positively and the other negatively. On inserting a sheet of unelectrified glass between the plates, no change in the charges is produced. But it will be found that the potential difference between the plates has been diminished. Since there is a diminution of $(V_A - V_B)$, and no change of q, it follows from (172) that the capacitance of the system is greater when the medium between the conductors is glass than when it is air. That quality whereby a substance introduced between the coatings of a condenser changes its electric capacitance is called the *inductive capacity* of the substance.

The ratio of the capacitance of a condenser when the space between the coatings is filled with a given medium, to the capacitance of the same condenser when the space between the coatings is vacuum, is called the *specific inductive capacity* of the given medium. Thus, if when the space between the plates of a certain condenser is filled with a given substance the electric capacitance of the system is represented by C_s, and when the space between the plates is vacuous the electric capacitance is C_v, then the magnitude of the specific inductive capacity of the given substance is

$$\text{Sp. Ind. Cap.} = \frac{C_s}{C_v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (173)$$

The specific inductive capacity of a medium is numerically equal to its dielectric constant.

Questions

1. Two bodies are rubbed together and then separated. It is found that they are electrified and have energy. What is the source of the energy?

2. Explain why a metal ball suspended by a silk thread between two bodies, one of which is charged positively and the other negatively, flies back and forth between the bodies.

3. Given an electroscope, a glass rod, a piece of silk, how would you proceed to test the sign of electrification of an unknown charge?

4. An uncharged conductor is brought near an insulated positively charged conductor. What change will be produced in the field of force? What further change will be produced if the first body is connected to the earth and then disconnected?

5. Assuming that oppositely charged bodies attract one another, show why any charged body will attract an uncharged body.

6. Explain why an initial repulsion is followed by an attraction when a small conductor with a considerable charge is brought gradually near to a large conductor with a small charge, the charges being of the same sign.

7. A perfectly insulated unelectrified conductor is brought into the neighborhood of an electrified body. While in this position the conductor is separated into two parts, and the two parts removed from the neighborhood. Describe the electric condition of the two parts.

8. Is it possible to have a positive and a negative charge on a conductor at the same time? On a non-conductor? Explain. Show how a body may have at the same time:

(*a*) a positive charge and be at zero potential;
(*b*) a negative charge and be at zero potential;
(*c*) a positive and a negative charge and be at high or low potential.

9. An isolated insulated metal vessel is positively charged. An uncharged metal ball supported by a silk thread is (*a*) introduced into the vessel without touching it; (*b*) then connected momentarily with the earth; (*c*) then removed to a distance. State the changes in the potential and in the charge of the ball during these operations.

10. A highly positively charged body is gradually brought from a distance toward a negatively charged electroscope without touching it. State and explain the phenomena observed.

11. If a gold leaf electroscope is charged negatively and a glass rod which has been rubbed with silk is moved toward the top of the electroscope, what will be the effect on the leaves? If a piece of electrified sulphur is brought near a negatively charged electroscope and the leaves diverge farther, is the electrification of the sulphur positive or negative?

12. Show clearly how a body may be charged positively by a negatively charged body.

13. A gold leaf electroscope is charged negatively. An electrified body is brought near the knob and the leaves separate. What is the nature of the charge of the body? Explain.

14. In charging an electroscope by influence, why must the finger be removed before the removal of the charged body?

15. An electroscope possesses a charge which causes the leaves to remain divergent, but you do not know whether the charge is plus or minus. How would you decide the point without taking any charge from the electroscope?

16. Show how it is possible to charge an uncharged body, (*a*) positively, (*b*) negatively, by means of a positively charged body.

17. Assume that you have a positively charged sphere and two uncharged insulated metal vessels, "A" and "B." Trace the changes in the potentials and the charges of the three bodies while the following operations are successively performed: (*a*) The sphere is placed inside "A" without contact; (*b*) the two are enclosed by the vessel "B" without contact; (*c*) "A" and "B" are joined by a conductor; (*d*) the contact is broken; (*e*) the three bodies are separated; (*f*) "A" is joined to the earth.

CHAPTER XIX

ELECTRICITY IN MOTION

§ 1. *Fundamental Laws of Direct Currents*

290. The Magnetic Field Surrounding Moving Electricity.—In 1820 Oersted observed that if a compass needle be near a current-carrying conductor, the needle will set itself perpendicular to the line from the needle to the conductor. In 1889 Rowland performed an experiment which showed that there is also a magnetic field of force about a charge fixed to a rotating non-conducting disk. We now know that a magnetic field is produced when a charge is moving, and also when the charge is changing on a body that is at rest. A magnetic field of force is produced wherever there is a changing electrostatic field.

The magnetic field about a charged particle moving through space is the same kind of a field as that about a current-carrying conductor. In all cases the direction of the magnetic field about a moving positive charge bears the same relation to the direction of motion of the positive charge that the direction of rotation of a wood screw bears to the direction of the advance of the screw.

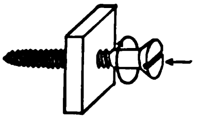

FIG. 268.

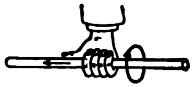

FIG. 269.

The direction of an electric current is taken to be the direction of motion of positive charges. Hence, *the positive direction of an electric current bears the same relation to the direction of the surrounding magnetic field of force that the direction of the advance of a wood screw bears to the direction of rotation of the screw.* This is called the Right-hand Screw Rule.

Another convenient rule for remembering the direction of the magnetic field about a current-carrying conductor is the so-called Right-hand Thumb Rule—*if one grasps the wire with the right hand so that the thumb points in the direction of the current, the fingers will point in the direction of the magnetic field.*

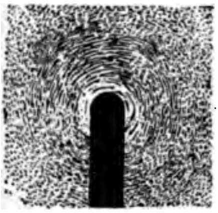

FIG. 270.

If a current flows through a wire passing perpendicularly through the plane of the paper, then the magnetic field of force, mapped by sprinkling iron filings on the paper, will be as shown in Fig. 270.

An iron filing diagram of the magnetic field about two parallel conductors carrying equal currents in *the same direction* is given in Fig. 271. An iron filing diagram of the magnetic field about two parallel conductors carrying equal currents in *opposite directions* is given in Fig. 272.

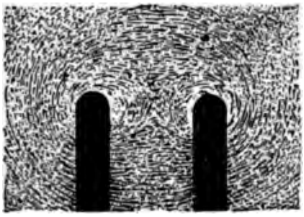

FIG. 271.

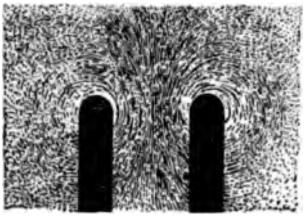

FIG. 272.

The direction of a current, or of a line of force, is conveniently represented by an arrow. When a current or a line of force is directed away from the reader, and normal to the plane of the paper, it is represented by an *,— suggesting the feathers seen on a retreating arrow. When normal to the plane of the paper and approaching the reader, a current or a line of force is represented by a dot,—suggesting the point of an approaching arrow. For example, the directions of the magnetic fields about a conductor normal to the plane of the paper when carrying current away

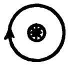

FIG. 273.

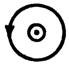

FIG. 274.

from the reader, and when carrying current toward the reader, are shown in Figs. 273 and 274, respectively.

291. Solenoids.—The magnetic field of force near a current-carrying conductor can be increased by bending the conductor into a cylindrical helix or *solenoid.* The intensity of the magnetic field at the center of the solenoid is proportional to the number of turns of wire per unit length in the solenoid. An iron filing map of the magnetic field about a solenoid is shown in Fig. 275.

To prepare this map a wire was threaded through two rows of holes in a card so as to form a solenoid with the card passing through the axis. By sprinkling iron filings on the card while a current was flowing in the solenoid there was produced on the card a map of the magnetic field in the plane passing through the axis of the solenoid. A comparison of this map with Fig. 244 shows that the magnetic field of a solenoid is the same as that of a bar magnet. Some lines of force in the plane of the card are drawn in Fig. 276. The small circles represent the cross-sections of the wire where it goes through the card. The crosses and dots in these small circles indicate the direction of the current.

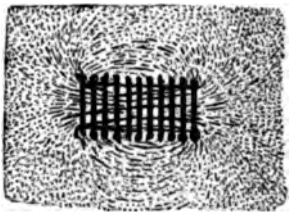

FIG. 275.

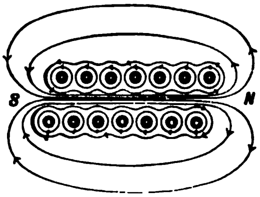

FIG. 276.

292. Electromagnets.—If a bar of iron be placed within a solenoid, the magnetic field of the current in the solenoid will magnetize the iron. The induction density in the iron core will be μ times as great as the magnetizing field. Magnets produced by the magnetic field of force about a current-carrying conductor are called *electromagnets.*

The polarity produced by a current in a given direction through the solenoid can be determined from a consideration of a line-of-force diagram such as Fig. 276. A convenient rule is the Thumb Rule which may be applied to a solenoid, either with iron or not, as follows: *Grasp the coil with the right hand so that the fingers point in the direction of the current in the wire. Then the thumb will point toward the north-seeking pole of the coil.* On applying this rule to Fig. 276, the rule will be found to be correct.

Electromagnets can be made that are much more intense than magnets produced in any other way. If the iron core of a solenoid is of soft iron, its magnetism will be reduced to nearly zero when the current ceases. For these reasons electromagnets are extensively used in electric bells, telephones, telegraph instruments, dynamos, motors, as well as in other instruments and machines.

293. The Simple Electric Telegraph.—Each station is provided with a source of electric current B, Fig. 277, a key K for quickly opening and closing the circuit, a switch S, and a sounder consisting of a small rod of iron pivoted above an electromagnet A. The figure represents two stations joined by a single wire. The current returns through the earth. E_1 and E_2 represent two metal plates buried in moist ground. When no current traverses the line, the pivoted iron rods, called "armatures," are held by springs against upper stops as shown in the figure. When current traverses the line, the armatures are pulled against lower stops. On striking either an upper or a lower stop, a distinct click is heard.

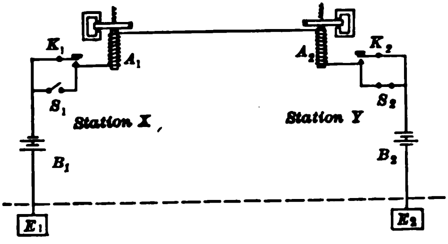

FIG. 277.

While the line is not in use, the switch at each station is kept closed and a current traverses the circuit. Suppose that the operator at station X desires to communicate with the operator at station Y. He first opens his switch S_1 and then makes and breaks the circuit by depressing and raising the key K_1. Every time K_1 is depressed, the sounders are traversed by a current and each armature is pulled down with a sharp click. On permitting the key to rise, the circuit is broken and each armature is pulled by its spring against the upper stop and another click is produced. A short interval between two clicks is called a "dot"; a longer interval is called a "dash." The various letters of the alphabet are represented by groups of dots and dashes.

294. Direction of the Force Acting upon two Bodies which Give Rise to Magnetic Fields.—It is found that two unlike magnet poles, Fig. 278, are urged toward one another; a current-carrying conductor and a magnet arranged as in Fig. 279 are urged toward one another; and two conductors with currents in the same direction, Figs. 280 and 281, are urged toward one another. On the other hand two like magnet poles, Fig. 282, are urged apart; a current-carrying conductor and magnet arranged as in Fig. 283 are urged apart; and two conductors with currents in opposite directions, Fig. 284, are urged apart. From an inspection of these figures it will be seen that *each body giving rise to a magnetic field is urged from the side where the component fields are in the same direction toward the side where the component fields are in opposite directions.*

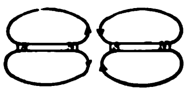

Fig. 278.

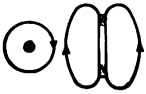

Fig. 279.

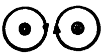

Fig. 280.

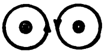

Fig. 281.

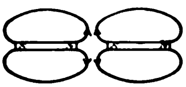

Fig. 282.

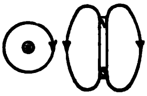

Fig. 283.

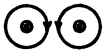

Fig. 284.

If the two component fields are perpendicular to one another there is no force action. For example there is no force acting upon a current-carrying conductor parallel to a magnetic field, Fig. 285. But in the case of the current-carrying conductor not parallel to the direction of the magnetic field in which it is situated, Fig. 286, there is a force on the upper side pushing the conductor downward, and a force on the lower side pulling it downward.

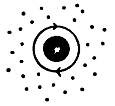

Fig. 285.

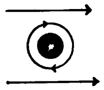

Fig. 286.

By the same method we can show that two non-parallel current-carrying conductors tend to turn till the conductors are parallel and the currents are in the same direction. Thus in Fig. 287, the component fields being in opposite directions within the angle *axc* and also within the angle *dxb*, *a* and *c* are urged toward one another, and *d* and *b* are urged toward one another. The component fields within the angle *axd* being in the same direction, and also the component fields within the angle *cxb*, *a* and *d* are pushed away from one another, and also *c* and *b* are pushed away from one another. If no forces oppose them, these actions will continue till the conductors are parallel and the currents are in the same direction.

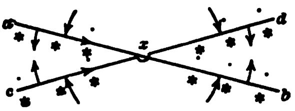

FIG. 287.

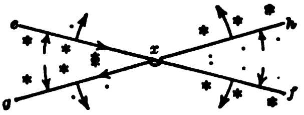

FIG. 288.

295. The Electromagnetic Unit of Current Strength.—A conductor carrying a current in a magnetic field is acted upon by a force tending to push it sidewise except when the conductor is parallel to the direction of the field. If the magnetic field be of unit intensity, and the direction of the conductor be perpendicular to the direction of the field, the force acting upon unit length of the conductor may be taken as the measure of the current flowing along the conductor. Using this arbitrary convention, it is found that the force acting on a current-carrying conductor placed perpendicular to a magnetic field varies directly with the length of the conductor, the intensity of the field, and the current. This is called Ampere's Law. In symbols,

$$F=clHI, \qquad (174)$$

where c is a constant depending upon the units adopted for the various quantities. We have already adopted the dyne as the unit of force, the centimeter as the unit of length and the gauss as the unit of intensity of magnetic field. The constant c will become unity if we adopt as the unit of current, *a current strength of such magnitude that one centimeter of the conductor along which it* *flows will be pushed sidewise with a force of one dyne when the conductor is in, and perpendicular to, a magnetic field of one gauss.*

This *absolute* electromagnetic unit of current strength is sometimes called the *abampere.* As the abampere is a rather large unit, currents are usually expressed in another unit just one-tenth as large. This *practical unit* of current strength is called the *ampere.*

296. The Electromagnetic Unit of Charge.—The absolute electromagnetic unit of charge is the quantity of electricity that passes a given point in one second when the conductor is traversed by an abampere (171). The *practical* unit of charge is the quantity of electricity that passes a given point in one second when the conductor is traversed by one ampere of current. The practical electromagnetic unit of charge is called the *coulomb.* The coulomb equals $3(10^9)$ electrostatic units of charge.

Measurements show that a flash of lightning is associated with about 30 coulombs.

QUESTIONS

1. A horizontal iron rod is to be magnetized so that the right end is a north pole. Explain fully how this can be done by means of a current.

2. A wire is stretched from east to west (magnetic). How can you test whether, and in what direction, an electric current is passing through it?

3. Explain why the filament of a lamp fed with alternating current will oscillate in a uniform magnetic field.

4. What is the cause of the increase of magnetic field when a piece of soft iron is placed in a coil bearing a current?

5. Would the currents in the neighboring turns of an electromagnet attract or repel each other? State how you would foretell.

6. A suspended wire is carrying a current toward you. An isolated north pole is placed above it. Does the wire swing to the right or to the left? How do you determine? The same magnet pole is placed below it. Does the wire swing to the right or to the left?

7. A wire hanging from a hook carries an electric current down into mercury surrounding the south pole of a vertical magnet. The north pole is some distance below. Draw a horizontal cross-section, viewing from above. Show the direction in which the wire will move, and tell how you determine it.

8. A wire hanging from a hook carries an electric current up out of mercury surrounding one pole of a vertical magnet. As seen from above, the wire rotates clockwise. Which pole of the magnet is uppermost? Explain clearly how you determine.

9. A flexible brush presses against a permanent magnet capable of rotation, Fig. 289. The negative pole of a battery is joined to this brush and the positive pole to the south pole of the magnet. Find whether the magnet will tend to rotate. If it will, find the direction.

10. An inverted U-shaped wire is mounted so as to be capable of rotation about one pole of a vertical permanent magnet, Fig. 290. The ends of the wire dip in a horizontal circular trough of mercury. If a battery be joined as indicated, will the wire tend to rotate? If so, find the direction.

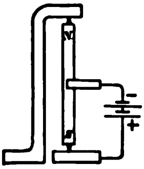

FIG. 289.

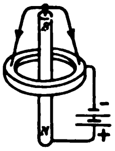

FIG. 290.

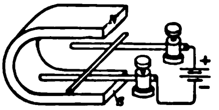

FIG. 291.

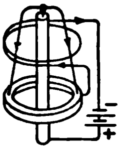

FIG. 292.

11. A knitting needle rests on two horizontal rails between the poles of a magnet as shown in Fig. 291. If a current traverses the knitting needle in the direction indicated, will there be any tendency for the needle to roll along the rails? If so, find the direction.

12. An inverted U-shaped wire is mounted so as to be capable of rotation about a vertical brass rod, Fig. 292. The ends of the wire dip in a horizontal circular trough of mercury. When a current traverses the rod, the U-shaped wire and a horizontal coil about the latter, will the U-shaped wire tend to rotate? If so, find the direction.

297. Electric Resistance.—When a conductor is traversed by a current it becomes heated. In order that the current may be maintained at a constant value electric energy must be supplied to the conductor. The property whereby a conductor absorbs the energy of the electric current and converts it into heat is called the *electric resistance* of the conductor. The resistance of a conductor is measured by the amount of heat developed in it by the passage of unit current for unit time.

By experiment it is found that the amount of heat developed in a wire of given material at a given temperature when traversed by unit current for unit time, that is, the resistance of the wire, is (*a*), directly proportional to the length of the wire, (*b*), inversely proportional to its area of cross-section, and, as a rule, (*c*), depends upon nothing else. Consequently, the resistance

$$R=\frac{\rho l}{A}, \quad \dots\dots\dots \quad (175)$$

where ρ is a constant for any given material at any given temperature. This constant is called the *resistivity* or *specific resistance* of the given material and is equal to the resistance between opposite faces of a centimeter cube of the material. The reciprocal of resistivity is called *conductivity.*

Engineers frequently employ the circular mil foot as a standard volume, in which case the resistivity is the resistance between opposite ends of a wire of circular section, 1 foot long and .001 inch in diameter. One mil is a thousandth of an inch. One circular mil is the area of a circle one mil in diameter. The area of a circle d mils in diameter is d^2 circular mils. When length is expressed in feet and sectional area in circular mils the numerical value of K in the equation

$$R\left[=\frac{\rho l}{A}\right]=\frac{Kl}{d^2} \quad \dots\dots\dots \quad (176)$$

is 10.8 for pure copper at 20° C.

The resistivity of some substances is affected by light and the presence of magnetic fields. At atmospheric temperatures a magnetic field of 27,500 gausses will increase the resistance of a bismuth wire transverse to the field about 2.54 times. This fact furnishes a method of measuring magnetic field intensities.

Not all of the cross-section of a conductor takes part equally in the conduction of a rapidly alternating current. The resistance of a copper wire to oscillating currents of the frequency of 10^6 alternations per second is about the same as that which would be offered by the outer layer 0.01 mm. thick. The resistance of a conductor varies as the square root of the frequency of alternation. For the alternating currents employed for lighting and power this " skin effect " is negligible; but in the case of the rapidly oscillating currents of lightning and wireless telegraphy it is considerable.

Part of the energy of an electric discharge or of an electric current is transformed into heat. The enormous electric discharges that occur in the air during storms develop sufficient heat along their path to produce brilliant lightning flashes.

The development of heat by the passage of electricity through a conductor is the basis of many domestic appliances such as the incandescent lamp, the electric cooking range, the electric flatiron, the electric toaster and the electric hair curler. In these devices the "resistor," or conductor in which the heat is developed, consists of a wire of high resistivity, either bare or wrapped with asbestos. Due to their high melting point, high resistivity and ability to withstand oxidation, alloys of nickel and chromium are much used for domestic devices. Tungsten is now most often used for the filaments of incandescent lamps.

A platinum wire or band heated to incandescence is frequently used by surgeons, instead of a knife, to remove tumorous growths. The hot wire will cut and also sear the wound so that dangerous bleeding is prevented.

If two pieces of metal be pressed together while a large current passes through them, there will be developed at the junction sufficient heat to fuse the two pieces together. In this manner metals can be welded that cannot be welded by the older processes.

298. Resistivity Changed by Temperature.—The resistivity of most substances increases when the temperature rises. For some pure metals, the increase of resistance from any standard temperature to any other temperature is proportional to the resistance at the standard temperature, and is approximately proportional to the change of temperature. Thus, if at 0° C. the resistance of a given wire made of one of those metals be R_o, then in rising to $t°$ there will be an increase of resistance $\beta R_o t$, where β is a constant of proportionality called the temperature coefficient of resistance. Consequently, at $t°$, the resistance will be

$$R_t = R_o + \beta R_o t = R_o(1 + \beta t). \quad . \quad . \quad . \quad . \quad . \quad (177)$$

For platinum, the temperature coefficient of resistance is approximately 0.0037 per degree centigrade. It will be noticed that this temperature coefficient of resistance is approximately numerically equal to the temperature coefficient of expansion of a perfect gas at constant pressure, (0.00366). Therefore, the resistivity of platinum varies approximately with the absolute temperature. If this law holds for all temperatures, the resistivity of platinum is nearly zero at the absolute zero of temperature.

Carbon, boron, glass, porcelain and non-metallic liquid conductors have negative temperature coefficients of resistance, that is, these substances diminish in resistance when the temperature is raised. At a red heat the resistance of a boron wire is only 0.00001 the resistance at room temperature.

The temperature coefficients of most alloys are much smaller than those of the pure metals of which they are composed. For example, the temperature coefficient of resistance of a certain alloy of nickel and steel is 0.00067 per ° C.; one of copper and nickel is -0.00001 per ° C.; and one of copper, nickel and manganese is zero from 40° C. to 50° C. For temperatures below 40° C. this last alloy has a small positive coefficient, and at temperatures above 50° C. it has a small negative coefficient.

Since electric resistance can be readily measured, the change of resistance with change of temperature furnishes a convenient means for the comparison of temperatures. Platinum may be used from the lowest temperature up to 1100° C. For temperatures below 200° C., pure nickel can be used.

299. Resistivity Changed by Light.—Selenium, and the mineral antimonite (antimony sulphide) possess to a considerable degree the property possessed by but few substances of changing in resistivity when exposed to light. Selenium and antimonite diminish in resistivity when exposed to light.

The resistance-light sensitivities of selenium and of antimonite are of the same order of magnitude and are higher than that of any other known substance. A "selenium cell" can be made by winding two parallel wires side by side about a sheet or rod of insulating material and then filling the narrow spaces between the convolutions with melted selenium. The resistance between the two wires is that of the very short length of the film of selenium extending from one wire to the other. A certain selenium cell that in the dark had a resistance of 10,000 ohms, had a resistance of 3000 ohms when exposed to a bright light. Another cell that in the dark had a resistance of 300,000 ohms, had a resistance of 20,000 ohms when exposed to a bright light. Selenium is most sensitive to yellow-green light.

The resistance-sensitivity of selenium is sometimes used by astronomers in the comparison of the brightness of stars.

300. The Electromagnetic Unit of Resistance.—From the definition of electric resistance it follows that the heat developed by the passage of a constant current for a given time is directly proportional to the resistance of the conductor. Thus, in symbols,

$$H \propto R.$$

From experiment it is found that the heat developed in any given time by a current traversing any conductor is directly proportional to the square of the current. Thus,

$$H \propto I^2.$$

It is also found that when a constant current traverses a conductor for different lengths of time, the quantity of heat developed is directly proportional to the time. Thus,

$$H \propto t.$$

Since it is found that the quantity of heat developed in the given conductors depends only upon the magnitudes of the resistance, current and time, the three variations expressed above can be represented by the equation

$$JH = I^2Rt = W, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (178)$$

where H is the energy expressed in heat units, I^2Rt is the energy expressed in electrical units, and W is the energy expressed in mechanical units. The constant of proportionality J is the mechanical equivalent of heat. The law represented by (178) is called Joule's Law.

On putting this equation into the form

$$R = \frac{JH}{I^2t}$$

the magnitude of the absolute unit of resistance can be easily deduced. Units of heat, current and time have already been adopted. Before the unit of resistance can be determined, a definite value must be assigned to J.

If H be expressed in calories and J in ergs per calorie, then JH will be the number of ergs of heat developed in the circuit. Hence, the absolute electromagnetic unit of resistance, sometimes called the abohm, is the resistance of a conductor in which one erg of heat is developed by the passage of one abampere for one second. This unit is seldom used.

If H be expressed in calories and J in joules per calorie, then JH will be the number of joules of heat developed in the circuit. If, in addition, I be expressed in amperes, and t in seconds, the unit of resistance is the resistance of a conductor in which one joule of heat is developed by the passage of one ampere of current for one second of time. This unit is called the *ohm* and is the practical unit of resistance. One ohm is 10^9 abohms.

Usually, I is expressed in amperes, R in ohms, t in seconds and H in calories. In this case $J=4.2$ joules per calorie and (178) may be written

$$4.2H=I^2Rt. \qquad \ldots\ldots \qquad (179)$$

Since the heat developed by the passage of a current through a conductor varies with the current strength according to a definite law (179), an electric current can be measured by means of the heat developed. Instead of measuring the heat developed, one can more easily measure the change in length of the conductor produced by the change in temperature. The principle of the "hot-wire ammeter" is shown in Fig. 293. A current in the fine wire BC produces a rise of temperature and an elongation of the wire. By means of a spring S attached to one end of a string wrapped about the axle P of the pointer, the elongation of the hot wire is magnified so as to produce a large indication on the scale.

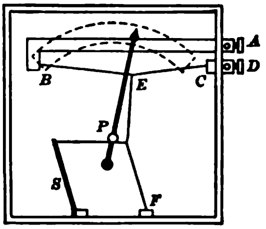

FIG. 293.

From (179) it is seen that the heat developed varies as the square of the current. That is, the heat is independent of the direction of the current. Consequently, the hot-wire ammeter can measure either direct or alternating currents.

QUESTIONS

1. A wire glows when a current is passed through it. Explain. If a part of the wire is cooled the remainder glows more brilliantly. Explain.

2. Show how by measuring the heat developed in a wire by a known cur-

rent, the mechanical equivalent of heat can be determined if we know the resistance of the wire.

3. If the resistances of a motor armature before and after operation be known, show how one can determine the change of temperature due to operation. Would this be a good way to determine the change in temperature? Why?

SOLVED PROBLEMS

PROBLEM.—No. 18 B. & S. gauge wire has a diameter of 0.04 inch. How many feet of copper wire of this size will there be in a 1000-ohm coil?

SOLUTION.—

$$R=\frac{Kl}{d^2}$$

$$1000=\frac{10.8l}{(40)^2}$$

$$l=\frac{1000(40)^2}{10.8}\text{ ft.}$$

PROBLEM.—The resistance of the line wire leading from a power plant to a given installation of lamps is 0.05 ohm. If the current in the line is 200 amperes, what is the loss of power in transmission?

SOLUTION.—From (178), power is

$$P=\frac{JH}{t}\text{ (watts)}=R\text{ (ohms) }I^2\text{ (amperes}^2\text{).}$$

Whence, remembering that 746 watts = 1 horse power (solved problem, p. 118), the loss of power in the present problem is

$$P=(0.05)(200)^2\text{ watts}$$

$$=\frac{(0.05)(200)^2}{746}\text{ H. P.}$$

301. Transformation of Energy in a Voltaic Cell.—A device in which heat of chemical reaction is transformed into electric energy is called a voltaic or galvanic cell. Consider a jar containing a solution of $CuSO_4$ and which has at the bottom a piece of copper, and at the top a piece of zinc, Fig. 294. Some of the $CuSO_4$ will dissociate into Cu and SO_4, and the copper will be deposited on the copper plate. Some of the zinc will go into solution and, by combining with SO_4, will form $ZnSO_4$. The dissociation of the $CuSO_4$ requires heat, and the formation of $ZnSO_4$ gives out heat. The evolution of heat produced by the formation of one gram equivalent of $ZnSO_4$ being greater than the absorption of heat accompanying the dissociation of a gram equivalent of $CuSO_4$ (Art. 225), the cell will rise in temperature.

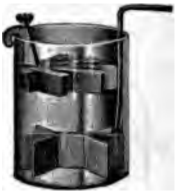

Fig. 294.

If, however, the two plates be connected by an electric conductor, there will be practically no temperature change and the circuit will be traversed by an electric current. We here have a transformation of heat into electric energy. As there is zero heat change in this cell when producing current, the electric energy developed equals the difference between the heat of formation of $ZnSO_4$ and the heat of dissociation of $CuSO_4$.

Some voltaic cells are warmer than the surroundings while supplying current. These cells do not transform into electric energy all of the thermal energy developed by the chemical reactions. Other cells when supplying current are lower in temperature than the surroundings. These cells absorb heat from the surroundings and transform it into electric energy.

Due to the transformation of heat into electric energy, positive charges are imparted to the copper plate, thereby raising its potential. While the two plates are joined by a conductor a current traverses the conductor from the copper plate to the zinc and traverses the cell from the zinc to the copper. Outside the cell, where there is no transformation into electric energy, the charge goes from high potential to low. Within the cell, where there is such a transformation of energy, the charge goes from low potential to high.

302. Transformation of Energy in a Thermoelectric Couple.—At ordinary temperatures the electric potentials of different substances are not the same. For example, the potential of iron is slightly higher than that of copper. The electric potential of a substance also depends upon the temperature. For example, the potential of hot copper is higher than that of cold copper, and the potential of cold iron is higher than that of hot iron.

Seebeck discovered in 1822 that with X, Fig. 295, kept at room temperature and Y at a somewhat higher temperature, there will be developed in the circuit a current of electricity that will flow through the heated junction from the copper to the iron.

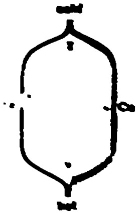
FIG. 295.

Since the current in the copper is from cold to hot, and in the iron it is from hot to cold, the direction of the current is from lower to higher potential in both the copper and the iron. Therefore there is a gain of electric energy as current passes through each branch of the circuit.

Again, since the potential of copper at ordinary temperatures is lower than that of iron, electric energy is also gained by the current at the heated junction, while electric energy is liberated at the cold junction. Whence, the net gain of electric energy in the thermoelectric circuit equals the sum of the electric energy gained in the two wires and at the heated junction, less that lost at the cold junction.

The Seebeck or thermoelectric effect is not limited to solids but is also found between two liquids and between a liquid and a solid.

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low. It is electromotive force that maintains a potential difference on a conductor. Electromotive force is independent of resistance and current, but the potential difference between two points of a closed circuit depends upon both.

It should be kept in mind that electromotive force is in no sense a force. It is energy per unit charge. The commonly used terms electric " pressure " and " tension " are also unsatisfactory. Any device which contains a seat of electromotive force is called an electric generator. It should be kept in mind, however, that electricity is in no sense " generated." It is simply set into motion. There are many cases of faulty terminology in electric nomenclature.

Since both electromotive force and potential difference are measured in volts, the term *voltage* is often used to express the magnitude of either.

304. Ohm's Law.—*First Form.* Consider a circuit of total resistance R on which is impressed a resultant electromotive force E. If a quantity of electricity q traverse the circuit, there must be supplied to the circuit an amount of electric energy (180)

$$W = Eq.$$

If this quantity q flows at a uniform rate for a time t, $q = It$. Whence, the energy supplied is

$$W = EIt.$$

During this same time, t, there is transformed into heat an amount of energy (178)

$$JH = I^2Rt.$$

If all the electric energy is turned into heat, we have therefore

$$EIt = I^2Rt.$$

Whence

$$E = IR, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

and

$$I = \frac{E}{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (182)$$

This equation shows that so long as the resistance of a closed circuit does not change, the current in the circuit is proportional to the electromotive force. This law was discovered experimentally by G. S. Ohm in 1827 and is called Ohm's Law.

Second Form.—In the above discussion we have considered a complete circuit which includes a seat of electromotive force. We shall now consider a portion of a circuit included between two points A and B, Fig. 296, between which there is no seat of electromotive force, but between which, due to some source of electromotive force outside the region considered, there is a constant current from A to B.

Fig. 296.

Due to the outside source of electromotive force, in time t a certain quantity of electricity q passes any point in the circuit. Since there is no seat of positive electromotive force between A and B, positive electricity must there move from a place of higher potential to a place of lower potential. The potential of A is therefore higher than that of B. Positive electricity moving away from A lowers the potential of A, and positive electricity arriving at B raises the potential of B. To maintain constant the potential difference between A and B, positive electricity must be transferred continually from B to A along some path not shown in the figure at the same rate at which electricity is flowing from A to B. To effect this transfer from low potential to high potential there must be an electromotive force. To carry unit positive charge from a point of potential V_B to a point of higher potential V_A requires an amount of work $V_A - V_B$ (Art. 279); and to carry q units, there is required q times as much work. Thus the amount of energy that must be converted into electric energy in time t in order to maintain constant the flow of electricity from A to B is

$$W[=(V_A - V_B)q] = (V_A - V_B)It,$$

or, representing the difference of potential $(V_A - V_B)$ by the more convenient symbol V_{AB},

$$W = V_{AB}It. \quad . \quad . \quad . \quad . \quad . \quad (183)$$

If the resistance from A to B is r, then in this part of the circuit there is transformed into heat during the time t an amount of electric energy (178),

$$JH = I^2rt.$$

If no electric energy is transformed into energy other than heat, the amount of electric energy imparted to AB must equal the heat energy developed in AB.
Whence,

$$V_{AB}It = I^2rt.$$

Consequently,

$$I = \frac{V_{AB}}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (184)$$

Thus when there is no seat of electromotive force between two points on a conductor, the strength of the current which flows along the conductor is obtained by dividing the potential difference of the given points by the resistance of the conductor between them. When there is no seat of electromotive force between two given points on a current-carrying conductor, the potential difference of these points equals the product of the current and the resistance between them. If the resistance per unit length of the conductor is uniform, the potential difference of any two points is directly proportional to the distance between them.

305. Relation between the Electromotive Force of a Generator and the Potential Difference at its Terminals.—The battery or other generator of electric energy is called the *internal* part of the circuit, whereas the conductor connecting its terminals is called the *external* part of the circuit. Represent the internal resistance of the circuit by r_i, and the external resistance by r_e. If the electromotive force of the generator is represented by E, there will be a current I throughout the circuit given by the equation, (182),

$$I = \frac{E}{r_i + r_e}.$$

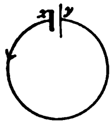

FIG. 297.

The potential difference at the terminals of the generator, $V_{xy} = Ir_e$, (184). Hence, the above equation may be put into the forms,

$$\left.\begin{aligned} E &= Ir_i + Ir_e, \\ E &= Ir_i + V_{xy}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

From these equations it is seen that if the electromotive force of a generator is constant, the potential difference between its terminals depends upon the relation between the internal and the external resistance. To bring out the relation we may write

$$V_{xy} = I r_e = \frac{E}{r_i + r_e} \cdot r_e = \frac{E}{\frac{r_i}{r_e} + 1}.$$

Thus if E remains the same, V_{xy} is small when r_e is small compared with r_i; and V_{xy} approaches E in magnitude when r_e increases. When r_e is infinite, $V_{xy} = E$. This condition exists when the circuit is *open*, that is, when there is no conducting connection from one terminal to the other. The ordinary method of determining the magnitude of the electromotive force of a generator is to open the circuit and then measure the potential difference between the terminals. In the case of a dynamo, the internal resistance is so small that the terminal potential difference V_{xy} is always very nearly equal to the electromotive force E. This is not true for a battery of voltaic cells.

Ohm's law shows that the current in a circuit is determined by the electromotive force and the resistance. If the resistance can be changed to the required value, the current in a circuit of any given electromotive force can have any desired value.

The ordinary telephone transmitter depends upon the change of current with change of resistance, and upon the change in the resistance of a column of pieces of carbon produced by a change of pressure. The transmitter T,

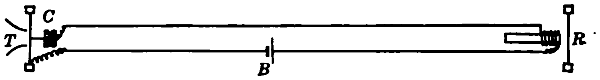

FIG. 298.

Fig. 298, consists of a diaphragm pressing against a capsule C filled with granules of carbon. An electric current flows through the capsule of carbon granules and a coil of wire about one end of a permanent magnet in the distant receiver R. When a man speaks into the transmitter, the sound waves cause the transmitter diaphragm to vibrate with the frequency of the sound and with amplitudes proportional to the loudness. The changes in the pressure on the carbon granules thereby produced, cause corresponding changes in the resistance, and consequently in the current flowing in the magnet coil. These current changes in the magnet coil, by altering the force of attraction on the iron diaphragm of the receiver, set the receiver diaphragm into vibrations similar to those of the transmitter diaphragm.

306. Magnitude of Energy and Power associated with Electric Currents.—From (180) and (171).

$$E=\frac{W}{q}=\frac{W}{It}.$$

Thus the electric energy developed in a circuit is given by

$$W=EIt. \qquad (186)$$

The power, or rate of development of energy is

$$P\left[=\frac{W}{t}\right]=EI. \qquad (187)$$

When E and I are expressed in absolute electromagnetic units, and t is expressed in seconds, W is expressed in ergs and P in ergs per second. When E is expressed in volts, I in amperes, and t in seconds, the unit of electric energy is the joule, and the unit of electric power is the watt.

The energy developed in one hour by an expenditure of energy at the rate of one watt is called the *watt-hour*. The watt-hour and the watt are the units of electric energy and power, respectively, ordinarily used in electrical engineering. One thousand watt-hours is called the *kilowatt-hour* or the *British Board of Trade Unit* of energy.

307. The Economy of High Electromotive Force in Transmission.—The rate at which energy is supplied to a circuit is measured by the product of the impressed electromotive force and the current (187). Hence a given power may be supplied either by a small or by a large current, so long as the product

(EI) is constant. But the rate with which electric energy is transformed into heat within a conductor is (178),

$$P=\left(=\frac{W}{t}=\frac{JH}{t}=\frac{I^2rt}{t}\right)=I^2r.$$

That is, for a given conductor, it varies as the square of the current. For this reason it is much more economical to supply electric power to a long transmission line by a small current and large electromotive force than by a large current and corresponding small electromotive force.

A maximum limit is set to the potential difference at the customer's end of the line by the danger of personal contact with high potential lines. A person in ordinary health can stand a current of 0.01 ampere for a short time without danger. The resistance of the body varies within wide limits depending upon the area of contact and the dryness of the surfaces of contact between the body and the conductor. Much the greater part of the total resistance is offered by the skin. The resistance of the human body from the dry finger tips of one hand to the dry finger tips of the other hand is of the order of magnitude of 50,000 ohms. In this case the maximum safe potential difference is

$$V_{ab}(=Ir)=0.01\times 50{,}000=500 \text{ volts.}$$

Street car motors are commonly operated at about this potential difference, but motors in shops and dwellings are usually operated at about either 110 or 220 volts. Incandescent lamps do not operate successfully at much above 120 volts.

In a succeeding chapter a method will be described by which an alternating current of any electromotive force may be economically transformed into another current of any desired electromotive force. By this device, power may be economically transmitted to a distant point by means of an alternating current of low current value and high electromotive force, and there transformed into another alternating current of practically equal power but of small electromotive force and corresponding high current value.

308. The Resultant Resistance of Several Conductors.—A number of conductors arranged so that the same current traverses all of them, one after another, as in Fig. 299, are said to be *in series.*

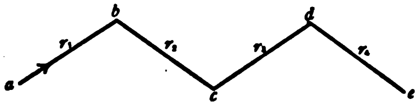

FIG. 299.

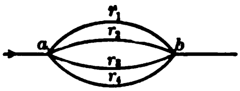

FIG. 300.

The resultant resistance R of a number of conductors of separate resistances r_1, r_2, r_3, r_4, arranged in series is

$$R = r_1 + r_2 + r_3 + r_4. \quad \ldots \ldots \quad (188)$$

A number of conductors so arranged that the current divides and a part traverses each branch, are said to be connected *in parallel* or in *multiple arc.* Let it be required to find the total resistance R between the points a and b, Fig. 300, due to four conductors of resistance r_1, r_2, r_3, and r_4, arranged in parallel.

Denoting the total current by I, and the currents in the branches by i_1, i_2, i_3, and i_4, respectively, we have,

$$I = i_1 + i_2 + i_3 + i_4.$$

Substituting for each current, its value as given in (184), we have

$$\frac{V_{ab}}{R} = \frac{V_{ab}}{r_1} + \frac{V_{ab}}{r_2} + \frac{V_{ab}}{r_3} + \frac{V_{ab}}{r_4},$$

or,

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}. \quad \ldots \ldots \quad (189)$$

309. The Division of Current in a Branched Circuit.—Let the current strength in the main line, in the branch r_1, and in the branch r_2 be denoted by I, i_1, and i_2, respectively. It is required to find the relations between I, i_1 and i_2.

From Ohm's Law, (184),

$$i_1=\frac{V_{ab}}{r_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (190)$$

and

$$i_2=\frac{V_{ab}}{r_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (191)$$

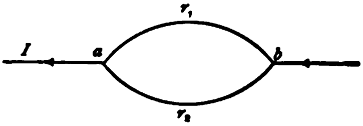

FIG. 301.

Whence,

$$\frac{i_1}{i_2}=\frac{r_2}{r_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (192)$$

That is, the ratio of the current strengths in the two branches equals the inverse ratio of the resistances of the two branches.

310. The Electromagnetic Unit of Capacity or Capacitance.— The ratio of the charge on a conductor to its potential is called the electric capacity or capacitance of the conductor (Art. 287). The electric capacity of a condenser is measured by the ratio of the charge given to one of the terminals to the potential difference between the terminals thereby produced. If charge be expressed in coulombs and potential difference in volts, the capacity will be expressed in a unit called the farad. The *farad* is numerically equal to the number of coulombs required to produce a potential difference of one volt. The capacity of the earth is about 0.000636 farad. As few conductors or condensers have capacities as large as one farad, the millionth part of a farad, called a *microfarad*, is taken as the practical electromagnetic unit of capacity.

QUESTIONS

1. When a copper wire and an iron wire of the same length and diameter are connected in parallel for the same length of time across the poles of a battery, the copper wire becomes hotter than the iron wire; but when they are joined in series across the poles of the same battery the iron wire becomes the hotter. Explain.

2. If the incandescent lamps in a building are all in parallel, and if each is to have always a potential difference of 100 volts between its terminals, how ought the potential difference between the terminals at the power plant to change when we turn on more lights? Why?

3. You are given two equal lengths of wire of the same thickness but of different materials. With a battery of known e.m.f. and negligible resistance

you are to make one of the wires as hot as possible. Which wire should you choose? Why?

SOLVED PROBLEMS

PROBLEM.—One liter of water at 10° C. is to be raised to the boiling point by means of heat developed by the passage of current through a 20-ohm coil attached to a 100-volt lighting circuit. If one-half of the heat developed is lost, find the time required to raise the temperature to the boiling point.

SOLUTION.—From (179) and (184)

$$4.2H = I^2Rt = \frac{V^2_{ab}}{R^2}Rt = \frac{V^2_{ab}t}{R}.$$

Whence,

$$4.2(1000\times 90)2 = \frac{100^2 t}{20},$$

or,

$$t = 1512 \text{ sec.}$$

PROBLEM.—A group of lamps is connected across a line operated by a dynamo of electromotive force 110 volts and internal resistance 0.03 ohm. The resistance of the line is 0.1 ohm, and the potential difference at the terminals of the group of lamps is 108 volts. Find the line current and the resistance of the group of lamps.

SOLUTION.—*First Method.* From (182) and (184)

$$I = \frac{E}{R} = \frac{V_{ab}}{r},$$

where r represents the resistance of the lamps,

or $$\frac{110}{0.03+0.1+r} = \frac{108}{r}. \quad \text{Whence } r = 7.0 \text{ ohms.}$$

Again,

$$I\left[=\frac{V_{ab}}{r}\right] = \frac{108}{7.0} = 15.4 \text{ amperes.}$$

Second Method.—From (184),

$$\text{Current} = \frac{\text{potential drop in dynamo and line}}{\text{resistance of dynamo and line}} = \frac{110-108}{0.03+0.1} = 15.4 \text{ amp.}$$

Again,

$$r\left[=\frac{V_{ab}}{I}\right] = \frac{108}{15.4} = 7.0 \text{ ohms.}$$

Problem.—A battery of 10 cells, each of constant electromotive force 1.1 volts and internal resistance 3 ohms, is joined, first, in series with a wire of 8 ohms, and then in multiple with the same wire. Find the current in each case.

Solution.—When joined in series, Fig. 302, the electromotive force of the battery equals the sum of the electromotive forces of the separate cells, and the resistance of the battery equals the sum of the resistances of the separate cells. Thus, when in series,

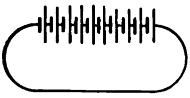

Fig. 302.

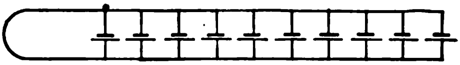

Fig. 303.

$$I_1\left[=\frac{E}{R}\right]=\frac{10(1.1)}{10(3)+8}=0.29 \text{ amp.}$$

When the cells are in multiple, Fig. 303, the electromotive force of the battery is that of a single cell, and the resistance of the battery, from (189), $r_b=3/10$. Whence, when in multiple,

$$I_2=\frac{1.1}{0.3+8}=0.13 \text{ amp.}$$

Problem.—The lighting circuit in a certain building has a resistance of two ohms. The building is to be connected to a power plant 500 ft. distant by a line such that the potential difference at the house terminals shall be 95 per cent of the potential difference at the dynamo terminals. Find the size of the line wire in circular mils.

Solution.—Representing the potential difference at the dynamo terminals by V_{ab} and that at the house terminals by V_{xy}, the resistance of the line by R and that of the lighting circuit by r, the line current by I, and the diameter and total length of the line by d and l, respectively, we have (184)

$$V_{xy}=Ir=I2.$$

$$V_{ab}=I(R+r)=I(R+2).$$

Dividing each member of the former equation by the corresponding member of the latter, and remembering that

$$V_{xy}/V_{ab}=0.95,$$

we have,

$$\frac{V_{xy}}{V_{ab}}=\frac{2I}{I(R+2)} \quad \text{or} \quad 0.95=\frac{2}{R+2}.$$

Whence, the line resistance is $R=0.105$ ohm.

From (176),

$$R=\frac{Kl}{d^2} \text{ where } K=10.8 \text{ for copper.}$$

Or

$$d^2\left[=\frac{Kl}{R}\right]=\frac{10.8\times1000}{0.105}=103{,}000 \text{ circular mils.}$$

From tables it will be found that this corresponds to B. & S. gauge No. 0.

Problem.—Two electric mains maintained at a potential difference of 110 volts are connected by four conductors in parallel having respectively resistances of 5, 8, 10 and 20 ohms. Find the total current and also the current in the 5-ohm conductor.

Solution.—Representing the total current by I, the total resistance by R, the resistance of the 5-ohm conductor by r, and the current in this conductor by i, we have, from (184).

$$I=\frac{V_{ab}}{R} \quad \text{and} \quad i=\frac{V_{ab}}{r}.$$

From (189)

$$\frac{1}{R}=\frac{1}{5}+\frac{1}{8}+\frac{1}{10}+\frac{1}{20}=\frac{19}{40}.$$

Hence,

$$I\left[=\frac{V_{ab}}{R}\right]=\frac{110\times19}{40}=52.25 \text{ amperes,}$$

and

$$i\left[=\frac{V_{ab}}{r}\right]=\frac{110}{5}=22 \text{ amperes.}$$

§ 2. *Electrolysis*

311. Ionization.—Experiments have shown that in the case of dilute solutions of a large class of compounds the osmotic pressure rises, the temperature of the boiling solution rises, and the freezing point of the solution falls, in direct proportion to the ratio of the number of molecules of solute to the number of molecules of solvent in the solution. In the case of dilute solutions of certain compounds it is found that the elevation of the osmotic pressure, the elevation of the temperature of the boiling solution, and the depression of the freezing point, is just double, triple, or quadruple that which occurs in other cases. This suggests that in solutions of this latter class of substances, each molecule of the solute is split into two, three, or four parts, respectively.

Now a molecule of any compound consists of atoms of two or more elements. And if such a molecule splits into two parts in such a manner that we have a separation of two dissimilar substances, we should expect one part to be charged positively and the other part negatively. If the solution contains positively and negatively charged parts of molecules, by placing in the solution a plate kept positively charged and another plate kept negatively charged, it would be possible to collect the negatively charged parts of the molecules about the positive plate and the positively charged parts of the molecules about the negative plate.

Experiments such as just indicated show that all solutions that give abnormal osmotic pressures, boiling points and freezing points contain charged parts of molecules, and that solutions which do not give abnormal osmotic pressure, etc., do not contain such charged parts of molecules. Consequently, we conclude that the molecules of some substances when dissolved split into parts that are electrically charged. The charged parts of molecules are called *ions*. The process of separation of a molecule into ions is called *ionization*. Positively charged ions are called *cations*, and negatively charged ions are called *anions*. The degree of ionization for any substance depends upon the dilution of the solution. In a concentrated solution there will be a greater number of molecules of solute ionized per unit volume of solution than there will be in a dilute solution. But in a dilute solution a greater fraction of all the molecules of the solute will be ionized than will be ionized in a concentrated solution.

A molecule is more apt to split into ions when dissolved in water than when dissolved in alcohol or ether because the force of electrostatic attraction between the charged parts of a molecule varies inversely with the magnitude of the dielectric constant of the surrounding medium. And since water has a much higher dielectric constant than ether or alcohol (Art. 277), therefore the force between the charged parts of a molecule is less when dissolved in water than when dissolved in ether or alcohol.

An ion may consist of one or many atoms. A monatomic ion must not be confused with an atom. On account of the charge possessed by the ion, the properties of an ion are very different than the properties of an atom of the same substance. For instance, a mass of potassium ions can exist in an aqueous solution whereas a mass of potassium atoms would combine with some of the water with explosive violence.

The ions of hydrogen and all of the metals are positively charged. The ions into which the molecules of some common compounds divide are indicated below. The + and − signs above the symbols indicate the relative amount of charge on the ions, the charge on a hydrogen ion being called unity.

Hydrochloric acid	$\overset{+}{H}$	$\overset{-}{Cl}$	Sulphuric acid	$\overset{+}{H}\overset{+}{H}$	$\overset{--}{(SO_4)}$
Nitric acid	$\overset{+}{H}$	$\overset{-}{(NO_3)}$	Copper sulphate	$\overset{++}{Cu}$	$\overset{--}{(SO_4)}$
Silver nitrate	$\overset{+}{Ag}$	$\overset{-}{(NO_3)}$	Zinc sulphate	$\overset{++}{Zn}$	$\overset{--}{(SO_4)}$
Water	$\overset{+}{H}$	$\overset{-}{(OH)}$	Auric chloride	$\overset{+++}{Au}$	$\overset{-}{Cl}\,\overset{-}{Cl}\,\overset{-}{Cl}$
Sodium hydrate	$\overset{+}{Na}$	$\overset{-}{(OH)}$	Platinic chloride	$\overset{++++}{Pt}$	$\overset{-}{Cl}\,\overset{-}{Cl}\,\overset{-}{Cl}\,\overset{-}{Cl}$

312. Electrolysis.—If two insoluble conducting plates be placed in a solution of copper sulphate, it is found that to keep one plate positively charged and the other negatively charged, charge must be continuously supplied to the plates. This means that electricity is passing from one plate to the other through the solution. It is also found that copper deposits on the negative plate, oxygen appears at the positive plate, and that sulphuric acid appears in the solution.

The two charged conducting plates are called *electrodes*. The positively charged electrode is called the *anode*, and the negatively charged electrode is called the *cathode*. Much of the copper sulphate ionizes into $\overset{++}{Cu}$ and $\overset{--}{(SO_4)}$, and a small amount of the water ionizes into $\overset{+}{H}$ and $(\overset{-}{OH})$. The positively charged ions (cations) move toward the negatively charged plate (cathode), and the negatively charged ions (anions) move toward the positively charged plate (anode). On reaching the cathode, the copper ions by combining with negative charge on the cathode will become uncharged. That is, the copper ions become copper atoms. A sulphion, $(\overset{--}{SO_4})$, on reaching the anode will give up its charge. The uncharged SO_4 is very unstable and will dissociate into SO_3+O. The oxygen will be deposited on the anode, while the SO_3 by combining with a molecule of H_2O will form a molecule of H_2SO_4.

The following reaction probably also occurs to a slight extent. During transit through the solution a sulphion, $(\overset{--}{SO_4})$, may encounter two hydrogen ions. The three ions will then combine and form an uncharged molecule of H_2SO_4. The remaining $(\overset{--}{OH})$ ions move toward the anode and there, by combining with positive charge, they become uncharged. But, after becoming uncharged, (OH) is an unstable substance that cannot exist in stable equilibrium. Two such aggregates will form water and free oxygen according to the relation

$$OH+OH=H_2O+O.$$

The process of separating different sorts of ions from one another by electric means is called electric analysis or *electrolysis*. A liquid containing ions is called an *electrolyte*.

313.—A conducting body can be coated or "plated" with metal by having it constitute the cathode of an electrolytic cell as described above. But in order that the solution may not become impoverished of the ions which are to be deposited, the anode is made of the same metal which is to be deposited.

For example, if a body is to be copper plated, it is suspended in a solution or "bath" containing copper ions, and kept negatively charged. A plate of copper is suspended in the same tank and kept positively charged. If a solution of copper sulphate be used, the action will be the same as described in the previous article with the addition that at the anode copper ions are forced into solution and when these ions come near any SO_4 ions molecules of $CuSO_4$ are formed. Due to this action the concentration of copper ions in the solution is maintained.

Iron pipes buried in moist ground are often rapidly destroyed by electrolytic corrosion due to electric currents. In ordinary electric street railway systems, the current goes to the cars over a wire, and after traversing the motors returns to the station through the rails. To ensure a good return circuit, the joints between the rails are joined by heavy copper bonds. If one of these bonds be broken, as at *B*, Fig. 304, current will leak to the earth and thence to the earthed connection at the power plant, or to some other point at a different potential than the rail at the broken bond. If there be an iron pipe extending from the break toward any point of the line at a different potential, the pipe will be traversed by the leakage current. Where the current leaves the pipe, as at *C* in the figure, there will be rapid corrosion.

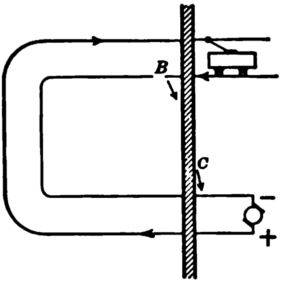

Fig. 304.

The ordinary method of producing aluminium is an electrolytic process. Al_2O_3 is melted with certain other minerals thereby forming an electrolytic solution. A very large current sent through the fused mass will cause aluminium to be separated at the cathode.

If a direct current be passed through acidulated water from one carbon or platinum plate to another, bubbles of oxygen will appear at the positive plate and hydrogen at the negative plate. Also, if the positive electrode be lead and the negative be aluminium, oxygen will appear as before on the anode and hydrogen on the cathode. But if the positive electrode be aluminium and the negative electrode be lead, the anode will almost instantly become covered with a film of aluminium oxide and the current will cease. Thus a current will traverse such a cell in one direction but not in the other. If an alternating electromotive force be impressed on the aluminium and lead plates, a current will flow in only one direction. The alternating current is thus "rectified." Again, the lead and oxide-covered aluminium plates in the liquid constitute a condenser which is charged when the aluminium plate is charged positively, and is discharged in the direction of the rectified current when the aluminium plate is charged negatively. Alternating electromotive forces of but a few volts arerectified by a cell containing acidulated water. But if the electrolyte consists of a saturated solution of phosphate of sodium, ammonium or potassium, alternating voltages up to 200 volts can be rectified. An electrolytic rectifier in which the electrolyte is a solution of a phosphate salt is often called a Nodon valve.

314. Electrolytic Separation of Metals.—In order that ions of any given metal may be deposited on the cathode of an electrolytic cell the potential difference between the anode and cathode must not be less than a certain minimum value, which is different for different metals. The minimum potential difference that will cause a given sort of ions to become discharged is called the *decomposition value* or *deposition value* of the given ions.

If two insoluble electrodes be placed in a solution containing several kinds of ions, all of the positively charged ions will tend to move toward the cathode. If the potential difference between the electrodes be less than the decomposition value of any of the ions, none of the ions will become deposited. If the potential difference between the electrodes be higher than the decomposition value of the ions of any metal in solution, ions of this metal will become discharged at the cathode and there will be deposited. If the decomposition values of all the other metallic ions are considerably higher than the potential difference between the electrodes, only this one metal will be deposited.

In this manner a metal can be separated from impurities. Practically all copper used for electric wires is electrolytically purified. A large mass of the impure copper forms the anode and a thin sheet of pure copper forms the cathode. The electrolyte is a solution of copper sulphate.

After all of one metal has been deposited, a second metal can be deposited by raising the potential difference between the electrodes to the decomposition value of the ions of another of the metals in the solution. And if no other ion in the solution has a decomposition value nearly equal to this one, this second metal will be deposited in a pure form at the cathode.

If, however, the potential difference between the electrodes exceeds the decomposition values of two metals, both metals will be deposited on the cathode simultaneously. In this way an object may be brass plated. In the case of brass plating, the concentration of zinc ions in the solution must be much greater than that of copper unless an electrode potential difference is used which is much greater than the decomposition value of zinc. An acid solution cannot be used with high potential differences, else hydrogen will deposit on the cathode, thereby causing the metallic deposit to be porous

315. The Quantitative Laws of Electrolysis.—Consider the mass of metal deposited when a given quantity of electricity passes through three solutions, one containing silver ions, another copper ions, and another gold ions. The number of ionic charges carried by ions of silver, copper, and gold are respectively one, two, and three. From the principles outlined in the previous Articles it is seen that a given quantity of electricity passing through various electrolytes either deposits or transfers to other combinations ions having the same total number of ionic charges. Consequently, in the present case, if the passage of unit charge causes a deposit of n ions of silver, it will cause a deposit of $n/2$ ions of copper and $n/3$ ions of gold. If now, the relative masses of these ions are 108, 63 and 197, respectively, it follows that the relative masses of the deposit of silver, copper, and gold produced by the passage through the different cells of the same quantity of electricity are respectively 108, 63/2, and 197/3. If instead of being unity, the charge passing through the solutions be q units, the masses of the various deposits will be q times as great as before.

These results may be tabulated as follows:

Ion	$\overset{+}{\mathrm{Ag}}$	$\overset{++}{\mathrm{Cu}}$	$\overset{+++}{\mathrm{Au}}$
Relative ionic mass	108	63	197
Relative ionic charge	1	2	3
Rel. No. of ions deposited by a given charge	1	1/2	1/3
Rel. mass of deposit by given charge	108	½(63)	⅓(197)

The relative mass of deposit produced by a given charge equals the quotient obtained by dividing the relative mass of an ion by the number of charges carried by the ion and is called the *chemical equivalent* of the substance. Or, in terms of the nomenclature of chemistry, the chemical equivalent of a substance is the ratio of its atomic mass to its valence.

It has now been shown that when a quantity of electricity, q, passes through a solution containing metallic ions of chemical equivalent y, there will be deposited a mass of metal

$$m \propto qy,$$

or

$$m = kqy. \qquad (193)$$

The facts included in this expression were originally enunciated by Faraday in the form of the two following laws:

1. The mass of an electrolyte decomposed by the passage of an electric current is directly proportional to the quantity of electricity that passes through it.

2. If the same quantity of electricity passes through different electrolytes, the masses of the different ions liberated at the electrodes are proportional to their chemical equivalents.

The ratio of the mass of substance deposited (or transferred to other combinations), to the total quantity of electricity carried by the ions of the substance, is called the *electrochemical equivalent* of the substance. Thus, representing the electrochemical equivalent by z,

$$z=\frac{m}{q}. \qquad (194)$$

For example, one coulomb (i.e., one ampere for one second) will deposit 1/96530 g. of hydrogen. Whence, the electrochemical equivalent of hydrogen is 1/96530 [=0.00001036] g. per coulomb.

If the electrochemical equivalent of a substance is z, the mass m deposited in t seconds by a current of I amperes is

$$m[=zq]=zIt. \qquad (195)$$

Knowing the chemical equivalents of two elements and the electrochemical equivalent of one of them, the electrochemical equivalent of the other can be found as follows: If m grams of substance of chemical equivalent y are deposited by a charge q, and m_1 grams of a substance of chemical equivalent y_1 are deposited by a charge q_1, then from (193)

$$\frac{m}{m_1}=\frac{qy}{q_1y_1}.$$

$$\frac{m}{q}=\frac{m_1y}{q_1y_1},$$

or,

$$z=z_1\frac{y}{y_1}. \qquad (196)$$

For example, the electrochemical equivalent of any substance of chemical equivalent y is

$$z=\frac{1}{96530}\left(\frac{y}{1}\right)=\frac{y}{96530}\text{ g. per coulomb. . . .} \quad (197)$$

The number of grams of any sort of ion which carries the same charge as one gram of hydrogen is called the *gram equivalent* of that ion. The charge carried by one gram equivalent of any sort of ion equals 96,530 coulombs. Therefore 96,530 coulombs will deposit a gram equivalent of any element.

Though in Physics, the electrochemical equivalent of a substance is defined as the ratio of the mass of substance deposited to the total charge carried by the ions of the substance, in Chemistry it is quite common to define it as the reciprocal of this quantity.

316. Computation of the Electromotive Force of a Cell.—For a cell in which the chemical reactions are known the electromotive force can be computed. For the general case in which there is a temperature change of the cell while supplying current, the computation involves a consideration of the properties of a reversible thermodynamic transformation. But for the special case in which there is no temperature change the computation is simple.

For instance, in a gravity cell (Art. 301) there is practically no temperature change. Hence, for every gram molecule of $ZnSO_4$ formed and of $CuSO_4$ dissociated, there are transformed into electric energy 50,130 calories (Art. 225). For one gram equivalent of zinc and of copper the electric energy produced would be $W=\frac{1}{2}(50{,}130)$ calories. Now the charge carried by one gram equivalent of any sort of ion equals 96,530 coulombs. Hence, the electromotive force

$$E\left[=\frac{W}{q}\right]=\frac{(4.2)(25065)}{96530}=1.09\text{ volts.}$$

317. The Accumulator or Secondary Cell.—The two poles of an electric cell must be different. In most cells the poles are plates of different metals. But two plates of the same material may be made different by electrolysis and thereafter function as an ordinary cell. For example, if electric current be passed through a cell consisting of two plates of lead sulphate immersed in a dilute aqueous solution of sulphuric acid, the anode will be changed into lead peroxide and the cathode into metallic lead. If the outside source of electric energy be removed and the two plates be joined, a current will be set up in the opposite direction. That is, in the outside circuit the current will be from the peroxide plate to the metallic lead plate. During the flow of this current, each plate will gradually be transformed back into lead sulphate. When both plates become alike the current will cease.

An electric cell in which the difference of the two plates has been produced by the action of a current is called an *electric accumulator*, *secondary cell*, or *storage cell*. Before an accumulator can produce a current it must be "charged" by means of an outside source of electromotive force. An accumulator can be repeatedly charged and discharged. A cell, such as heretofore considered, which does not need to be charged is often called a "primary" cell.

When a storage battery is being charged, the charging electromotive force must be greater than the counter electromotive force of the battery.

318. Electrolytic Conduction.—When oppositely charged electrodes are placed in a solution containing ions, there are set up two processions of ions moving through the solution in opposite directions toward the electrodes. With the transfer of matter there is also a transfer of electricity. This transfer of electricity constitutes the electric current through the liquid. Except in the case of liquid metals, all electric conduction through liquids is due to ions, and liquids without ions are non-conducting. The degree of conductivity of a solution depends upon the number and speed of its ions. The speed of an ion depends largely upon the number of molecules of the solvent that adhere to the ion and so impede its movement.

A solution without ions is nonconducting. A solution of sugar in water is non-conducting. A solution of sodium chloride in water is conducting. The degree of ionization of a solute in solution is measured by the conductivity of the solution.

SOLVED PROBLEMS

PROBLEM.—Find the mass of copper (atomic mass 63.2, valence 2), that will be deposited in one hour by a current of 5 amperes traversing a solution which contains copper ions.

SOLUTION.—From (197) the electrochemical equivalent of copper is

$$z=\frac{\frac{1}{2}(63.2)}{(96530)}=0.00032738 \text{ g. per coulomb.}$$

From (195) the mass deposited in one hour by a current of 5 amperes is

$$m(=zIt)=0.00032738(5)(3600)=5.89 \text{ g.}$$

PROBLEM.—In order that a deposit of nickel may not be granular the current must not exceed 0.003 amp. per sq. cm. surface of the article being electroplated. Using a current of this density, find how long a time is required to deposit on an object having a surface of one square meter a coating of nickel 0.006 cm. thick. The density of nickel is 8.9 g. per c.c., the atomic mass is 58.6, and the valence is 2.

SOLUTION.—From (195),

$$t=\frac{m}{zI}.$$

The mass of deposit,

$$m[=dV]=8.9(\overline{100}^2\times 0.006)=534 \text{ g.}$$

The electrochemical equivalent of nickel, (197),

$$z\left[=\frac{y}{96530}\right]=\frac{\frac{1}{2}(58.6)}{96530}=0.0003 \text{ g. per coulomb.}$$

The allowable current,

$$I=0.003(100)^2=30 \text{ amp.}$$

Hence, the time of deposit,

$$t\left[=\frac{m}{zI}\right]=\frac{534}{0.0003\times 30}=58700 \text{ sec.}$$

§ 3. *Instruments for Measuring Current, Potential Difference and Resistance*

319. Current Measuring Instruments.—Currents can be measured by (*a*) the amount of metal deposited from a solution in a given time (Art. 315); (*b*) the heat developed in the current-carrying conductor (Art. 300); (*c*) the force acting on the current-carrying conductor when situated in a magnetic field (Art. 295). The last is the most common method. It depends upon the fact that when a current-carrying conductor is in a magnetic field, the conductor and the field tend to move relative to one another. We have instruments having (1) a movable magnet near a fixed current-carrying conductor; (2) a movable current-carrying conductor near a fixed magnet; (3) a movable current-carrying conductor near a fixed current-carrying conductor.

Examples of the various types will now be considered.

320. The Tangent Galvanometer.—A galvanometer is an instrument for measuring electric current. In Art. 295 it was shown that a current-carrying conductor in a magnetic field, and perpendicular to it, is acted upon by a force, and that the magnitude of the current can be expressed in terms of this force, the intensity of the magnetic field, and the length of the conductor situated in it. The tangent galvanometer is an instrument by which electric current can be determined from the force between a current-carrying conductor and a compass needle suspended in the earth's magnetic field. It consists, Fig. 305, of a circular coil of wire of large radius at the center of which is a delicately suspended compass needle of short length.

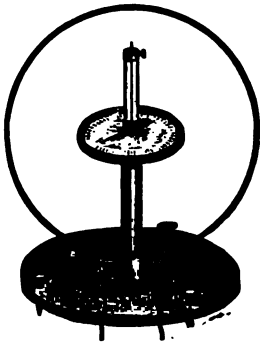

Fig. 305.

In using the instrument the plane of the coil is placed in the earth's magnetic meridian. Due to the earth's magnetic field, the needle tends to set itself in the plane of the coil. Due to the magnetic field of the current in the coil, the needle tends to set itself perpendicular to the plane of the coil. The resultant force acting on each pole of the needle will be inclined to the magnetic meridian of the earth at some angle Θ. Consequently, the needle will turn till its magnetic axis is in the line of action of this resultant force. That is, the needle will be deflected from the earth's meridian through an angle Θ. The relation between the current and this deflection will now be determined.

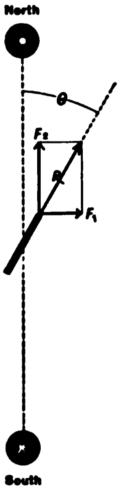

FIG. 306.

Fig. 306 represents a horizontal section through the center of the coil. The force acting on each pole of the needle due to the current in the coil is represented by the symbol F_1 and the force due to the earth's magnetic field by the symbol F_2. When the needle is in equilibrium under the action of these forces

$$\tan \Theta = \frac{F_1}{F_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (198)$$

The values of F_1 and F_2 will now be found and substituted in this equation.

The force acting upon the needle due to the current-carrying conductor equals the force acting upon the current-carrying conductor due to the needle. Consequently, (174),

$$F_1 = lH'I,$$

where l represents the total length of the circular conductor, H' represents the intensity of the field at the conductor due to the pole m, and I represents the magnitude of the current.

If the coil consist of n turns of radius r, then

$$l = 2\pi rn.$$

If the pole strength of the needle be m, then from (Art. 265), since the pole is in air,

$$H' = \frac{m}{r^2}.$$

Therefore, when a current I traverses the coil, there is a force acting on the magnet pole situated at the center of the coil, and a reacting force acting on the coil, of the magnitude

$$F_1(=lH'I)=\frac{2\pi nmI}{r}. \quad . \quad . \quad . \quad . \quad . \quad (199)$$

If the horizontal component of the earth's field be denoted by H, then there is a force acting on each pole of the needle in the north and south direction (163),

$$F_2=mH. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

Consequently (198) becomes,

$$\tan \Theta\left[=\frac{F_1}{F_2}\right]=\frac{2\pi nI}{rH}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (201)$$

Whence,

$$I=\left(\frac{rH}{2\pi n}\right)\tan \Theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

expressed in C. G. S. electromagnetic units of current. If current is to be expressed in amperes, the above value must be multiplied by 10.

In deriving this equation it has been assumed that the magnetic field is uniform throughout the region in which the needle is situated. This requirement is fulfilled by making the diameter of the coil large compared with the length of the needle.

321. The Sensitive Suspended Needle Galvanometer.—An inspection of (201) shows that when a magnet needle is at the center of a coil of current-carrying wire, the deflection produced by a given current will be increased by

(1) increasing the number of turns of wire in the coil;

(2) diminishing the radius of the coil;

(3) diminishing the effect of the earth's magnetic field.

The third condition may be met by reducing the magnetic field at the place where the needle is situated by means of a weak compensating magnet. Another method consists in using a pair of needles, each magnetized so highly that it will be acted upon strongly by the magnetic field of a neighboring current, and so fastened together that the system will be but slightly acted upon by the magnetic field of the earth. A consideration of Fig. 307 will make clear this method. The two needles are mounted in the same vertical plane on a very thin rigid rod. If the two needles are alike and point in exactly opposite directions, the resultant couple due to the earth will be zero. But if the poles of one needle are stronger than those of the other, the resultant couple due to the earth will equal the difference between the separate couples acting on the two needles. By making the two needles nearly alike, the resultant controlling couple may be made as small as desired. Two nearly equal needles mounted in this way constitute an "astatic pair." The magnet system is suspended by a minutely thin fiber of quartz or silk of very small rigidity.

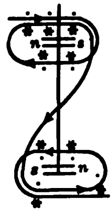

Fig. 307.

Each needle is at the center of a coil of wire of many turns and small radius. The two coils are so connected that the current traverses one coil in the clockwise direction and the other in the opposite direction. Consequently, the deflecting couples on the suspended magnets due to the current-carrying coils will be in the same direction, while the opposing couples due to the earth's magnetic field will almost neutralize one another.

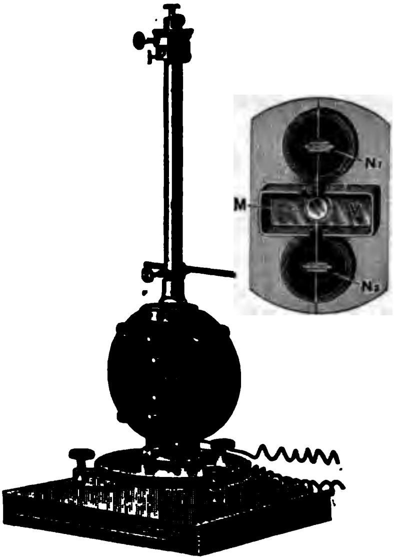

Fig. 308.

The deflection of the suspended system is measured by observing the image of a horizontal scale reflected from a small mirror mounted on the same rod that supports the magnetic needles. A sensitive astatic galvanometer is represented in Fig. 308. To the right of the instrument is shown the suspended system consisting of the two needles, N_1 and N_2, the mirror M, and the vane V. The swinging of this vane in a box but little larger than itself damps out vibrations and brings the suspended system quickly to rest. Behind the needles are shown two halves of the coils.

As the conditions underlying (202) are not fulfilled in the sensitive moving-needle galvanometer, this equation cannot be applied to this instrument.

322. The Suspended Coil or d'Arsonval Galvanometer.—Due to the fact that the magnitude of the deflection of a suspended needle galvanometer depends upon the direction and intensity of the earth's magnetic field, this type of galvanometer can be relied on only when precautions are taken to maintain the constancy of the earth's magnetic field. The indications of a sensitive suspended needle galvanometer would be considerably modified by the change in the earth's magnetic field produced by the motion in the neighborhood of the instrument of a person with a bunch of iron keys in his pocket, the motion of a dray near the building, or the motion of an electric car at a distance of a couple of blocks.

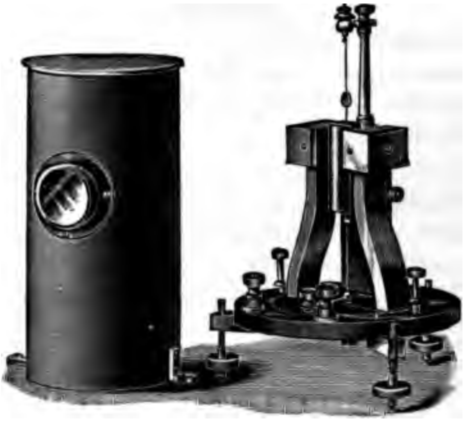

Fig. 309.

The suspended coil galvanometer is free from this serious disadvantage. This instrument, Fig. 309, consists of a coil of fine wire suspended in a strong magnetic field due to a permanent magnet. The coil is supported above and below by a very fine phosphor bronze or steel wire that serves to connect the coil with the source of the current that is to be measured.

The action of the magnetic field due to the permanent magnet and the current-carrying coil can be studied in Figs. 310, 311, and 312, which represent a single rectangular loop of wire capable of rotation about an axis normal to the plane of the paper. The loop is within a gap between the poles of a permanent magnet. Imagine an electric current flowing in a loop of wire as indicated in the following diagrams.

With the current-carrying conductor in the position shown in Fig. 310, the resultant magnetic field above *A* is stronger than that below *A*; whereas the resultant magnetic field above *B* is weaker than that below *B*. Consequently, *A* will be urged downward and *B* upward. Therefore, the loop is acted upon by a torque in the clockwise direction. When in the position shown in Fig. 311 the torque acting on the loop has a maximum magnitude and is in the same direction as before. When in the position shown in Fig. 312 the torque is zero. If, by means of some outside agent, the loop be turned past the position shown in Fig. 312, the loop will be acted upon by a torque in the counter-clockwise direction which will urge the loop back into the position shown in Fig. 312. It is thus seen that with the current flowing from *A* toward *B*, there will be developed a torque that will tend to rotate the loop till its plane is perpendicular to the direction of the external magnetic field. This torque is opposed by another torque developed in either a straight suspending wire, Fig. 309, or a flat spiral spring.

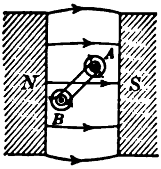

Fig. 310.

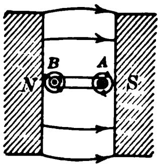

Fig. 311.

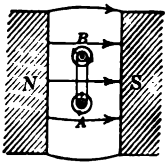

Fig. 312.

If the intensity of the magnetic field in which the coil is situated is uniform and the lines of force parallel, as represented in the above diagrams, the lever arm of the force acting upon the current-carrying conductor will vary from a maximum when the plane of the coil is parallel to the magnetic field, Fig. 311, to zero when the plane of the coil is perpendicular to the field, Fig. 312. Under these conditions the deflection will not be proportional to the current in the coil. If the lever arm of the force and also the intensity of the magnetic field were constant, then the deflection would be proportional to the current in the suspended coil. This result can be attained by having the suspended coil in an annular air gap between a stationary cylindrical soft iron core and coaxial pole pieces, as in Fig. 313. With this arrangement the magnetic field in which the coil is situated is radial and of constant strength. Hence the lever arm and force are constant. One type of portable galvanometer using this device is shown in Fig. 314.

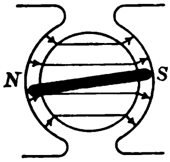

FIG. 313.

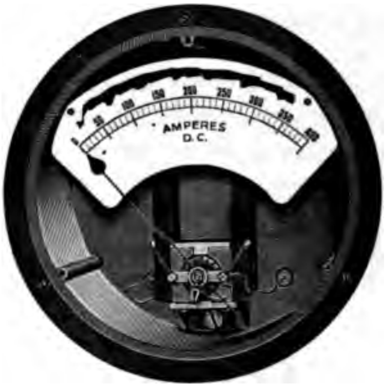

FIG. 314.

The magnetic field of force within which the coil is suspended is so intense compared with the earth's magnetic field that it would be inappreciably affected by any alteration of the earth's field. This instrument can be used where the earth's field is altered by masses of moving iron, moving electric machinery, or variable electric currents. Though not so sensitive as the most sensitive suspended needle galvanometers, the suspended coil galvanometer is much more generally useful.

323. The Electrodynamometer.—Fig. 315 represents two coils of wire with the planes of the coils vertical and inclined to one another. The large coil is stationary. The smaller coil is suspended within the larger coil by means of two fine wires of such rigidity that it can easily turn.

If a current traverses the coils in the directions indicated, the inner coil will tend to turn till it is in the plane of the larger coil and the currents in the adjacent conductors are in the same direction. The torque which tends to produce rotation depends upon the current.

If the direction of the current be reversed, there will be a torque which will tend to turn the inner coil in the same direction as before. If the direction of the current be reversed many times per second, there will be a torque acting on the inner coil as before.

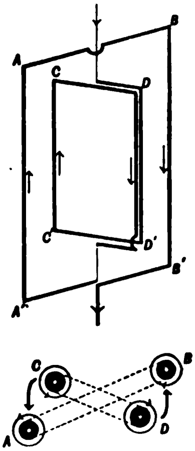

FIG. 315.

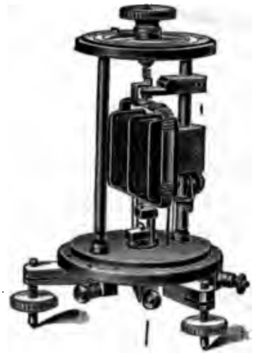

FIG. 316.

An instrument making use of this action for the measurement of a current is called an *electrodynamometer*. As it contains no iron it is not sensitive to outside magnetic disturbances. The electrodynamometer can be used for the measurement of direct or alternating currents, but it is more often used for alternating currents.

If an alternating current produces the same torque as an unvarying direct current I, the alternating current is said to be of the magnitude I.

An instrument of this type is shown in Fig. 316. The instrument here shown consists of an inner stationary coil of heavy wire and outer coil of fine wire capable of rotation about a vertical axis. The current traversing the two coils develops a torque which is opposed by a spiral spring. The current is indicated either by the amount of rotation of the movable coil, or by the amount the upper end of the control spring must be turned to keep the movable coil in the zero position.

324. The Weston Electromagnetic Galvanometer.—In Fig. 317, the current traverses a coil C of few turns of heavy wire. Concentric with the coil are two strips of soft iron,—the outer one, A, stationary, the inner one D, movable about an axis perpendicular to the plane of the diagram. A current traversing the coil will develop magnetic poles of the same sort on the edges of the iron strips which face the reader. A pole of the opposite sort will be developed on the edges away from the reader.

Due to the repulsion of similar poles, the movable strip of iron will rotate clockwise. The direction of rotation will be the same if the direction of the current be reversed. By attaching a pointer to the movable strip of iron and opposing the rotation by a spring, deflections will be obtained which depend upon the currents employed.

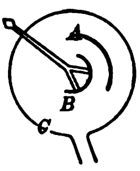

FIG. 317.

325. The Thompson Inclined Coil Galvanometer.—A third device which will measure either alternating or direct currents is represented in Fig 318. This device consists of either one or two strips of soft iron a and b attached to an axle inclined to the axis of the current-carrying coil CC. When a current traverses the coil there is developed within the coil a magnetic field in the direction of the axis of the coil. The two soft iron strips tend to set themselves into the direction of the field, that is, axial to the coil as represented in Fig. 318. The direction of rotation from the zero position will be the same whichever be the direction of the current.

Rotation is opposed by a spiral spring not shown in the diagram. The torque acting upon the movable system is indicated by a pointer, P, which moves in a plane normal to the plane of the diagram.

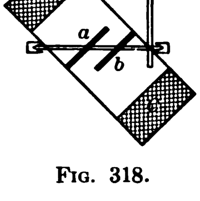

FIG. 318.

326. Galvanometer used Ballistically.—The quantity q of an electric discharge can be measured by means of a galvanometer having a suspended system which will not move appreciably from the zero position until the entire quantity has traversed the coil. The required slowness in getting started is effected by making large the moment of inertia of the moving system. A galvanometer having a moving system of such great moment of inertia that it can be used for the measurement of the quantity of an electric discharge is called a *ballistic galvanometer.* Any galvanometer of sufficient sensitivity can be arranged so as to be used ballistically.

327. The Ammeter and the Voltmeter.—The scale of any galvanometer may be divided so as to indicate directly in amperes the current passing through it. The resistance of the instrument may be such that on introducing the galvanometer into the circuit, the change thereby produced in the current will be negligible. A galvanometer provided with a scale divided so as to indicate amperes directly, is called an *ammeter.*

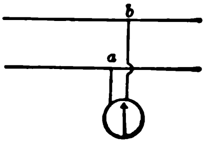

FIG. 319.

If a galvanometer be joined to two points a and b, Fig. 319, between which a potential difference exists, a current will traverse the instrument. Representing the potential difference between a and b by V_{ab}, and the galvanometer resistance by r_g, the current through the galvanometer will be, (184),

$$i_g = \frac{V_{ab}}{r_g}.$$

This current will produce a certain galvanometer deflection. If the galvanometer current be expressed in amperes, and the galvanometer resistance in ohms, then the potential difference between a and b, expressed in volts, will be

$$V_{ab} = i_g r_g.$$

For example, if the current through the galvanometer were 0.005 ampere, and the galvanometer resistance were 100 ohms, then the potential difference at the galvanometer terminals would be

$$V_{ab} = (0.005)100 = 0.5 \text{ volt.}$$

If each scale division of the galvanometer be marked, not in the amperes necessary to produce the given deflection, but the product of that current and the galvanometer resistance, then the divisions of the scale will indicate the potential differences at the galvanometer terminals which will produce the various deflections. Thus the place where the pointer came to rest when used as above would not be marked 0.005 ampere, but would be marked 0.5 volt. A galvanometer having a scale divided so as to indicate volts directly, is called a *voltmeter.*

A voltmeter indicates the potential difference at its terminals. If it be required that the potential difference between two points shall not be appreciably altered by being joined to the terminals of a voltmeter, the resistance of the instrument must be large.

328. The Shunted Ammeter.—In order that the torque acting on the moving system of a galvanometer may be fairly large, either the coil must contain many turns or the current must be large. In order that the moving system of a moving coil galvanometer may not be too heavy, the wire must be of small diameter. A coil of many turns of fine wire will have a rather high resistance. The usual resistance of ammeter and voltmeter coils is 100 ohms. Only small currents can be passed safely through coils of fine wire on account of the heat developed in the coil. Again, the resistance of such an instrument would appreciably alter the current which it is desired to measure. We thus see that the resistance of the coil of a moving coil galvanometer must be large, while the resistance of the instrument must be small.

These conditions are met by connecting to the terminals of the coil a by-pass or " shunt " through which the greater part of the current will pass. The current traversing the shunted galvanometer coil is so small that the coil is not overheated. By adding a proper shunt to the galvanometer, the resistance of the instrument can be made so small with respect to that of the remainder of the circuit that the introduction of the instrument will cause no appreciable change in the current.

Sometimes galvanometers are provided with several shunts of such resistances that without any shunt, a deflection across the whole scale will be produced by a current of 0.1 ampere; with the first shunt the same current will traverse the coil and produce the same deflection when the line current is one ampere; with the second shunt, the same current will traverse the coil when the line current is 10 amperes; and so on. In other words, without a shunt, the deflection indicates the current in the line. With the first shunt, the line current is 10 times the scale reading, and so on. The number by which the current through a galvanometer must be multiplied in order to obtain the value of the current in the main line is called the *multiplying power* of the shunt.

SOLVED PROBLEMS

PROBLEM.—A galvanometer is to be provided with a shunt so as to be used as an ammeter. Find the resistance of the shunt, compared with that of the galvanometer, so that the multiplying power of the shunt shall be 100.

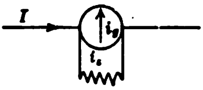

FIG. 320.

SOLUTION.—Since the fall of potential V_g through the galvanometer equals the fall of potential V_s through the shunt, $V_g = V_s$. Representing the current in the line by I, that in the galvanometer by i_g, and that in the shunt by i_s, we may write, (192),

$$i_g r_g = i_s r_s \quad \text{or} \quad \frac{r_s}{r_g} = \frac{i_g}{i_s}.$$

We shall now find a value for the right-hand member of this equation in terms of the quantities given in the problem.

$$I = 100 i_g,$$

and

$$I = i_g + i_s.$$

Hence

$$100 i_g = i_g + i_s,$$

or

$$\frac{i_g}{i_s} = \frac{1}{99}.$$

Substituting this value in the above equation, we obtain,

$$\frac{r_s}{r_g}\left[= \frac{i_g}{i_s}\right] = \frac{1}{99}.$$

Consequently, in order that the multiplying power may be 100, the resistance of the shunt must have the value

$$r_s = \frac{r_g}{99}.$$

Problem.—Find the resistance which an ammeter must have in order that in introducing the instrument into a circuit of resistance 20 ohms, the current shall not decrease more than one-tenth of one per cent.

Solution.—The current I in the circuit before the introduction of the instrument is

$$I = \frac{E}{20}.$$

The current I' after the introduction of an instrument of resistance r_g, is

$$I' = \frac{E}{20 + r_g}.$$

Dividing each member of the former equation by the corresponding member of the latter, and remembering that $I' \geqq 0.999I$,

$$\frac{1}{0.999} \geqq \frac{20 + r_g}{20}.$$

Whence,

$$r_g \geqq 0.02 \text{ ohm}.$$

329. The Voltmeter Multiplier.—Suppose that 0.01 ampere flowing through a 100-ohm galvanometer produces a deflection across the entire scale. Then, the maximum potential difference that can be measured will be, (184),

$$V_{ab}[= i_g r_g] = (0.01)100 = 1 \text{ volt}.$$

The same deflection would be produced on connecting the instrument to two points having a potential difference of 10 volts if there were put in series with the galvanometer a resistance r_m of a value given by the equation (184)

$$(0.01)\ (100 + r_m) = 10 \text{ volts}.$$

That is, the range of the instrument would be increased 10 times by putting in series with the galvanometer coil a resistance $r_m = 900$ ohms.

In general, if a current i_g will cause a given galvanometer of resistance r_g to give a full scale deflection, the galvanometer can be used to measure potential differences up to a value V'_{ab} by placing in series with the galvanometer coil a resistance r_m having the magnitude given by the relation, (184),

$$V'_{ab} = i_g(r_g + r_m). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (203)$$

The device of resistance r_m placed in series with a galvanometer coil for the purpose of increasing the range of the potential differences that can be measured is called a *voltmeter multiplier*.

In case one does not know the maximum current, i_g, but does know the maximum voltmeter reading without the multiplier, V, the above equation assumes the form

$$V'_{ab} = \frac{V}{r_g}(r_g + r_m). \quad . \quad . \quad . \quad . \quad . \quad . \qquad (204)$$

SOLVED PROBLEMS

PROBLEM.—A voltmeter has a coil of resistance r_g and a multiplier of resistance r_m. Find the value of the resistance R which must be added to the instrument to increase the range of the scale n fold.

SOLUTION.—A given deflection is due to the same current in the galvanometer coil whatever the potential difference at the terminals of the instrument. If a potential difference n fold as great as another is to produce the same current in the galvanometer coil, the total resistance of the instrument must be increased n fold. Therefore, there must be *added* to the instrument a resistance $(n-1)$ times as great as the original resistance. Thus the required added resistance R has the value

$$R = (n-1)(r_m + r_g).$$

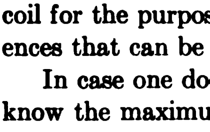

FIG. 321.

PROBLEM.—A galvanometer is to be provided with a series resistance so as to be used as a voltmeter. Find the resistance r_m of this multiplier, compared with the resistance of the galvanometer, so that the multiplying power shall be 100.

SOLUTION.—From the condition of the problem,

$$V_{ab} = 100V_{xy}.$$

or, (203),

$$i_g(r_g + r_m) = 100 i_g r_g.$$

Whence,

$$r_m = 99 r_g.$$

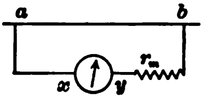

FIG. 322.

PROBLEM.—The potential difference is to be measured at the ends of a wire of 10 ohms forming part of a circuit of constant electromotive force and total resistance 200 ohms. Find the resistance which a voltmeter must have in order that on introducing the instrument, the potential difference between the given points shall not be diminished more than one-tenth of 1 per cent.

SOLUTION.—Representing the required resistance of the voltmeter by r, the potential difference between the points a and b before and after the introduction of the voltmeter by V_{ab} and V'_{ab}, respectively, and the current in the main line in the two cases by I and I', respectively, we have, (184) and (189),

$$V_{ab}=I10,$$

$$V'_{ab}=I'\left(\frac{10r}{10+r}\right).$$

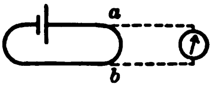

FIG. 323.

Dividing each member of the former equation by the corresponding member of the latter and remembering that from the conditions of the problem $V'_{ab}\geqq 0.999V_{ab}$,

$$\frac{1}{0.999}\geqq\frac{I}{I'}\,\frac{10+r}{r}. \qquad \text{(205)}$$

A value for the ratio I/I' will now be obtained and substituted in this equation. From (182)

$$E=I200,$$

and

$$E=I'\left(190+\frac{10r}{10+r}\right)=I'\left(\frac{1900+200r}{10+r}\right).$$

Dividing each member of the former equation by the corresponding member of the latter

$$\frac{I}{I'}=\frac{19+2r}{2(10+r)}.$$

Substituting this value of I/I' in (205),

$$\frac{1}{0.999}\geqq\frac{19+2r}{2(10+r)}\,\frac{10+r}{r}\geqq\frac{19+2r}{2r}.$$

Whence,

$$r\geqq 9490 \text{ ohms}.$$

330. The Wattmeter.—The power, expressed in watts, supplied to any part of a circuit equals the product of the current, expressed in amperes, and the potential difference, in volts, between the two points considered (187). The current, and the potential difference can be measured by an ammeter and voltmeter, respectively. A single instrument, however, can be constructed which will give a deflection depending upon the product of the current and the potential difference. An instrument which indicates directly the power, expressed in watts, is called a *wattmeter*.

The torque acting on the moving coil of an electrodynamometer, Figs. 315 and 316, depends upon the current in the fixed coil and also the current in the movable coil. If the instrument be so connected to a circuit that one coil is traversed by the entire line current, and the other coil is traversed by a current which depends upon the potential difference at the points considered, then the torque, and consequently the deflection, will depend upon the watts expended between the points considered.

Fig. 324 represents the stationary or current coil *BB* of a wattmeter of the electrodynamometer type connected in series with a circuit, and the movable or potential coil *A* connected across the points *a* and *b* of the circuit under examination. In order that the deflection of the movable coil may be proportional to the line current, the current traversing the potential coil is passed through a compensating coil wound over the current coil in such a direction that the magnetic field set up by the current in the compensating coil just neutralizes that produced by the part of the current in the current coil which is due to the current taken from the line at *ab*.

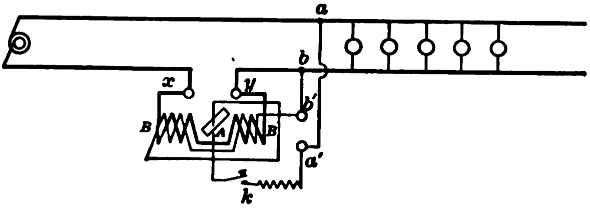

FIG. 324.

331. The Wheatstone Bridge.—This is the device most commonly used for the comparison of electric resistances. Consider two conductors *ABC* and *ADC*, Fig. 325, joined in parallel to the terminals of a battery. Corresponding to any point *B* on the conductor *ABC*, there is a point *D* on the conductor *ADC* which is at the same potential. If two such points be joined by a conductor BD, no current will flow along this bridge wire. Represent the potentials at the points A, B, C and D, by the symbols V_A, V_B, V_C and V_D, respectively. Let the resistances of the arms AB, BC, AD and DC be denoted by r_1, r_2, r_3 and r_4, respectively. Let the current intensities in these arms be i_1, i_2, i_3 and i_4, respectively. In the case considered, since $V_B = V_D$,

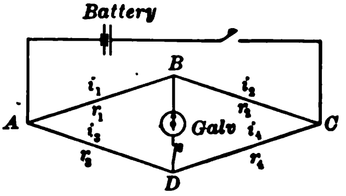

FIG. 325.

$$V_A - V_B = V_A - V_D,$$

and

$$V_B - V_C = V_D - V_C.$$

Or, expressing these potential differences in terms of current and resistance,

$$i_1 r_1 = i_3 r_3$$

and

$$i_2 r_2 = i_4 r_4.$$

Also, when $V_B = V_D$, no current traverses the bridge wire BD. Then $i_1 = i_2$ and $i_3 = i_4$. Whence dividing each member of the former equation by the corresponding member of the latter,

$$\frac{r_1}{r_2} = \frac{r_3}{r_4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (206)$$

Thus, if any three of the resistances are known when no current traverses the bridge wire, the remaining unknown resistance can be determined. A galvanometer in the bridge wire indicates the presence or absence of current. The usual form of Wheatstone bridge consists of a box containing three groups of coils of known resistance, with convenient arrangements for altering the resistance of each group. When made for laboratory use, the box containing the coils is separate from the galvanometer and the battery.

If one arm of a Wheatstone bridge include a very thin strip of metal of high temperature-resistance coefficient, very small changes of temperature can be measured. This device, called the bolometer, is much used for measuring radiance. The resistances of the bridge arms are first adjusted till no current traverses the galvanometer when the bolometer strip is screened from radiance. If now the bolometer strip be exposed to radiance, it will absorb energy and rise in temperature and in resistance. The bridge being thereby unbalanced, a current will traverse the galvanometer. The deflection of the galvanometer depends upon the radiance incident on the bolometer strip.

The tungsten daylight recorder includes a Wheatstone bridge circuit in one arm of which there is a group L of tungsten lamps. While light is excluded, the resistance of the arm BC is adjusted till there is no galvanometer deflection. On exposing the tungsten lamps to light, the resistance of the tungsten increases and the galvanometer is deflected. By means of a relay, not shown in the figure, combined with the galvanometer, a slide s is moved till the bridge is again in balance. It is the displacement of this slide that is recorded by a pin on a moving strip of paper.

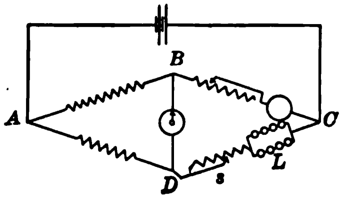

FIG. 326.

Sound waves in air sweeping past a thin heated wire cause a slight diminution of temperature. Variations of sound intensity can be measured by means of a bolometer circuit in which such a hot wire constitutes one arm. This is the basis of a sound ranging method developed by the U. S. Army.

332. The International Electric Units.—In preceding Articles the absolute and the practical C. G. S. electromagnetic units of electromotive force, current, resistance, etc., have been defined. But all of these units are difficult to realize experimentally. For the purpose of electrical measurements and as a basis of legislation, in 1908 an International Conference on Electrical Units and Standards recommended for adoption the following units of resistance, current and electromotive force. To a high degree of precision these units are of the same magnitude as the corresponding practical C. G. S. electromagnetic units and have the great advantage of being easily realized experimentally.

The International Ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area, and of a length of 106.300 centimeters.

The International Ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water (in accordance with specifications attached to the resolutions) deposits silver at the rate of 0.00111800 of a gram per second.

The International Volt is the electromotive force which, when steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere.

All of the other international electric units are derived from these three by means of the ordinary relations.

CHAPTER XX

ELECTROMAGNETIC INDUCTION

§ 1. *Transformation of Mechanical Work into Electric Energy*

333. The Discovery of Henry and Faraday.—In 1831, soon after the discovery that a current-carrying conductor in a magnetic field, and not parallel to it, tends to move across the field, Joseph Henry in America and Michael Faraday in England discovered the reverse phenomenon that an electric current tends to be produced in any conductor when moved across a magnetic field.

If a magnet *ns*, Fig. 327, be thrust into a coil of wire *S* connected to a current-indicating instrument *G*, the latter will indicate the presence of an electric current in the coil during the time the magnet is moving relative to the coil. On withdrawing the magnet, there will be a current in the opposite direction. If the magnet be reversed and the above operations repeated, there will be currents in the opposite directions. If the speed of the displacement be increased or diminished, the current strength will be changed in the same proportion.

If a current-carrying coil *P*, Fig. 328, be moved either toward, or away from, the coil *S*, current will be set up in the latter coil as before.

If while both coils are at rest, the magnitude of the current in *P* be changed, a current will flow in the other coil during the time the current in *P* is changing.

If while both coils are at rest and the current in *P* is constant, a rod of iron be either thrust into or withdrawn from the coil *P*, there will be a momentary current set up in the other coil.

In general, whenever a magnetic field sweeps across a conductor, an electromotive force will be induced which will tend to produce an electric current. The phenomenon of the development of an electromotive force by relative motion of an electric conductor and a magnetic field is called *electromagnetic induction.*

In each of the above cases, the magnetic flux which crosses the turns of wire in the coil *S* is the flux which is produced or destroyed inside of *S*. It equals the product of the area of one turn, the number of turns and the change of intensity of magnetic field within the coil. In the case of a straight conductor of length *l* moving a distance *x* perpendicularly to a magnetic field of intensity *H*, the flux which crosses the wire is *lxH*.

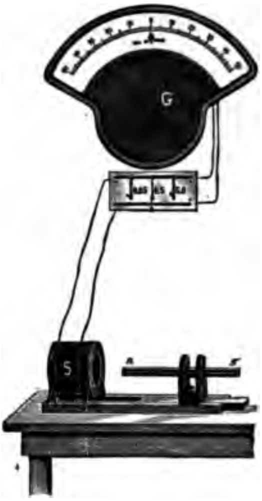

Fig. 327.

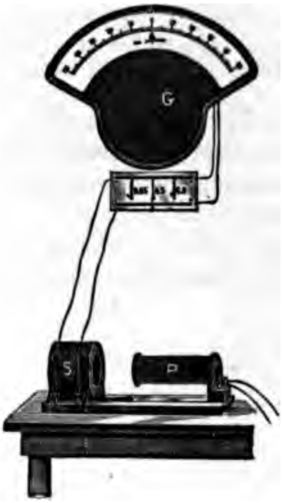

Fig. 328.

If a magnet pole be moved along the axis of a circular metal ring, an electromotive force will be induced which will produce a current. But between no two points of the circuit will there be a potential difference. This is an example of electromotive force with zero potential difference. Kamerlingh-Onnes has performed the experiment with a metal ring of such minute resistance that the current continued with slight diminution for several hours after the magnetic pole had come to rest relative to the ring.

The ship channels leading to many harbors are so narrow and crooked that, until the development of a recent device depending upon electromagnetic induction, these channels could not be used safely during the night or during foggy weather. This device includes an insulated wire laid are equally loud.

334. The Direction of an Induced Ele Induced Current. Lenz's Law. Though of electromagnetic induction is unknown, y tive force is induced, the direction of the inferred. An electric current implies an el the latter implies an absorption of some f

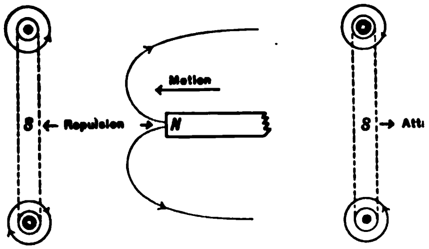

FIG. 329.

transformation into electric energy. When there is always a stress in such a direction as ment which produced them. For example magnet and a neighboring conducting circuit opposes their approach or separation. This p in Lenz's Law. *when a current is induced* motion, it is necessary that on the side of the wire toward the advancing magnet, the magnetic field of the induced current shall be in the same direction as the magnetic field of the magnet. With an advancing north pole the direction of the induced current will be as indicated.

Fig. 330 represents a magnet being pushed away from a closed loop of wire. In order that there shall be a force opposing the motion, it is necessary that on the side of the wire toward the retreating magnet, the magnetic field of the induced current shall be in the direction opposite to the magnetic field of the magnet. With a retreating north pole the direction of the induced current will be as indicated.

Fig. 331 represents a current-carrying loop *P* being pushed toward a closed loop of wire *S*. The inducing current is called the *primary current*, and the conductor *P* in which it flows, the *primary conductor*. The induced current is often called the *secondary current*, and the conductor *S* in which it flows is called the *secondary conductor*. In order that there shall be a force opposing the approach of the two conductors, it is necessary that on the approaching sides of the two conductors the magnetic field due to the induced current shall be in the same direction as that due to the primary current. With the current in the advancing primary conductor in the direction indicated, the direction of the induced current will be as shown.

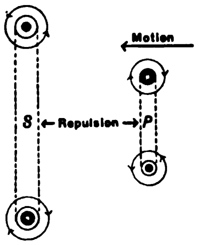

FIG. 331.

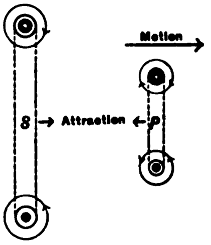

FIG. 332.

Fig. 332 represents a current-carrying conductor *P* being pushed away from a closed loop of wire *S*. By applying Lenz's along the middle of the bed of the channel, together with an apparatus on the ship consisting of two telephone receivers, each connected to a coil of wire on the outside of the hull. One terminal of an alternating current generator is connected to the submarine conductor, and the other terminal is connected to the ground. When the ship is to one side of the submarine conductor, the alternating currents produced in one of the coils on the ship are greater than the currents induced in the other coil Consequently, the sound produced by one telephone will be louder than that produced by the other. The ship is steered so that the sounds produced by the two telephone receivers are equally loud.

334. The Direction of an Induced Electromotive Force and Induced Current. Lenz's Law.—Though the ultimate nature of electromagnetic induction is unknown, yet when an electromotive force is induced, the direction of the current can be readily inferred. An electric current implies an electromotive force and the latter implies an absorption of some form of energy and its transformation into electric energy. When currents are induced there is always a stress in such a direction as to oppose the movement which produced them. For example, between a moving magnet and a neighboring conducting circuit there is a stress that opposes their approach or separation. This principle is formulated in Lenz's Law, *when a current is induced, its magnetic field must be in that direction which will oppose the change which produces it.*

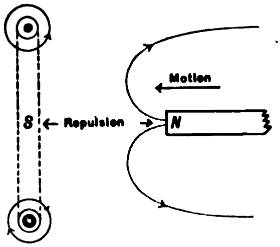

FIG. 329.

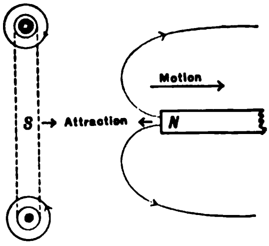

FIG. 330.

To illustrate the application of Lenz's Law we shall now use it for the determination of the direction of the induced current in the cases described in the preceding Article.

Fig. 329 represents a magnet being pushed toward a closed loop of wire *S*. In order that there shall be a force opposing the motion, it is necessary that on the side of the wire toward the advancing magnet, the magnetic field of the induced current shall be in the same direction as the magnetic field of the magnet. With an advancing north pole the direction of the induced current will be as indicated.

Fig. 330 represents a magnet being pushed away from a closed loop of wire. In order that there shall be a force opposing the motion, it is necessary that on the side of the wire toward the retreating magnet, the magnetic field of the induced current shall be in the direction opposite to the magnetic field of the magnet. With a retreating north pole the direction of the induced current will be as indicated.

Fig. 331 represents a current-carrying loop *P* being pushed toward a closed loop of wire *S*. The inducing current is called the *primary current*, and the conductor *P* in which it flows, the *primary conductor*. The induced current is often called the *secondary current*, and the conductor *S* in which it flows is called the *secondary conductor*. In order that there shall be a force opposing the approach of the two conductors, it is necessary that on the approaching sides of the two conductors the magnetic field due to the induced current shall be in the same direction as that due to the primary current. With the current in the advancing primary conductor in the direction indicated, the direction of the induced current will be as shown.

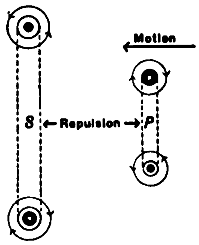

FIG. 331.

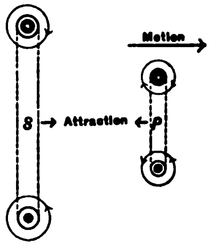

FIG. 332.

Fig. 332 represents a current-carrying conductor *P* being pushed away from a closed loop of wire *S*. By applying Lenz's

Law the direction of the induced current will be found to be as indicated.

Figs. 333 and 334 represent a stationary primary conductor P, and a stationary secondary conductor S. An increasing current in P produces in S the same effect as if P were approaching S, and a decreasing current in P produces in S the same effect as if P were receding from S. Therefore the directions of the induced currents in Figs. 333 and 334 are the same as in Figs. 331 and 332.

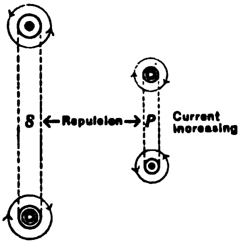

FIG. 333.

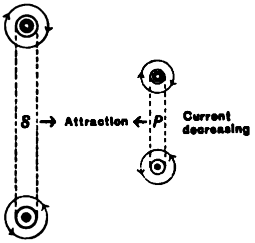

FIG. 334.

335. Magnitude of Induced Electromotive Force.—Suppose a wire XY slides for a distance x with a velocity v along a wire $ABCD$ whose plane is perpendicular to a magnetic field of intensity H. Let the direction of this magnetic field be perpendicular to the plane of the page and toward the reader. Since the sliding wire is crossing a magnetic field, there is an induced electromotive force in the circuit $XYCBX$. The direction of the current produced by this electromotive force can be foretold by means of Lenz's Law (Art. 334). This current will be in such direction that its magnetic field of force reacting upon the original field H will develop a force F in opposition to the motion of the slider XY.

If the distance between the parallel conductors AB and DC be l, then since the sliding conductor is perpendicular to the magnetic field H, the force opposing the motion of the sliding conductor will be $F = lHI$ (Art. 295). Consequently, a displacement of the sliding conductor through a distance x involves an absorption of mechanical energy by the system equal to

$$W[=Fx] = lHIx.$$

Therefore, the mean electromotive force has the magnitude, (180),

$$E_m\left[=\frac{W}{q}=\frac{Fx}{It}=\frac{lHIx}{It}\right]=\frac{lxH}{t}.$$

It is customary to call the induced electromotive force positive when the magnetic flux which it causes through the loop $XBCY$ has the same direction as that which was there to begin with. In Fig. 335 the electromotive force is therefore in the positive direction.

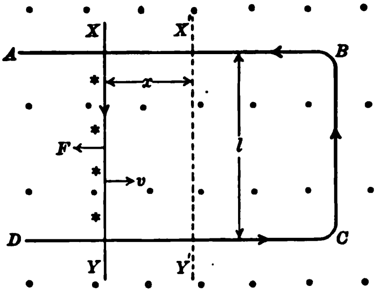

FIG. 335.

Now lx is the decrease in the area of the loop, lxH is the decrease in the magnetic flux through the loop, and lxH/t is the rate at which the magnetic flux through the loop is decreasing. If we use $\phi_2-\phi_1$ to represent the increase in the magnetic flux through the loop, we can write

$$E_m\left[=\frac{lxH}{t}\right]=-\frac{\phi_2-\phi_1}{t}.$$

That is, the mean value of the electromotive force induced in a single loop equals the mean rate at which the magnetic flux through the loop is decreasing.

If instead of a single turn of wire, the circuit consists of n turns in each of which the magnetic flux increases from ϕ_1 to ϕ_2 in time t, then during this time the coil will be the seat of an induced electromotive force of mean value

$$E_m=-\frac{n(\phi_2-\phi_1)}{t}.$$

A change of magnetic flux in a single turn of wire at the rate of

The product $n(\phi_2-\phi_1)$ is called t
flux linkings. If the change of magne
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337. Electromotive Force Induced in a Coil Rotating in a Magnetic Field.—Consider a single wire loop *abc* of area A that can be rotated about an axis xy. Let xy be in the plane of the loop and normal to a uniform magnetic field of intensity H. When the plane of the loop is normal to the direction of the magnetic field, the magnetic flux within the loop equals AH (164). When the plane of the loop makes an angle θ with the former position, the component of the area perpendicular to the flux is $A \cos \theta$, and the flux is

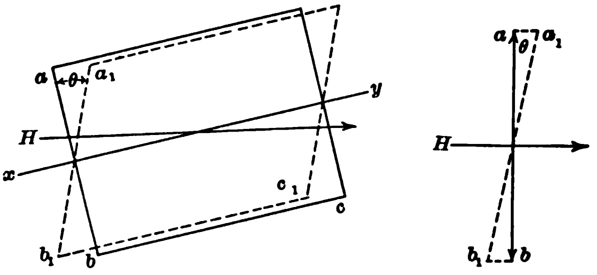


FIG. 336.

$$\phi_1 = A \cos \theta \cdot H.$$

If the loop be rotated through a small angle $d\theta$ in the small time dt, the flux through the loop at the end of this time will be

$$\phi_2 = AH \cos (\theta + d\theta)$$

and the increase of flux during the time dt is

$$\phi_2 - \phi_1 = AH[\cos (\theta + d\theta) - \cos \theta].$$

Representing $\phi_2 - \phi_1$ by $d\phi$, we obtain on expanding $\cos (\theta + d\theta)$,

$$d\phi = AH[(\cos \theta \cos d\theta - \sin \theta \sin d\theta) - \cos \theta].$$

If $d\theta$ is very small, $\cos d\theta \doteqdot 1$, and $\sin d\theta \doteqdot d\theta$. In this case the above value for the change of flux during the time dt may be written

$$d\phi = AH[\cos \theta - \sin \theta d\theta - \cos \theta] = -AH \sin \theta d\theta.$$

one maxwell per second develops one abvolt. Since one volt equals 10^8 abvolts,

$$E_m = -\frac{n(\phi_2 - \phi_1)}{t} \text{ abvolts} \quad . \quad . \quad . \quad . \quad (207)$$

$$= -\frac{n(\phi_2 - \phi_1)}{10^8 t} \text{ volts.} \quad . \quad . \quad . \quad . \quad . \quad (208)$$

The product $n(\phi_2 - \phi_1)$ is called the change of flux-turns or flux linkings. If the change of magnetic flux in each turn of wire during the infinitesimal time interval dt be denoted by $d\phi$, then the instantaneous value of the electromotive force is

$$E = -n\frac{d\phi}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (209)$$

336. Mutual Induction.—Whenever the magnetic flux-turns through a conducting circuit are changed, an electromotive force is set up in the conductor. In the case of two neighboring circuits, a change of the current in the first will change the flux-turns through the second and will therefore induce in the second an electromotive force. The current thereby produced will, in turn, produce another current in the first circuit. That property of a pair of circuits which causes a current to be induced in each circuit when there is a change of flux-turns in either circuit is called *mutual induction.*

The magnitude of the electromotive force of mutual induction depends upon the rate of change of flux-turns within the two circuits. For a given change of current in one coil, the magnitude of the electromotive force of mutual induction is increased, (*a*) by decreasing the distance between the two coils; (*b*) by moving the coils so that their axes are more nearly parallel and in the same line; (*c*) by inserting iron into the coils.

A familiar example of mutual induction is furnished by the disturbances set up in telephone circuits by variable currents in neighboring wires. This trouble is overcome by having the outgoing and the return wires of each telephone circuit side by side and twisted about one another. It is left as an exercise for the student to explain how this device produces the desired result.

337. Electromotive Force Induced in a Coil Rotating in a Magnetic Field.—Consider a single wire loop *abc* of area A that can be rotated about an axis xy. Let xy be in the plane of the loop and normal to a uniform magnetic field of intensity H. When the plane of the loop is normal to the direction of the magnetic field, the magnetic flux within the loop equals AH (164). When the plane of the loop makes an angle θ with the former position,

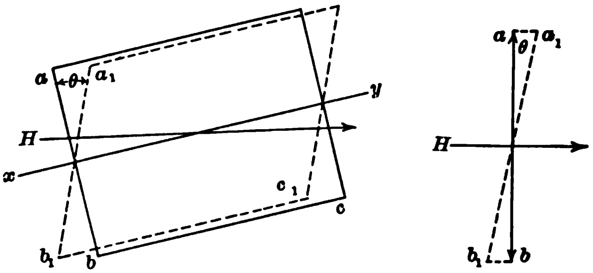


FIG. 336.

the component of the area perpendicular to the flux is $A \cos \theta$, and the flux is

$$\phi_1 = A \cos \theta \cdot H.$$

If the loop be rotated through a small angle $d\theta$ in the small time dt, the flux through the loop at the end of this time will be

$$\phi_2 = AH \cos (\theta + d\theta)$$

and the increase of flux during the time dt is

$$\phi_2 - \phi_1 = AH[\cos (\theta + d\theta) - \cos \theta].$$

Representing $\phi_2 - \phi_1$ by $d\phi$, we obtain on expanding $\cos (\theta + d\theta)$,

$$d\phi = AH[(\cos \theta \cos d\theta - \sin \theta \sin d\theta) - \cos \theta].$$

If $d\theta$ is very small, $\cos d\theta \doteqdot 1$, and $\sin d\theta \doteqdot d\theta$. In this case the above value for the change of flux during the time dt may be written

$$d\phi = AH[\cos \theta - \sin \theta d\theta - \cos \theta] = -AH \sin \theta d\theta.$$

From (209), the magnitude of the electromotive force during the time dt is

$$E\left[=-1\frac{d\phi}{dt}\right]=AH\sin\theta\frac{d\theta}{dt}.$$

If during this time the angular velocity of the loop be uniform and of magnitude w, the above expression may be put into the form

$$E=AHw\sin\theta.$$

If, instead of a single loop, there be a coil of n parallel loops

$$E=nAHw\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (210)$$

Consequently, when a coil of wire rotates steadily in a uniform magnetic field, there will be induced in the coil an electromotive force which at any instant is proportional to the sine of the angle between the plane of the coil and a plane normal to the magnetic field.

An inspection of this equation makes evident the facts that when $\theta=0°$ or $180°$, the magnitude of the electromotive force is zero; that when $\theta=90°$ there is a maximum electromotive force in one direction; and that when $\theta=270°$ there is an equal electromotive force in the opposite direction. This alternating electromotive force will set up an alternating current in the coil and in any conducting circuit connected in series with the coil.

338. Mutual Inductance.—If a current I_1 in one circuit causes a magnetic flux ϕ_2 through a second circuit, it is found that a current I_1, in the second causes a magnetic flux ϕ_2 in the first. The *mutual inductance* or the coefficient of mutual induction of two circuits is the ratio of the magnetic flux ϕ_2 through one of them produced by a current in the other, to the intensity I_1 of this current. Thus, the mutual inductance

$$M=\frac{\phi_2}{I_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

Since a change of flux $d\phi_2$ in time dt produces an electromotive force E'_2, we may write, (209),

$$E'_2\left[=-\frac{d\phi_2}{dt}\right]=-M\frac{dI_1}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (212)$$

Whence, the practical unit of mutual inductance, called the *henry*, is the mutual inductance of two circuits when a change of current in one circuit at the rate of one ampere per second induces in the other circuit an electromotive force of one volt.

339. Self Induction.—We have electromagnetic effects not only between separate circuits but also between the parts of a single circuit. Due to a current change in any circuit, an opposing electromotive force will be set up in the same circuit. If the original current be increased, the electromotive force thereby induced will oppose the increase. If the original current be decreased, the electromotive force thereby induced will oppose the decrease. That property of an electric circuit that opposes, by means of a counter electromotive force, any variation of the strength of current traversing it, is called *self induction.*

By winding part of the wire of a circuit into a coil, the self induction of the circuit is increased. By inserting an iron core in the coil, the self induction is farther increased. If this core is a completely closed iron circuit, the self induction will be greater than if the core contains an air gap.

A "choke" or "impedance" coil may be used to diminish the magnitude of an alternating current. One form of choke coil of variable self-induction is diagrammatically represented in Fig. 337. When the iron armature *A* is in contact with the pole pieces of the iron yoke *BB'*, the self induction of the choke coil is a maximum and the magnitude of the alternating current in the circuit is a minimum. By separating this armature from the yoke, the self induction of the choke coil is diminished and the magnitude of the alternating current is increased.

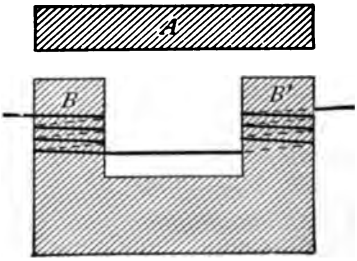


Fig. 337.

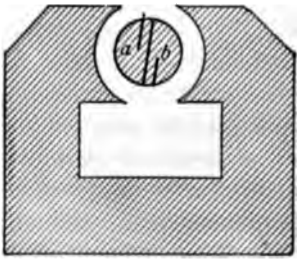


Fig. 338.

Another type of commercial choke coil of variable self induction is represented in Fig. 338. This consists of a cylindrical iron armature wrapped longitudinally with a current-carrying conductor and supported between the ends of a yoke of soft iron. When the plane of the coil is as shown in the figure, a change of current will produce the maximum change in the magnetic flux within the coil; whereas, when the plane of the coil is 90° from the position indicated in the figure, the same change of current will produce the minimum change in the magnetic flux within the coil. Now, the electromotive force of self induction varies directly with the rate of change of magnetic flux. Consequently if a source of alternating electromotive force be connected to *ab*, the magnitude of the current will be smallest when the plane of the coil is as shown in Fig. 338, and will be greatest when the plane of the coil is 90° from the position indicated.

340. Self Inductance.—The coefficient of self induction, or the *self inductance* of a circuit, is measured by the ratio of the magnetic flux through it due to the current in it, to the current strength. Thus, the self inductance

$$L=\frac{\phi_1}{I_1}. \qquad (213)$$

Since a change of flux, $d\phi_1$ in time dt, produces an electromotive force E', we may write

$$E'\left[=-\frac{d\phi_1}{dt}\right]=-L\frac{dI_1}{dt}. \qquad (214)$$

Whence, the practical unit of self inductance is that self inductance of a circuit in which an electromotive force of one volt is induced when the current in the circuit varies at the rate of one ampere per second. The unit of self inductance is called the *henry*.

341. Eddy or Foucault Currents.—If a metallic ring be pushed into a magnetic field as represented in Fig. 339, an electromotive force will be induced in the ring which will produce a current in the direction shown. If the ring be pushed out of the magnetic field as represented in Fig. 340, a current will be set up in the direction there shown.

If the ring be replaced by a metallic disk currents will be set up as before. But in this case the currents spread throughout the disk, Figs. 341 and 342. Currents induced in a mass of material by a magnetic flux alternating in direction are called *eddy* or *Foucault* currents.

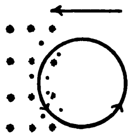


FIG. 339.

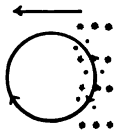


FIG. 340.

When the body is large in which eddy currents occur, the electric resistance may be so small that the current is very large. The mechanical stress between the original magnetic field and that due to this current will strongly oppose the relative motion of the two fields. Continuation of this relative motion will involve a large expenditure of mechanical work. This work is transformed into heat within the conducting body. The strong opposition offered to the relative motion of the parts of such a system has been applied in one type of street car brake which consists of a massive metal disk keyed to an axle and revolving between the poles of a powerful electromagnet. On energizing the electromagnet, there will be set up in the rotating disk currents in such directions and of such magnitudes as to produce a strong opposition to the rotation of the axle.

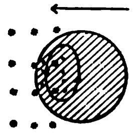


FIG. 341.

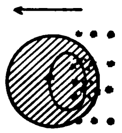


FIG. 342.

Eddy currents can be prevented by dividing the body into thin layers by nonconducting laminæ normal to the direction that the eddy currents would otherwise flow. It is customary to thus " laminate " those parts of electric machinery that are subject to rapid variations of magnetic flux.

242. Table of Units of Commonly Employed Electric Quantities.—

Name of Quantity	Symbol	Defining Equation	Name of Practical Unit
Pole strength	m	$F=\frac{m_1m_2}{\mu r^2}$	
Field strength	H	$F=mH$	gauss
Induction density	B	$B=\mu H$	gauss
Magnetic flux	ϕ	$\phi=AH$	maxwell
Current strength	I	$F=0.1lHI$	ampere
Quantity	q	$q=It$	coulomb
Resistance	R	$W=I^2Rt$	ohm
Electromotive force	E	$E=\frac{W}{q}$	volt
Potential difference	V	$W=Vq$	volt
Capacitance	C	$q=CV$	farad
Inductance	L	$E=L\frac{dI}{dt}$	henry

QUESTIONS

1. A wire circle is supported in a horizontal position and a vertical bar magnet is dropped through it, the south end going down first. Discuss the e.m.f. which is induced in the wire.

2. If an iron core be quickly withdrawn from its magnetizing helix, what will be the momentary effect on the reading of an ammeter in the circuit?

3. A loop of wire is being moved through a magnetic field. Under what circumstances (*a*) is no e.m.f. induced in it? (*b*) does no current flow in it? (*c*) is a large e.m.f. induced in it? (*d*) does a large current flow in it?

4. A flat loop of wire lying on a horizontal table is quickly moved in the plane of the table. Is an e.m.f. set up in the wire? Does a current flow in the wire? Explain fully.

5. In what direction would a current have to flow in a wire wrapped around an iron trolley pole to make the end in the ground a south pole? If the pole were thus magnetized, what would be the direction of a current in a metal hoop dropped over the end?

6. At the absolute zero of temperature a conductor is of nearly zero resistance. Would it be difficult or easy to move a piece of copper wire across a magnetic field under these conditions? Why?

7. The coil of a d'Arsonval galvanometer is swinging. Show by diagram the direction of the e.m.f. induced in the coil. If the coil is short circuited, this e.m.f. may cause a very appreciable current to flow. Find whether this current is in such direction as to aid or hinder the swinging of the coil. Show how you find out.

§ 2. *Electric Machines*

343. The Simple Alternating Current Generator.—Figs. 343, 344 and 345 represent a loop of wire rotating about an axis normal to the plane of the paper in the magnetic field between the poles of a magnet. Equation (210) shows that in such a loop an alternating electromotive force will be induced which will produce an alternating current in any closed circuit joining the ends of the loop. This action will now be further considered and the method described by which it is utilized in the construction of commercial electric " generators " for setting electricity into motion.

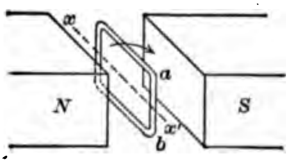


FIG. 343.

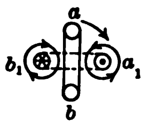


FIG. 344.

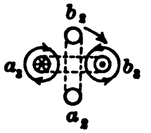


FIG. 345.

From Lenz's Law (Art. 334), when a current is produced by induction its magnetic field must be in that direction which will oppose the change that produces it. Consequently, if the loop be rotated clockwise about an axis xx', and the direction of the magnetic field be as indicated in the figures, then the induced current in the side a_1 of the loop, Fig. 344, will be directed toward the reader. When, however, the loop has rotated 180° from this position, the current in this side of the loop, a_3, Fig. 345, will be directed away from the reader. Thus, the direction of the current induced in the loop changes twice every revolution. If the ends of the loop be joined to two *collector rings*, RR' Fig. 346, fastened to, but insulated from the shaft, and conducting brushes BB' be pressed against these collector rings, then the current can be led off to operate incandescent lamps, or any other mechanism that can be operated by an alternating current.

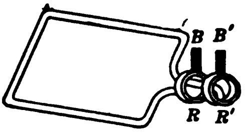


FIG. 346.

In order that the induced electromotive force may be large, the rate of change of magnetic flux through the rotating loop must be large. The intensity of the magnetic field should therefore be large, and the flux within the loop should change rapidly. The required intense magnetic field within the space in which the loop rotates is produced by an electromagnet, Fig. 347, energized or " excited " by some source of constant current. To increase the intensity of this magnetic field, the rotating loop is wrapped about a soft iron core. One way to produce a rapid change in the magnitude of the flux through the rotating loop is to rotate the loop at high angular speed. Another way is to have more than one pair of poles to produce the magnetic field. When multiple poles are used, the adjacent poles are of opposite polarity, Fig. 348.

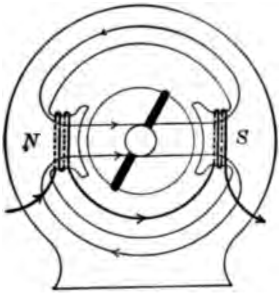


Fig. 347.

The magnet which produces the inducing magnetic field is called the *field magnet.* The system consisting of an iron core and the copper conductor in which electromotive forces are induced, is called the *armature.* A machine, such as described in this Article, which produces an electromotive force that reverses in direction each time the direction of the flux through the armature is reversed, is called an *alternating current generator* or *alternator.*

In the machine above described the armature is rotated by some mechanical means and the field magnets remain stationary. Alternators are also made in which the field magnets are rotated and the armature remains stationary. A comparison of Figs. 348 and 349 will make clear the difference in construction and the similarity in action. For simplicity of representation in each figure the cross-section of a single turn of the armature winding is shown, and the collector rings and brushes are omitted. In the rotating armature type, Fig. 348, the constant exciting current of relatively low electromotive force is led into the field coils by means of fixed conductors, and the induced alternating current of high electromotive force is led from the machine through collector rings and brushes. In the rotating field type, Fig. 349, the exciting current is led into the machine through collector rings and brushes, and the induced current of high electromotive force is led away from the machine by fixed conductors. With alternators of the rotating field type there is less trouble due to sparking at the

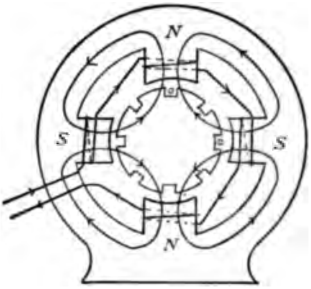


FIG. 348.

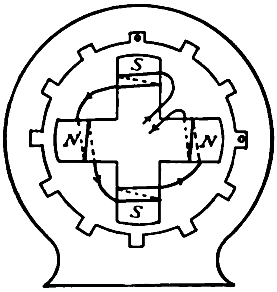


FIG. 349.

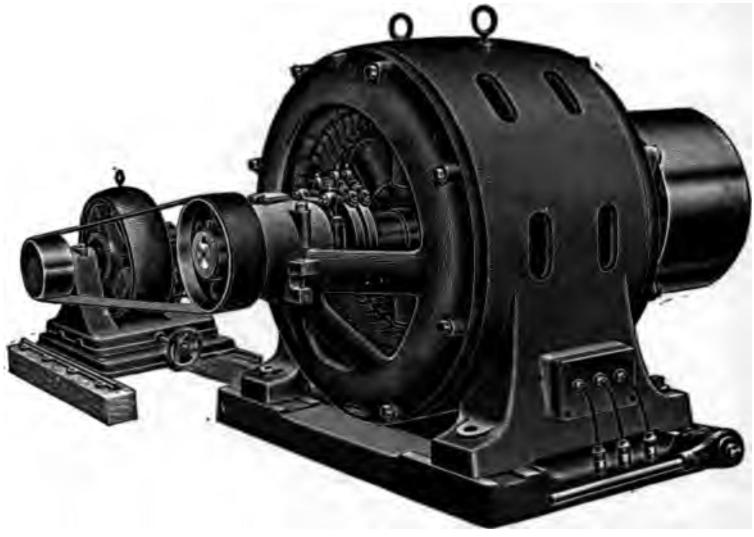


FIG. 350.

brushes, and there is more room for armature coils, than with alternators of the rotating armature type.

The magnetic flux through the core of an armature changes rapidly in direction and in magnitude. To prevent the production of large eddy currents and the consequent heat losses, the cores of armatures are always laminated.

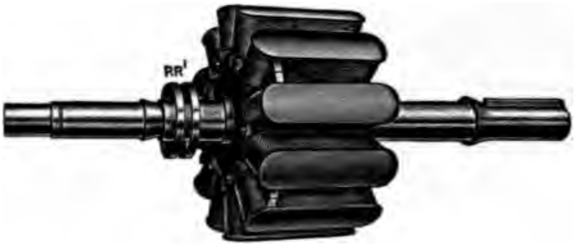


Fig. 351.

Fig. 350 represents a modern alternator of the rotating field type, together with its direct current "exciter." The separated field magnets and armature of this alternator are shown in Figs. 351 and 352 respectively. The brushes press on the rings RR'.

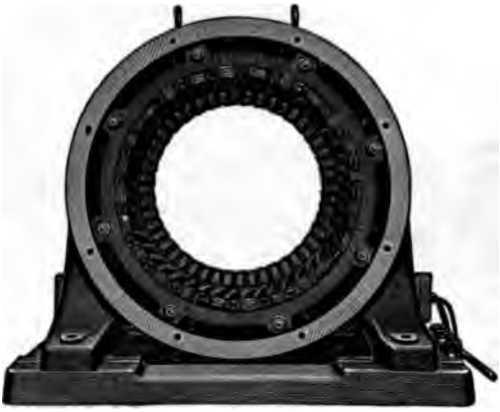


Fig. 352.

344. The Elements of the Direct Current Generator.—While a conducting loop is rotating at a constant angular velocity in a uniform magnetic field there is induced in the loop an electromotive force that changes with the position of the loop in the manner expressed by (210) and represented graphically by the curve in Fig. 353. If the period of alternation is not too great, the fact that the current alternates in direction is of no disadvantage in operating lights and certain other devices. But for electrolytic and certain other operations a current of constant direction is necessary. The alternating pulses induced in a loop rotating in a magnetic field can be rendered unidirectional by means of a device now to be described called a "commutator."

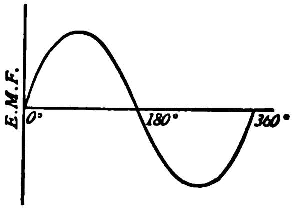


FIG. 353.

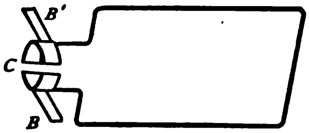


FIG. 354.

The commutator consists of a split ring, *C*, Fig. 354, attached to, but insulated from the shaft of the armature. If the armature has but one coil, the commutator will have two segments, each attached to one end of the coil. Fastened to the frame of the machine are two conducting "brushes," *B*, *B'*, which press against the rotating commutator. The brushes are the terminals of the external circuit which is to be supplied with current. The brushes are so placed that they slip from one commutator strip to another at the instant when the current in the connected armature coil reverses in direction. An inspection of Figs. 356 and 357 will

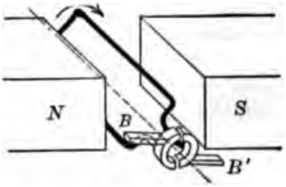


FIG. 355.

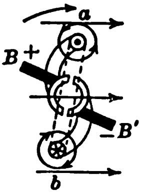


FIG. 356.

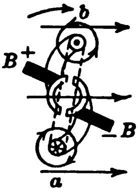


FIG. 357.

show that with the direction of the magnetic field and the direction of the armature rotation as shown, the upper brush will be positive and the lower brush will be negative whatever commutator strips are in contact with them. That is, the current that goes into the external circuit is always in the same direction. Twice during a revolution of the armature, the current rises from zero to a maximum value and falls to zero, but the direction of the current is constant. Such a current is said to be *unidirectional* or *direct*.

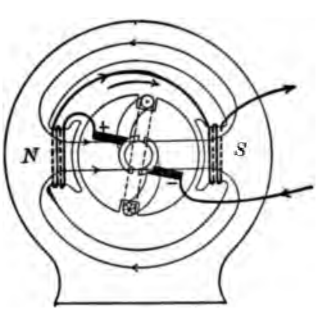


FIG. 358.

The magnetic field in which the armature revolves is produced by a powerful electromagnet energized by the current generated in the rotating armature. If all of the current from the armature is passed through the field coils as shown in Fig. 358, the machine is called a series-wound direct-current dynamo, or generator.

SOLVED PROBLEM

PROBLEM.—Diagram the windings and connections of a simple series-wound direct-current generator that will meet the following specifications—left pole south, rotation of the armature clockwise, positive brush on the upper side of the commutator.

SOLUTION.—The given specifications are represented in Fig. 359. The development of an electromotive force in the coil *ab* implies an absorption of

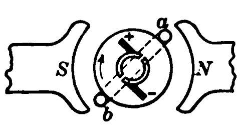


FIG. 359.

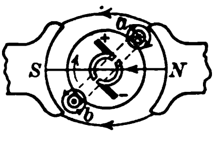


FIG. 360.

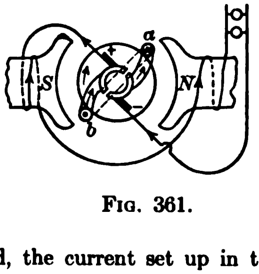


FIG. 361.

energy. In order that energy may be absorbed, the current set up in the moving coil must be in such a direction that the rotation of the coil with its magnetic field shall be opposed by the magnetic field due to the poles *N* and

S, (Lenz's Law). Consequently, the field of force about the conductor *ab* due to the induced current must be in the direction indicated in Fig. 360. The current is coming toward the reader at *b* and going from the reader at *a*.

In order that the upper brush may be positive, the current must leave the machine at this brush. Consequently, the end *b* of the coil is connected to the commutator strip in contact with the positive brush as shown in Fig. 361.

To give the poles the required polarity, the current-carrying conductor must be wrapped about the field magnets as indicated in Fig. 361.

345. Multiple-coil Armatures.—When a one-coil armature provided with a commutator is rotating, the electromotive force developed goes through a series of changes that for one revolution is represented graphically in Fig. 362. Though the electromotive force is unidirectional, the variations of the current may be so great as to cause a noticeable flickering in the light of any incandescent lamp to which the dynamo might be connected.

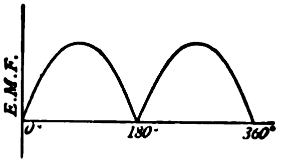


FIG. 362.

To avoid such great fluctuations of current, armatures of commercial direct current dynamos are always provided with several coils equally spaced throughout the periphery of the armature. If an armature were provided with two coils at right angles to one another and joined to a two-part commutator as shown in Fig. 363, while one coil is passing the position in which zero electromotive force is being induced, the other coil is passing through the position in which a maximum electromotive force is being induced. As the two coils are in series, an electromotive force is being induced in the conductor at all parts of a revolution.

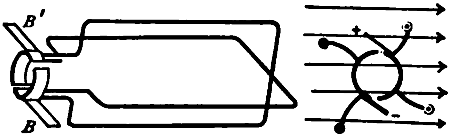


FIG. 363.

If the two loops were not joined together, an electromotive force would be induced in one of them that would be represented by the light full line in Fig. 364, and an electromotive force would be induced in the other loop that would be represented by the dotted line. When the two loops are joined, the resultant electromotive force is obtained by compounding these two curves. The heavy line represents the variation of the electromotive force impressed on the brushes. It will be observed that this electromotive force never reduces to zero, nor does it vary through such wide limits as does the electromotive force induced by a one-coil armature. By covering the periphery of the armature core with many coils, equally spaced, the fluctuations are reduced to inconsiderable dimensions. The arrangement of the windings on one type of armature having two coils is shown in Fig. 365. The corresponding arrangements for four coils and for six coils are shown in Figs. 366 and 367. If these armatures be rotated clockwise in a magnetic field directed to the right, the currents thereby induced will be in the directions indicated.

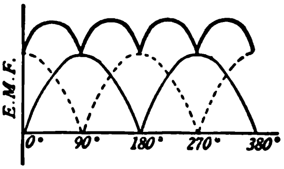


FIG. 364.

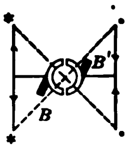


FIG. 365.

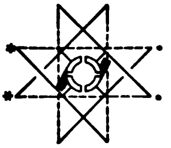


FIG. 366.

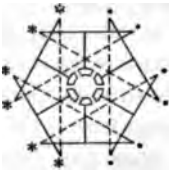


FIG. 367.

A modern direct current generator is illustrated in Fig. 368. The armature of this machine is shown separately in Fig. 369.

346. The Value of the Electromotive Force of a Generator.—In the case of a two-pole dynamo, a conductor on the periphery of the armature crosses and recrosses the magnetic field every revolution. If the magnetic flux cut by the conductor be ϕ maxwells and the angular speed be N revolutions per second, then during one revolution the mean value of the electromotive force induced in the conductor will be $2N\phi$ abvolts. If, instead of a single conductor, there be n conductors arranged in two groups connected in parallel, each group consisting of $n/2$ conductors connected in series, then the mean electromotive force developed in the armature will be

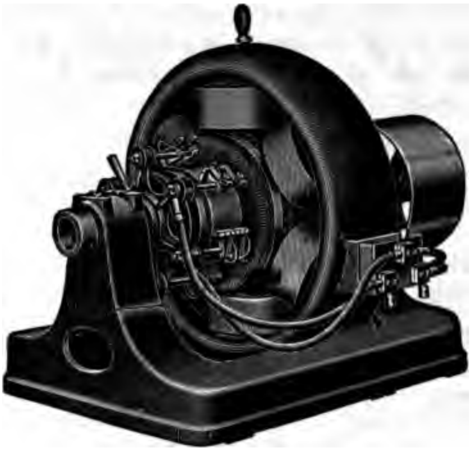


FIG. 368.

$$E_m = nN\phi \text{ abvolts} = \frac{nN\phi}{10^8} \text{ volts.} \quad . \quad . \quad . \quad (215)$$

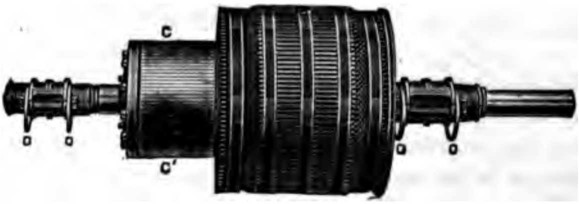


FIG. 369.

347. Series-, Shunt- and Compound-wound Machines.—In the series-wound machine heretofore considered, the armature and field coils are in series as represented in Fig. 370.

On open circuit, that is, with infinite resistance, the magnetic flux through the armature coils is due solely to the residual magnetism of the field magnet cores, and the induced electromotive force is small. But on closed circuit, the field cores are magnetized by all of the current set into motion by the machine. The variation of electromotive force with line current for a series-wound generator is represented by the curve *AB*, Fig. 373. This curve shows that the electromotive force of a series machine at first rapidly increases with the current-load, but that after a certain current-load a farther increase of load is accompanied by a decrease of electromotive force. This lack of constancy of electromotive force renders series generators unsuited for incandescent lighting.

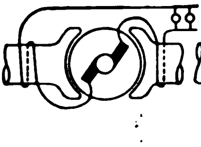


Fig. 370.

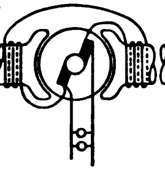


Fig. 371.

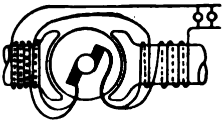


Fig. 372.

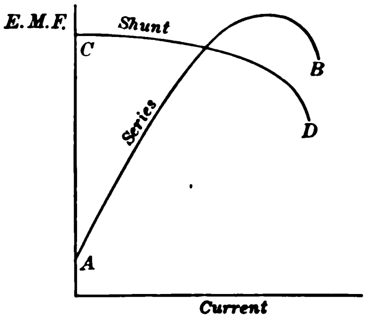


Fig. 373.

In the shunt-wound machine, Fig. 371, the current from the armature divides, part traversing the field coils and the remainder traversing the external circuit. On open circuit the armature and field coils are in series, and the electromotive force is maximum. On closed circuit the electromotive force will depend upon the amount of current traversing the field coils. With increase of current-load in the external circuit, the current in the field coils diminishes and consequently the electromotive force generated by the machine diminishes. The relation between the line current and the electromotive force of a shunt generator is represented by the line *CD*, Fig. 373.

In a shunt-wound generator, the electromotive force diminishes with increase of current-load, while in a series-wound generator the electromotive force increases with load. Hence by using on the field magnets a series coil and also a shunt coil it is possible to produce a machine that will develop an electromotive force which, within certain limits of current-load, is nearly independent of load. Such a machine is called a compound-wound generator and is represented in Fig. 372.

QUESTIONS

1. Draw a diagram of the windings of a two-pole direct current series dynamo having the positive brush on the upper side of the commutator, the left pole north, and the rotation counterclockwise.
2. At full load, the electromotive force of a series-wound dynamo is higher than at zero load. For a shunt-wound dynamo, the reverse is true. Explain.
3. Show why a sudden increase in the line resistance in connection with a shunt-wound dynamo causes a sudden increase in the potential difference at the brushes.
4. What effect does an increase in the resistance of the field circuit of a shunt-wound dynamo have on the potential difference at the brushes?
5. Decreasing the line resistance of a series generator causes an increase in the electromotive force and also the power developed by the machine. Explain.

348. The Elements of the Simple Direct-current Series Motor.—It has been seen (Art. 295) that a current-carrying conductor in a magnetic field is acted upon by a force which urges it sidewise except when the conductor is parallel to the magnetic field in which it is situated. In the electric motor this fact is utilized for the continuous transformation of electromagnetic energy into mechanical work.

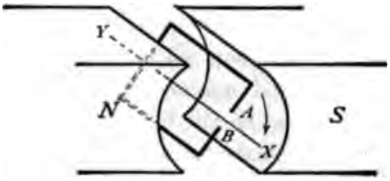


FIG. 374.

In Fig. 374 consider a rectangular loop of wire within the magnetic field between the poles of a powerful magnet and capable of rotation about an axis XY. In Figs. 375, 376, 377 and 378 the loop is represented in four successive positions. Let an electric current from some outside source flow from A through the loop to B, Fig. 374.

With the current-carrying loop in the position shown in Fig. 375, the field above A due to the current in the loop and the field due to the poles are in the same direction. Below A the two fields are in opposite directions. Hence the side A is urged downward. In the same manner we see that the side B is urged upward. Therefore the loop is acted upon by a torque in the clockwise direction. The lever arm of the couple producing the torque is BC. When the loop is in the position A_1B_1, Fig. 376, the torque has the maximum magnitude and is in the same direction as before. When in the position A_2B_2, Fig. 377, the lever arm, and consequently the torque, are zero. If, by means of some outside agent the loop be turned past this position into that of A_3B_3, Fig. 378, the loop is acted upon by a torque in the counterclockwise direction which urges the loop back into the position shown in Fig. 377. It is thus seen that with the current flowing from A toward B, there is developed a torque that rotates the loop till its plane is perpendicular to the direction of the external magnetic field. If the direction of the current be reversed when the loop has passed the perpendicular to the direction of the external magnetic field, then the loop will rotate another 180° in the same direction. If now the direction of the current be again reversed, the loop will rotate another 180°. The direction of the current in the loop can be automatically reversed at the proper times by means of a commutator such as is used in the direct-current generator.

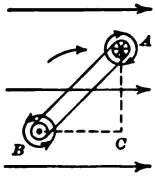


Fig. 375.

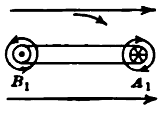


Fig. 376.

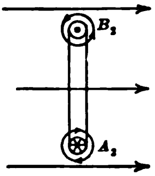


Fig. 377.

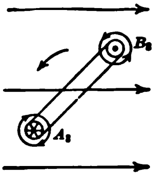


Fig. 378.

When the loop is in the position shown in Fig. 379, the current enters at the end *A* by way of the positive brush, and emerges from the end *B* by means of the negative brush. When the loop is rotated 180° from this position, the commutator strips under the brushes are changed, thereby reversing the direction of the current through the loop. By this means the loop is caused to rotate continuously in the same direction.

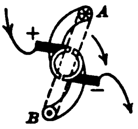


FIG. 379.

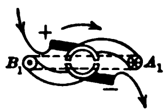


FIG. 380.

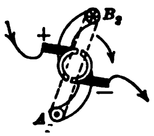


FIG. 381.

The torque acting on the loop is proportional to the current in the loop, and also to the intensity of the magnetic field in which the loop rotates. The magnetic field is produced by means of an electromagnet, Fig. 382, energized by the same current that traverses the loop. To increase the intensity of the magnetic field in which the loop rotates, the space within the loop is filled with iron of high permeability.

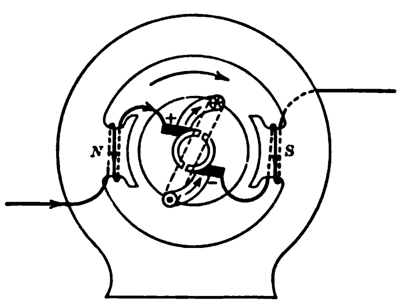


FIG. 382.

With an armature of a single coil, such as has been thus far considered, the torque is not the same at different parts of a revolution. But if the armature be provided with many equally spaced loops, properly connected to commutator strips, the torque will be practically constant throughout a revolution. The windings of direct-current motors may be the same as those of direct current generators. Motors may have any even number of poles. Any direct-current generator may be used as a motor.

A consideration of Fig. 382, shows that if the direction of the current be reversed, the direction of rotation of the armature will be unaltered. This shows that the armature of a series motor will rotate continuously when connected to an alternating-current circuit. To avoid the heating due to eddy currents set up in the armature core by the reversal of the current in the armature conductors the core must be laminated. If the field coils are traversed by alternating current, the field cores must also be laminated.

349. Back Electromotive Force of a Motor.—When an armature rotates in a magnetic field there is induced an electromotive force. It makes no difference whether the rotation be produced by a steam engine or by the interaction of the magnetic fields due to the field magnets and the current-carrying armature conductors. When the armature is that of a motor, the direction of the induced electromotive force is opposite that of the circuit or impressed electromotive force. Hence, the electromotive force induced by a rotating motor armature is called the *back* or *counter* electromotive force. The difference between the impressed or circuit electromotive force and the back electromotive force is the net or resultant electromotive force which causes the current through the armature. At high speeds the back electromotive force is large, thereby causing the resultant electromotive force, and therefore the current, to be small. At zero speed the back electromotive force is zero and the current through the armature is very large.

The product of the circuit voltage and the current through the motor is the *input* or power supplied to the machine. The product of the armature current and the back electromotive force is the power exerted in rotating the armature.

350. Speed of an Electric Motor.—We shall now consider the factors affecting the speed of a motor. In a motor, as in a dynamo, there is induced by the rotating armature an electromotive force of the value, (215),

$$e=\frac{nN\phi}{10^8}\text{ volts}=kN\phi\text{ volts,}$$

where k takes the place of the constant quantity $n/10^8$.

In a motor, the electromotive force which sends current through the armature is the difference between the impressed electromotive force E and the induced electromotive force e. It follows that

$$E-e=i_a r_a,$$

where i_a and r_a stand respectively for the current through the armature and the resistance of the armature.

From these two equations we find for the speed of the armature

$$N\left[=\frac{e}{k\phi}\right]=\frac{E-i_a r_a}{k\phi}\text{ rev. per min.} \quad . \; . \; . \quad (216)$$

351. Series-, Shunt- and Compound-wound Direct-current Motors.—Direct-current motor, like direct current dynamos, may be series-, shunt- or compound-wound. From Ampere's Law, it follows that the torque produced by a motor armature depends upon the number of armature conductors, the armature current and the field flux. If the armature current of a series-wound motor be increased, the field current and consequently the flux are increased almost proportionally. Hence the torque varies nearly as the square of the current. On closing the circuit, the counter electromotive force is negligible, the current is high and the torque is large. The large starting torque makes series-wound motors especially well suited for street car operation.

The speed of a series-wound motor is much affected by the load. The immediate effect of reduction of load is increase of speed. With increase of speed, the counter electromotive force increases, the armature current decreases, the field decreases, and the speed still farther increases. The " racing " of an unloaded series-wound motor may seriously injure the machine.

The resistance of the field coils of a shunt-wound motor is large compared with the resistance of the armature. Consequently if the armature current of a shunt-wound motor be increased, the field current is slightly affected. Hence the torque developed by a shunt-wound motor is almost proportional to the armature current.

The speed of a shunt-wound motor is much less affected by changes of load than is the speed of a series-wound motor. If the load be increased the armature will slow down, thereby causing successively a diminution of the counter electromotive force, an increase in the armature current, an increase in the counter electromotive force, a decrease in the field flux, and consequently an increase in the armature speed. By this series of actions the speed is maintained nearly constant.

By wrapping on the field magnets of a shunt-wound motor a few turns of wire in series with the outside circuit, and in opposition to the shunt windings, a compound-wound motor can be made which will give either constant speed for all loads within a certain range or a speed that will slightly increase with load. The numbers of turns in the shunt and series windings are so selected that the weakening of the field produced by the series turns either just compensates, or slightly over-compensates, for the diminution of speed due to the increased load.

In starting a shunt-wound or a compound-wound motor it must be remembered that the shunt coils are of much higher resistance and inductance than the armature. If while the armature is at rest the brushes were to be connected directly to the operating circuit, the current through the armature would be so great as to dangerously overheat the armature conductors. After the armature is in rotation, there will be a sufficiently great counter electromotive force to diminish the armature current to a safe value. To prevent a dangerous rise of current in the armature on starting a shunt- or a compound-wound motor a " starting box " is always employed. A common form is so arranged that on turning a lever the following operations occur in succession. Current first traverses the shunt winding. Later a current is connected through the armature in series with a resistance which is gradually diminished as the motion of the lever is continued. By the time the resistance has been reduced to zero, the armature will be in motion and there will be a counter electromotive force sufficient to prevent an excessive current.

352. The Integrating or Kilowatt-Hour Meter.—The energy supplied in one hour at the rate of one kilowatt is called a *kilowatt-hour*. This is the unit usually employed in industry for the measurement of electric energy. An integrating meter or kilowatt-hour meter is essentially a motor so designed that the speed of rotation is proportional to the electric power supplied to the line at that instant. One common form resembles the watt-meter (Art. 330), in that it comprises movable coils A, Fig. 383, and stationary coils BB.

If there were no opposing torque, the armature of the kilowatt-hour meter would begin to race as soon as it was connected into the circuit. But if the instrument be so designed that there is an opposing torque proportional to the speed of rotation, then the speed of rotation will be always practically proportional to the driving torque. For the armature will rotate steadily at such a speed that the opposing torque equals the driving torque. If the driving torque becomes greater than the opposing torque, the armature will speed up until the two are equal; and if the driving torque becomes smaller than the opposing torque, the armature will slow down till the two are equal.

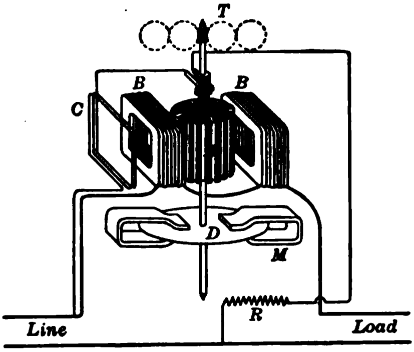


Fig. 383.

The required counter torque may be produced by the magnetic drag acting on a light metal disk attached to the armature shaft when the disk rotates between the poles of a pair of permanent magnets, *MM*, Fig. 383. The magnetic flux through the disk *D* being constant, the eddy current induced in the rotating disk is proportional to the speed of rotation. But the counter torque is proportional to the product of the flux and the eddy current. It follows that the speed of the disk is proportional to the driving torque.

Since the driving torque is proportional to the electric power that is being supplied to the line, it follows that in any given time the number of turns made by the disk is proportional to the electric energy supplied to the line. Geared to the shaft of the disk is a revolution counter *T*. The face of the dials of the revolution counter is shown in Fig. 384. The reading of these dials is 207700 kilowatt-hours.

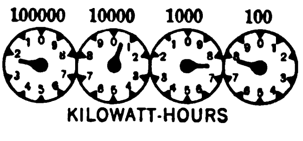


Fig. 384.

Questions

1. Draw a diagram of the windings of a direct-current two-pole shunt motor whose armature will rotate in the clockwise direction when the left pole is north and the brush on the upper side of the commutator is negative.

2. Explain why a series motor with laminated field and armature cores is more efficient when operated on direct current than when operated on alternating current.

3. Show (a), that for given connections a series-wound direct-current motor and dynamo will run in opposite directions; (b), that a shunt-wound machine will run in the same direction whether used as a motor or as a dynamo; (c), that the direction of rotation of a differential compound-wound motor will depend upon the relative strengths of the series and shunt field coils.

4. For all modern motors, weakening the field raises the speed. But if the armature be of sufficient resistance, weakening of the field will decrease the speed. Explain.

5. The fields of a certain motor are separately excited by means of a battery. On joining the motor brushes with the brushes of a direct-current series-wound dynamo, it is found that the direction of rotation of the motor armature reverses periodically. Explain.

6. Show how the counter electromotive force of a motor can be determined.

7. When a given motor is running more rapidly, less power is absorbed than when it is running slowly. Explain.

353. Rotary Converters.—By means of an electric motor connected to a dynamo, energy from an electric circuit can be transformed into mechanical work and retransformed into electric energy of different electromotive force. Such motor-generators are also used for the transformation of alternating currents into direct, or the reverse.

The motor and the dynamo need not be separate machines. Machines are in common use consisting of a stationary multipolar field energized by a separate direct current, and a revolving armature provided with a commutator on one end and collector rings on the other. Such a machine is called a rotary converter. Rotary converters are made which have an efficiency as high as 94 per cent. They are much used in interurban electric railway substations for transforming high-tension alternating current into lower tension direct current.

354. Transformers.—For transforming an alternating current into another of different electromotive force we have a very simple device with no moving parts. Consider a closed iron ring wrapped with two coils of insulated wire. Let one coil be connected to a source of alternating electromotive force. This coil is called the " primary " coil, and the other is called the " secondary " coil. While the current in the primary is changing in magnitude there will be a change of the magnetic flux in each turn of both the primary and the secondary coils.

If the change of magnetic flux in each turn be the same, the electromotive force thereby induced in each turn will be the same. (To ensure each turn of wire of the two coils being subjected to the same change of flux, the two coils are not apart as represented in the diagram, Fig. 385, but are either wound one on top of the other, or the turns of one distributed amongst the turns of the other.) Representing the number of turns in the primary and in the secondary coils by n_1 and n_2, respectively, and the electromotive forces induced in the two coils by e_1 and e_2, respectively, it follows that if the rate of change of flux is the same for each turn of wire,

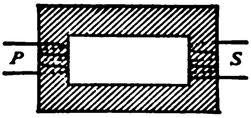


Fig. 385.

$$\frac{e_1}{e_2}=\frac{n_1}{n_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (217)$$

The electromotive force e'_1 impressed on the primary coil by the outside source sends current through this coil in opposition to the back electromotive force of self induction e_1. If the resistance of the primary coil be small, the energy lost in it by heat will be negligible, and the back electromotive force e_1 will be nearly equal to the impressed electromotive force e'_1. That is,

$$\frac{e'_1}{e_2}\doteq\frac{n_1}{n_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (218)$$

That is, by using the proper number of turns of wire in the two coils of such a device, it is possible to develop any desired electromotive force in the secondary by means of any given electromotive force impressed on the primary.

A device consisting of two coils wrapped on an iron ring by which an alternating electromotive force of one value can be transformed into an electromotive force of a different value is called a *stationary transformer*. If n_1 be greater than n_2, e_2 will be smaller than e'_1, and the transformer is said to "step-down" the electromotive force. If [illegible] is less than [illegible], [illegible] will be larger than [illegible], and the transformer is said to "step-up" the electromotive force. The same transformer may be used either as a step-down or as a step-up transformer. The primary of a step-down transformer is usually called the "high-tension coil," and the secondary is called the "low-tension coil." In the case of a step-up transformer the primary is the low-tension coil and the secondary is the high-tension coil. A modern transformer without the containing case is shown in Fig. 386.

In practice the primaries of transformers supplying various buildings are connected in parallel across the distributing lines from the central station. Lamps and motors are connected in parallel across the secondary. When the secondary circuit is open, that is, when no current is flowing in the secondary circuit, the primary acts as a choke coil and the current in the primary is consequently very small.

Fig. 386.

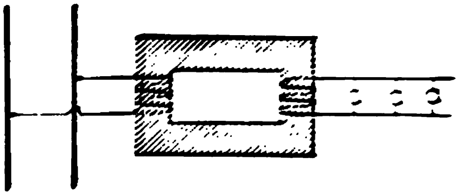


Fig. 387.

On connecting lamps or motors across the secondary line an alternating current will flow through the secondary coil. It can be shown that while the primary current is rising in value, the secondary current is falling; and while the primary current is falling in value, the secondary is rising. The changing magnetic flux, produced by the varying secondary current, induces in the primary coil an electromotive force in the direction opposite to the back electromotive force of self induction in that coil. This diminution in the opposition to the impressed electromotive force permits a greater current to flow in the primary coil. Thus, when current is taken from the secondary, the primary current increases and more power is taken from the main line. Whatever amount of power is taken from the secondary, a practically equal amount is taken from the main line. In this sense a stationary transformer is said to be "self-regulating." Stationary transformers are made in which the loss of power due to resistance, eddy currents and hysteresis is so small that at certain loads the power delivered by the secondary is as much as 98 per cent of the power supplied to the primary.

Since induced electromotive force is proportional to the rate of change of magnetic flux, and the rate of change of magnetic flux is proportional to the frequency, it might be supposed that the secondary electromotive force of a transformer would depend upon the frequency. But it must be remembered that with increase of frequency there is an increase in the primary back electromotive force and a consequent diminution of primary current and magnetic flux. In actual transformers the diminution of flux due to increased frequency is such that the secondary electromotive force is independent of the frequency.

355. Stationary Transformer Current Relations.—Since the power in the secondary coil so nearly equals that in the primary, we may write

$$e_1' i_1' \doteq e_2 i_2,$$

where i'_1 is the current in the primary coil due to the impressed electromotive force e'_1.

Whence,
$$\frac{i'_1}{i_2} \doteq \frac{e_2}{e'_1}.$$

Consequently, from (218)

$$\frac{i'_1}{i_2}\left[\doteq \frac{e_2}{e'_1}\right] \doteq \frac{n_2}{n_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (219)$$

Or, in words, the ratio between the current in the primary coil due to the electromotive force impressed by the outside source, and the current in the secondary, is very nearly the same as the ratio between the number of turns of wire in the secondary coil and the number of turns in the primary.

356.—The current relations in a stationary transformer will now be considered in more detail. The current in the secondary coil of a stationary transformer sets up a magnetic flux opposed to the flux due to the current in the primary coil. The decrease thereby produced in the back electromotive force of self induction e_1 in the primary coil will cause an increase in the primary current. This current will increase until the increase of the magnetic flux thereby produced develops a back electromotive force of self induction in the primary coil which is again nearly equal to the impressed electromotive force, that is, until the resultant magnetic flux in the core has nearly the value it has when the secondary current is zero. Now, when the secondary current is zero, that is, when the secondary coil is on open circuit, the core flux is due to the magnetizing current, i_1, produced by the resultant of the impressed electromotive force and the back electromotive force of self induction in the primary coil. Therefore, whatever the value of the secondary current, the core flux is nearly the same as that which would be produced by a current i_1 in the primary coil.

When the secondary coil is closed, the alternating electromotive force e_2 will develop in it an alternating current i_2. The magnetic flux thereby produced will induce in the primary coil a current i'_1 in the same direction as i_1, and in the opposite direction to i_2. The core flux is now equal to the sum of the fluxes due to i_1 and i'_1 diminished by the flux due to i_2. And since the combined magnetic effect of the two coils is the same whatever the current in the secondary; and since when the secondary circuit is open, the core flux is that due to the magnetizing effect of i_1; we can write,

Total core flux at any instant [= flux due to i_1 + flux due to i'_1 − flux due to i_2] = flux due to i_1.

Whence, the flux due to i'_1 = flux due to i_2.
That is, the magnetizing effect of i'_1 equals that of i_2.

Now the magnetizing effect of i'_1 is measured by $n_1 i'_1$, and that of i_2 by $n_2 i_2$. Whence, ignoring algebraic signs,

$$n_1 i'_1 \doteqdot n_2 i_2$$

or

$$\frac{i'_1}{i_2} \doteqdot \frac{n_2}{n_1}, \qquad \text{. . . . . (219)}$$

where i'_1 equals the current in the primary coil due to the electromotive force impressed on the coil by the outside source, i_2 represents the current in the secondary coil, and n_1 and n_2 represent the number of turns of wire in the primary and secondary coils, respectively.

357. The Number of Turns in the Primary Coil.—An inspection of (218) might lead one to suppose that the number of turns in the primary coil of a stationary transformer is unimportant so long as the number of turns in the secondary is properly selected with reference to the primary. There is nothing in this equation to suggest that it might not be entirely satisfactory to have the primary coil of a step-up transformer consist of a single turn. A little consideration, however, shows that for a given transformer it is not well to diminish the primary turns below a certain number.

The core flux must be sufficient to induce in the primary coil a back electromotive force practically equal to the impressed electromotive force. The magnitude of the core flux depends upon the magnetizing current in the primary coil, the number of turns in the primary coil, and upon the magnetic quality and sectional area of the iron core. As the number of turns in the primary is decreased, the magnetizing current and the sectional area of the core must be increased. A large magnetizing current implies a large power absorption even with the secondary on open circuit. A core of large section implies a transformer of large initial cost. It is therefore expedient to use such a number of primary turns as will avoid the large magnetizing current and large core that otherwise would be necessary.

358. The Ruhmkorff Induction Coil.—A direct current of moderate electromotive force can be transformed into either an alternating or a unidirectional intermittent current of high electromotive force by means of a device now to be described. The device consists of a straight iron core of soft iron wires on which is wrapped a primary coil of few turns of thick insulated copper wire, and a secondary coil of many turns of finer wire thoroughly insulated from the primary coil. The primary circuit includes an automatic contact breaker, one form of which consists of an iron hammer *H*, Fig. 388, attached to one end of a flat spring, and an adjustable screw *A*. The ends P_1 and P_2 of the primary coil are joined to the terminals of a battery or other source of direct current.

On bringing the adjustable screw *A* into contact with the hammer *H*, the primary circuit is completed, the core is magnetized, an electromotive force is induced in the secondary, the hammer is attracted toward the core, the primary circuit is broken, and an electromotive force in the reverse direction is induced in the secondary. The hammer then springs back into contact with the screw *A* and the same series of actions is repeated. An alternating current of high electromotive force is thus produced. The instrument is essentially a step-up stationary transformer supplied with an intermittent unidirectional current. The magnitude of the secondary electromotive force depends upon the electromotive force impressed on the primary, the ratio of the number of turns in the secondary coil to the number in the primary coil, and upon the suddenness of the change of the core flux on the " make " and on the " break " of the primary current.

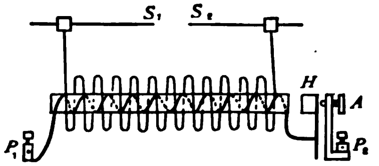


FIG. 388.

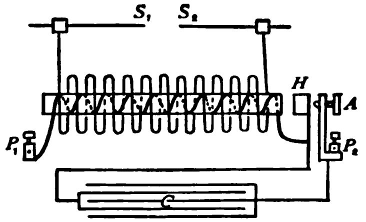


FIG. 389.

The suddenness of the alteration of the core flux is greatly diminished by the opposition to the change of primary current developed by the self induction of the primary coil. In fact, on the " make " an electromotive force is induced in the direction opposite to the impressed electromotive force. This retards the increase of flux and so diminishes the electromotive force induced in the secondary. And on the " break," an electromotive force is induced in the primary in the direction of the current being broken. This causes a spark across the widening gap between the hammer and the adjusting screw. Thus the diminution of core flux is retarded and the electromotive force induced in the secondary is kept low. These effects on the " break " are minimized by means of a condenser connected across the spark gap as illustrated in Fig. 389.

A spark will not jump across a gap until the potential difference at the terminals of the gap reaches a certain magnitude depending upon the width of the gap. Without a condenser across the spark gap, a spark will follow the retreating hammer. But with a condenser across the gap, the condenser plates must be raised to the sparking potential difference before a spark will occur. With a condenser of proper capacity, before the condenser plates have been raised to a sufficiently high potential difference, the gap will have become so wide that a spark will not occur. The primary circuit being still open, as soon as the condenser is charged, it will discharge through the primary coil in the direction opposite to the current being broken. The primary current being thus suddenly reversed, the core flux is thereby suddenly reduced to zero and reversed in direction. Consequently, on the " break," with a condenser across the spark gap, the core flux changes very suddenly from a high value in one direction to a high value in the opposite direction. The electromotive force of the secondary on the " break " is consequently very large. On the " make," the condenser exerts an inappreciable effect on the magnitude of the secondary electromotive force.

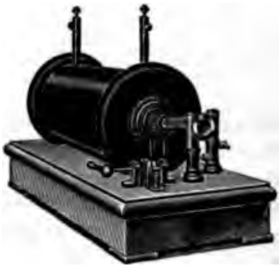


FIG. 390.

With a small distance between the terminals of the secondary, a thin spark will pass when the primary current is made, and a much thicker spark in the opposite direction when the primary current is broken. With a longer distance between the terminals of the secondary, sparks will pass only when the primary is broken.

The speed of the change of the core flux is also increased by the use of the straight iron core rather than by an iron ring such as is used in the stationary transformer.

In making large induction coils, manufacturers use in the secondary coil about one mile of wire per inch of spark length between its terminals.

359. The Autotransformer.—For small transformers of low transformation ratio, economy of construction and efficiency of operation can be obtained by having all of the turns of both primary and secondary coils in series, as illustrated in Fig. 391. A transformer in which the same coil is used for both the primary and the secondary is called an autotransformer. If used as a step-down transformer, the entire coil *abc* would constitute the primary, and the part *bc* the secondary. Representing the number of turns in *abc* by n_1, and the number in *bc* by n_2, (217),

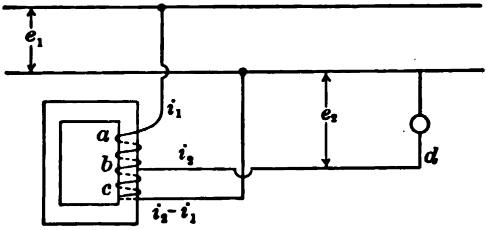


Fig. 391.

$$\frac{e_1}{e_2} \doteq \frac{n_1}{n_2}.$$

Representing the current in *ab* by i_1, and that in *bd* by i_2, (219),

$$\frac{i_2}{i_1} \doteq \frac{n_1}{n_2}.$$

n_2 is less than n_1. Hence, when the secondary is loaded, i_2 is greater than i_1, and the current in *bc* will have the value i_2-i_1.

Step-down autotransformers are used for reducing the potential difference applied to alternating current motors during the time of starting. A step-up autotransformer constitutes a part of a common form of ignition magneto used on automobile and other gasolene engines.

360. Electric Resonance.—In Fig. 392 are represented a few turns of thick wire *P* connected in series with a condenser *C* and the secondary of an induction coil *I*. Within the helix of thick wire there is another coil *S*. Each time the primary circuit of the induction coil is broken, the condenser *C* becomes charged. If the potential difference of the two condenser plates becomes sufficiently great, a discharge occurs at the gap *G*. The discharge is so sudden that a considerable electromotive force is induced in the secondary coil *S* even though the number of turns in this secondary be not very great.

Under certain conditions now to be considered, the electromotive force developed in the secondary coil may be very greatly increased. Suppose that just before the discharge takes place across the gap *G*, the upper plate of the condenser is charged positively and the lower plate negatively. If there were no inductance in the circuit, the discharge would bring both plates to the same potential and would then cease. But the inductance of the coil *P* keeps the current flowing until the upper plate of the condenser becomes charged negatively and the lower plate positively. This new charge in the condenser soon stops the discharge to the left through *G*, and starts another discharge to the right. The inductance of *P* keeps that discharge going until the condenser is again charged as in the beginning. This action may be several times repeated, the frequency with which the discharge alternates in direction depending on the inductance and capacitance. A large inductance tends to keep the current flowing for some time in one direction. If the capacitance is large, a considerable

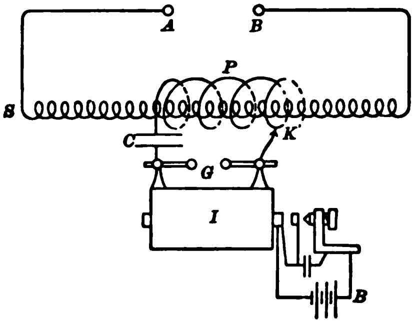


Fig. 392.

charge must flow into the condenser before the current is reversed. Thus, the smaller the capacitance and the smaller the inductance, the more frequent will be the electric oscillations in the primary circuit *P*.

If the natural period of the oscillations in the secondary is the same as the period of the oscillations in the primary, the amplitude of the oscillations in the secondary will be very great. Now the period of an electric oscillation depends upon the capacitance and self inductance of the circuit. By moving the contact point *K* along the primary coil, the self inductance of the primary circuit can be " tuned " to the period of the secondary—that is, can be adjusted till the two circuits are in electric resonance. Such an air-core transformer capable of producing high-frequency high potential discharges is called a Tesla coil or oscillation transformer. By this device, potentials can readily be obtained that are 10,000 times as great as that of the battery *B*.

CHAPTER XXI

THE ELECTRON HYPOTHESIS

361. Discharge through Gases.—At atmospheric pressure, the potential difference required to cause an electric discharge across a given distance depends upon the nature of the gas and upon the shape of the terminals of the gap. To cause a spark to start in air across a gap of one centimeter requires a potential difference of 8670 volts if the gap terminals are metal points, 20,670 volts if spheres of 0.25 cm. radius, and 27,810 volts if spheres of one centimeter radius. To maintain sparking requires much smaller potential differences than to start a spark. At higher pressures, the potential differences necessary to produce a discharge are proportional to the product of the spark length and the gas pressure. If the gas pressure is gradually reduced below the atmospheric pressure, the spark potential at first decreases, and then at lower pressures increases. At very low pressures no discharge occurs.

The appearance of a discharge through a gas at pressures much below that of the atmosphere changes considerably with change of pressure. If a sufficient electromotive force be impressed on electrodes sealed in a glass tube and the tube be gradually evacuated, there will be observed a carmine streak of light extending from one electrode to the other when the pressure is about 60 mm. of mercury. At a pressure of about 5 mm. of mercury the cathode, (negatively charged electrode), is surrounded by a faint violet glow and a faint pink haze extends from the anode (positively charged electrode), nearly to the cathode glow. The dark gap separating the cathode glow *G* from the positive column *C*, Fig. 393, is called Faraday's dark space.

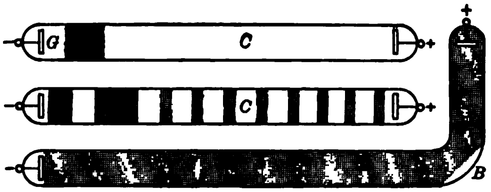
At a lower gas pressure, say one millimeter of mercury, a second gap called Crooke's dark space is observed. This gap *C*, Fig. 394, is between the cathode and the cathode glow. The positive column occupying three-fourths or more of the length of the tube now consists of alternate bright and dark striæ.

As the gas pressure is further diminished, the Crooke's dark space lengthens, till at pressures as low as 0.001 mm. of mercury it fills the entire tube. The space within the tube is now nonluminous, but the walls of the tube opposite the cathode are brilliantly

Fig. 393.

Fig. 394.

Fig. 395.



phosphorescent. A bright spot *B*, Fig. 395, directly in front of the cathode, is especially striking.

362. Electrons and Cathode Rays.—When electricity is passed between two electrodes in a glass tube evacuated to about one millionth of an atmosphere, a bright fluorescent spot appears on the glass surface opposite the cathode. An obstacle placed between the cathode and the fluorescent spot casts a shadow and is acted upon by a force directed away froom the cathode. If a suitable electrode joined to an electroscope be placed in front of the fluorescent spot, the electrode will be found to be negatively charged. These experiments indicate that negatively charged particles leave the cathode and proceed in straight lines. If the tube be placed in either a magnetic field or an electric field, the fluorescent spot will move to one side, and in the direction required by the above hypothesis.

Numerous other experiments seem to indicate definitely the existence of minute negatively charged particles. They have a mass of 1/1845 that of a hydrogen atom, or about 10^{-27} gram. They have a diameter of about $2(10^{-13})$ cm., and a charge of about $5(10^{-10})$ electrostatic units. Their speed varies as the square root of the potential difference of the electrodes. With high potential differences, speeds as high as one-third that of light have been measured. These minute negatively charged particles are called *electrons.* A stream of electrons set into rapid motion of translation by an electric field of force is called a *cathode ray.*

At a spot where cathode rays impinge on matter there are developed heat,* light, force, a negative charge and Roentgen or X-rays. The passage of a cathode ray through a gas greatly increases the electric conductivity of the gas. A cathode ray is accompanied by a magnetic field in circles in planes normal to the ray. Since electrons are charged bodies, electrons and cathode rays are acted upon by forces when in electrostatic fields. Electrons are repelled by other electrons and by all negatively charged bodies. They are attracted by all positively charged bodies.

Electrons are emitted by any body charged to a sufficiently low electric potential and by bodies raised above a red heat. Electrons emitted by a hot body are often called *thermions.* The rate of emission of electrons is increased when the temperature is increased, when the electric potential of the substance is decreased, and when the pressure of the surrounding gas is decreased. Electrons are emitted by conductors when exposed to light of sufficiently high frequency. This phenomenon is called the photoelectric effect. Radium spontaneously emits electrons, called beta particles, having speeds varying from about 0.3 to 0.98 that of light. Electrons are also emitted during some chemical reactions.†

The ultimate nature of the electron is unknown. In very general terms it is sometimes described as a localized condition of the ether. This means that the electron is conceived to consist of a portion of ether different in some unspecified manner from the surrounding ether. A portion of ether having different density or different elasticity than the surrounding ether, and a portion of the ether in vortex motion, are examples of localized conditions of the ether. None of these, however, serves any useful purpose in picturing an electron.

In diagrams, we indicate the direction of an electric current by an arrow-head. It is convenient to indicate an electron flow by a double-pointed arrow-head. For a long time the direction of motion of positive charges has been called the direction of the current. Thus the direction of the electric current is opposite to the electron flow.

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* By the impact of cathode rays platinum (melting-point about 1750° C.) and tungsten (melting-point about 3300° C.) have been melted.

† For example, during the oxidation of phosphorus in moist air, and during the combination of nitric oxide with large excess of chlorine.

363. Matter and Electricity.—If whatever opposes any change of motion be called inertia, then the opposition which an electric current offers to a change in magnitude, called self-induction, may be considered to be due to inertia. Since electricity possesses this fundamental characteristic of matter, an electric charge is said to possess electromagnetic mass. From theoretical considerations, Maxwell showed that electromagnetic mass increases when the speed of the charge increases, becoming very great when the speed approaches that of light. It is experimentally found that the inertia of electrons is measurably greater at speeds approaching that of light than at lower speeds. It may be that the entire mass of an electron is of electromagnetic origin. Many facts suggest that matter and electricity are identical, but at present we are not certain.

364. Vacuum Tube Rectifiers.—The fact that electrons are emitted by metals raised above red heat is utilized in devices for rectifying alternating currents. In Fig. 396 is represented a highly evacuated bulb provided with a lamp filament F operated by a battery B_1, and a metal plate P connected to the positive pole of a battery B_2. When the filament is heated above redness electrons are emitted. The positively charged plate, by attracting these electrons, establishes an electron flow from the filament to the plate. This means that an electric current is flowing from the plate to the filament. If, however, the plate be negatively charged, no electrons will be attracted toward the plate and there will be no electron flow and no electric current. If the battery B_2 be replaced by a source of alternating current, there will be an electron flow only when the plate is charged positively. Thus, by eliminating the alternate oscillations, this vacuum tube device rectifies or renders undirectional an alternating current. Highly exhausted tubes of this sort, called kenetron tubes, can be obtained for rectifying alternating currents up to 180,000 volts. The current carried by a single tube is but a fraction of an ampere, but tubes can be arranged in multiple so that the group will carry several amperes.

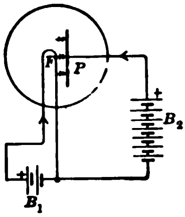

FIG. 396.

With the filament temperature constant, a constant stream of electrons will be emitted by the filament. The rate at which electrons will flow toward the plate depends upon the electric potential of the latter. With the plate at a certain potential, the electrons will flow toward the plate as fast as emitted by the filament. The maximum electron flow is called the saturation current of the tube for the particular filament temperature. For a higher filament temperature the saturation current will be greater.

365. Three-electrode Vacuum Tube.—By adding to the two-electrode vacuum tube, just described, a grid of wire or perforated metal G, which can be charged to various potentials, Fig. 397, the electron flow between the filament and plate, and consequently the electric current in the plate circuit B_2PFB_2, can be controlled within wide limits. An increase of the grid potential causes an increase in the speed of the electron flow and consequently an increase in the plate current; whereas a decrease of the grid potential, by producing a diminution in the speed of the electron flow, causes a decrease in the plate current. Thus, the relation between the plate current and the electromotive force of the battery B_2 is not the simple one expressed by Ohm's Law. The plate current is not proportional to the electromotive force of B_2.

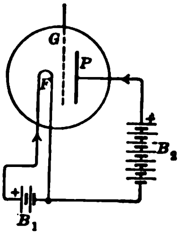

FIG. 397.

For a particular three-electrode vacuum tube with the plate maintained at a given potential and the filament maintained at a given temperature, the relation between the plate current and the potential of the grid relative to the filament is shown in Fig. 398. An inspection of this curve shows that if the relative grid potential be that represented by A, a given diminution of grid potential will produce a much greater change in the plate current than will an equal increase of the grid potential. Again, if the relative grid potential be that represented by B, a given diminution of the grid potential will produce a much smaller change in the plate current than will an equal increase of the grid potential. Thus, when operating at either bend of the curve, Fig. 398, if there be impressed on the grid an alternating electromotive force, the current in the plate circuit will be almost completely rectified.

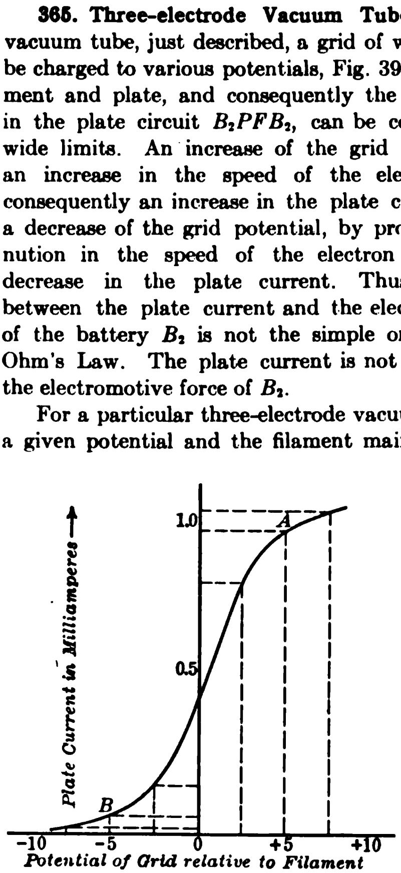

FIG. 398.

If the grid were removed, the space formerly occupied by the grid would be at a certain electric potential. When the grid is present, suppose its potential be maintained higher than this value. The charged grid now increases the rate of electron flow, and hence the electric current, in the plate circuit. The rectified electromotive force and current in the plate circuit B_2PFB_2 may be made many times as great as the alternating electromotive force and current supplied by A, Fig. 399. The three-electrode vacuum tube is much used as an amplifier of minute currents in long distance telephony and in wireless telegraphy. One common form is called the "audion."

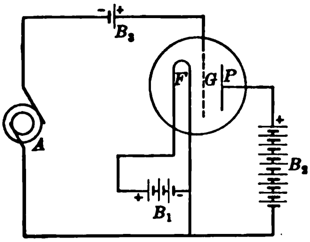

Fig. 399.

366. Structure of the Atom.—A satisfactory atom model must correlate such diverse facts as the relation between the chemical properties and the atomic weights of the elements shown in Mendeléeff's Periodic Table, the laws of electricity, magnetism, radiation, as well as the arrangement of the lines in the spectrum. Although no atom model thus far imagined has met with general acceptance, the atom models built of electrons are not only very promising, but even at the present time are also highly useful for the purpose of fixing the ideas upon an easily apprehended mechanism.

All atom models built of electrons consist of a positively charged nucleus and one or more negatively charged electrons. In an uncharged atom the total negative charge of the associated electrons equals the charge of the positive nucleus. As we do not know whether an electron is a speck, a thin ring, or some other shape, we can refer to the diameter in only an arbitrary sense. But assuming that the mass of an electron is entirely electrical, we can compute the diameter of the sphere over which the known charge of an electron would need to be distributed in order that it may have the known mass of an electron. This diameter is found to be about $4(10^{-13})$ cm. Now the diameter of a hydrogen molecule is supposed to be about $2(10^{-8})$ cm. Consequently, the "equivalent diameter" of an electron is about 0.00002 the diameter of a hydrogen molecule. There are reasons for believing that the

" equivalent diameter " of the positive nucleus of no atom is larger than 0.0001 the diameter of the atom. The atom is highly porous. Nearly all of the volume occupied by an atom is cavity. If a number of bees are flying about a sparrow poised at the center of a spherical cavern 2000 ft. in diameter, the relative sizes of cavern, sparrow and bees are about the same as those of an atom, its positive nucleus and its electrons.

The number of negative electrons in an atom is conceived to vary from one to nearly 100, depending upon the atomic weight. The hydrogen atom is supposed to consist of a single negative electron and a positive nucleus with an equal charge. The gold atom is supposed to consist of 79 negative electrons and a positive nucleus with a charge equal to the total charge of the associated electrons. It may be that the positive nucleus of an atom consists of positively charged particles separated from one another by considerable distances, and each with a charge equal to that on a negative electron.

The reason why the negative electrons of an atom do not fall into the positive nucleus is assumed to be that the electrons are revolving about the nucleus like the planets about the sun. Different electrons are assumed to be revolving in orbits of different radii. These orbits can be changed. A diminution in the radius of an orbit of an electron is assumed by Bohr to be accompanied by a loss of energy of rotation. Radiation is considered to be due to such a diminution in the radii of the orbits of electrons.

Electrons close to the positive nucleus of an atom would be more strongly held than electrons farther away. The loosely attached electrons that can be readily drawn away by outside forces may either become attached to other atoms or remain free. An atom that has lost an electron is positively charged, and one that has gained an electron is negatively charged. A charged atom or group of atoms is called an ion. Gases may be ionized by cathode rays, X-rays, hot bodies and radioactive substances.

367. X-Rays.—In 1895 Roentgen discovered that at the place where cathode rays strike a solid there is emitted something which is invisible but which affects photographic plates, causes certain crystals to fluoresce, and which is readily transmitted by many substances that do not transmit light. The nature of these new rays being unknown, he called them "X-rays." The fact that metals, bones and some organs are more opaque to X-rays than other tissues of the body is now much used by surgeons in the location of bullets and in the examination of fractures and diseased tissues. For example, if a fractured arm be placed for a few seconds between a photographic plate and a source of X-rays, and the plate be developed, the bones will show as dark shadows sharply distinguished from the surrounding flesh. Fig. 400 is an engraving

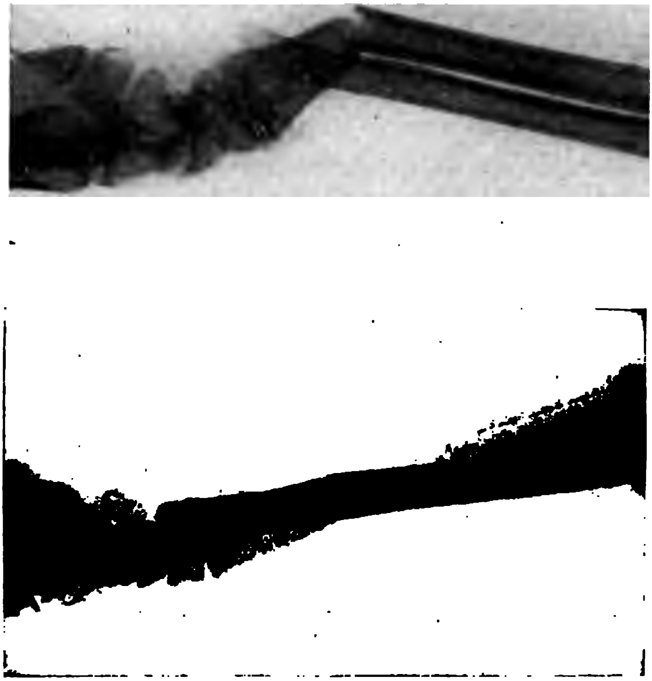

Fig. 400.

from X-ray photographs of a broken wrist. The upper picture shows exactly what was the matter with the wrist, and the lower picture shows the success of the reduction of the fracture.

When required in physical examination, X-rays are produced by a special form of vacuum tube. The penetrating power or " hardness " of the rays is increased by diminution of gas pressure within the tube and by an increase in the potential difference at the electrodes. At the present time the most powerful producer of highly penetrating X-rays is the Coolidge X-ray tube illustrated in Fig. 401. This is exhausted to a pressure less than one micron of mercury.* The cathode *C*, shown on a larger scale in Fig. 402, consists of a flat spiral of tungsten wire. When at a temperature of 2000° C., or above, and at a low electric potential, this cathode emits electrons copiously. For the purpose of concentrating the cathode ray on a small spot of the anode *A*, the spiral cathode is placed at the bottom of a shallow cylindrical cup. The anode, also called the anti-cathode and the target, is a single piece of tungsten. With the tube in the position shown in Fig. 401, the X-rays will be directed downward in the form of a solid cone having its vertex on the anode.

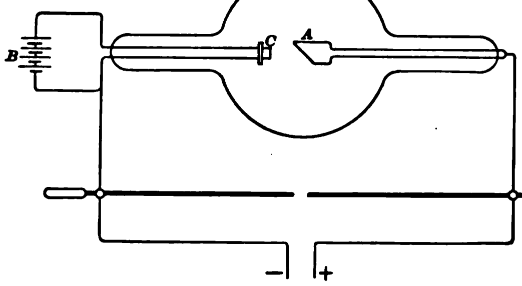

FIG. 401.

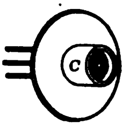

FIG. 402.

Besides being used by surgeons for the examination of broken bones, the location of foreign objects in the body, the detection of diseased lung tissue and the location of tumors and abscesses,

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* A micron is 0.001 mm.

X-rays are also used by customs inspectors for the detection of smuggled articles within innocent-appearing packages, the detection of flaws in metals, the distinguishing of diamonds from imitations, and the killing of bacteria.

For a long time the exact nature of X-rays has been in doubt. But it now appears to be well established that X-rays are very short transverse wave motions in the ether. They seem to differ from light waves only in the fact that their wave-lengths are much shorter. The wave-lengths of X-rays, thus far measured, lie between 10^{-9} cm. and 10^{-7} cm. The wave-lengths of light lie between $4(10^{-5})$ cm. and $7(10^{-5})$ cm. The shorter X-rays are more penetrating than the longer ones.

368. The Electron Hypothesis of Electricity.—Physicists are now agreed as to the following properties of electrons.

(*a*) All electrons have the same mass and this mass is very small compared with the mass of any atom.

(*b*) All electrons carry equal negative charges numerically equal to the charge carried by a monovalent ion in electrolysis.

(*c*) In gases electrons move from a region of low electric potential to a region of high potential with a speed depending upon the potential gradient (volts per cm.) and upon the pressure of the gas.

(*d*) A gas is rendered conducting by the passage of a stream of rapidly moving electrons.

(*e*) The impact of rapidly moving electrons on matter develops heat, light and mechanical force.

(*f*) Electrons are repelled by other electrons and by all other negatively charged bodies.

(*g*) Electrons are attracted by positively charged bodies.

(*h*) Electrons in motion are accompanied by a magnetic field which is in circles having their planes normal to the line of motion.

(*i*) Electrons are emitted by bodies at a sufficiently high temperature and by bodies at a sufficiently low electric potential. They are emitted spontaneously by a few substances at ordinary temperature even when uncharged. They are emitted during some chemical reactions and also by some substances when exposed to X-rays or to light waves of short wave-length.

From these properties and certain attributes ascribed to the electron, together with the mechanism of the electron atom, already considered, there has been constructed a very useful model of electrical processes called the electron hypothesis of electricity. Some applications of this hypothesis to the explanation of the simpler electrical phenomena will now be mentioned.

It is postulated that electrons are attracted with different forces by the positive nuclei of the atoms of different kinds of substance. From this would follow the fact that when dissimilar substances are separated from intimate contact, each body becomes charged, one positively and the other negatively to the same degree.

If a positively charged body be placed near an uncharged body, any loosely attached electrons in the latter would be drawn from one atom to the next toward the positively charged body. The end toward the positively charged body would be thereby charged negatively and the opposite end positively. This is the phenomenon of electrostatic induction.

An electric conductor is a substance containing loosely attached electrons. In a perfect insulator there would be no loosely attached electrons.

Electric conduction is assumed to consist in the passage of electrons. The positively charged ions from which the free electrons have parted do not contribute appreciably to the conduction because these relatively large bodies have negligible motion of translation in the direction of the electric force. Current strength is measured by the number of electrons passing any section of the conductor in unit time.

Gases are rendered conducting by the impact of cathode rays and X-rays. This effect is conceived to be due to the formation of free electrons by a separation of electrons from the atoms of the gas struck by the ray.

Electric resistance is the property whereby a conductor absorbs the kinetic energy of electrons and converts it into heat.

The property of a system which tends to set electrons into motion is called electromotive force. It is measured in ergs per unit charge.

The explanation of the electromotive force of a galvanic cell may be briefly indicated as follows. When plates of dissimilar metals are placed in water or certain solutions, each plate is assumed to go into solution in the form of positive ions. The name "electrolytic solution pressure" is given to the impelling force which is supposed to push ions into solution. When positive ions leave a plate, the latter becomes negatively charged. If the rates of electrolytic solution of the two substances composing the plates are different, one plate will send more positively charged ions into the solution than the other and in consequence will acquire a greater negative charge. Within the cell some ions will combine and form molecules, and some molecules will dissociate into ions. Some of these reactions involve an absorption of energy, and others involve a liberation of energy. In some cells the energy absorbed exceeds the energy liberated during these reactions. It is assumed that this excess of energy causes electrons within the cell to move toward the more negatively charged plate. If the two plates of such a cell be joined by an outside conductor, there will be a flow of electrons in the conductor from the more negatively charged plate to the less negatively charged plate. A galvanic cell or other seat of electromotive force is really an electron pump which sets electrons into circulation. There is no production of electricity. The electrons are simply set into motion.

It is assumed that acting on two charges traveling side by side through the ether there is a force urging them toward one another when the charges are similar, and a force urging them apart when the charges are dissimilar. The forces acting upon two charges due to their motion through the ether oppose the electrostatic forces acting upon them. It is supposed that the forces due to the steady motion of charged bodies through the ether are what are usually called magnetic forces. The magnetic forces acting upon moving charges increase with the speed of the charges. They become equal to the electrostatic forces acting upon the charges when the speed of the charges becomes equal to the speed of light.

The electrostatic force acting upon each of two parallel current-carrying conductors is zero because each conductor contains as many positive ions as free electrons. Due to the motion of the electrons, the two conductors are urged toward one another if the two electron streams are in the same direction, and they are urged apart if the two electron streams are in the opposite direction.

An electron moving across a magnetic field is acted upon by a force urging it out of its path. Hence electrons possess energy of position, or potential energy. The potential energy of an electron is found to be proportional to the magnitude of the magnetic field. Hence, a change in the magnitude of the magnetic field across which an electron is moving involves a change in the potential energy and in the speed of the electron. This change of speed constitutes a change of current. This is electromagnetic induction. The change of speed is accompanied by a change in the magnetic field about the moving electron, and is opposed by the inertia of the electron. The change in the magnetic field due to the changing speed is in such a direction as to be opposed by the change in the magnetizing field which produces it. This is the Lenz Law of electromagnetic induction.

369. Paramagnetism and Diamagnetism.—With the exception of iron, nickel and cobalt, all substances have a magnetic permeability near unity. A substance having a magnetic permeability less than unity is called diamagnetic, and one more than unity is called paramagnetic. The very strongly paramagnetic substances iron, nickel and cobalt are said to be ferromagnetic. The magnetic induction at a given place in a magnetic field is decreased by the presence of a diamagnetic substance, and increased by the presence of a paramagnetic substance.

In the electron hypothesis all magnetic phenomena are due to the motion of electrons. Magnetism is an aspect of electricity and is not a separate entity. When a substance is placed in a magnetizing field, two effects can occur. The speed of rotation of the electrons in their orbits may be changed, and the planes of the electron orbits may be turned. The resultant magnetic condition is conceived to be due to the combination of these two effects. The diameters of the orbits are not supposed to be changed sufficiently to produce any noticeable effect.

The possible motions, at any moment, of all the electrons of a given unmagnetized body can be resolved into two equal and oppositely directed coplanar rotations and a simple harmonic motion along a straight line normal to the plane of these circular components, Fig. 403. In this and in the two following diagrams, the axis of the circles has been taken parallel to the outside magnetic field.

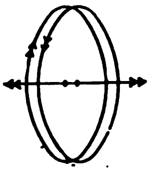

FIG. 403.

Due to the two component rotations there are two component magnetic fields, h_1 and h_2, Fig. 404, along the axis of the electron orbits. If the specimen is unmagnetized, $h_1=h_2$. On bringing the unmagnetized specimen into a magnetizing field H, Fig. 405, the moving electrons in each circular orbit will be acted upon by an electromagnetic inductive effect which will produce a decrease of the current in the orbit A and an increase in B. Corresponding to these changes, the axial magnetic field due to the electron flow in A diminishes to h'_1, and that in B increases to h'_2.

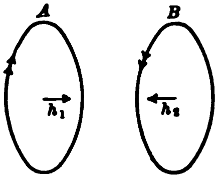

FIG. 404.

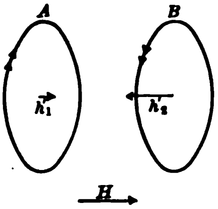

FIG. 405.

The magnetic field due to the rectilinear axial component of the electron motion, Fig. 405, is perpendicular to the magnetizing field H. Hence this magnetic field is unaffected by H. It follows that the magnetic induction due to changes in the speed of electrons in their orbits produced by a magnetizing field H equals $H+h'_1-h'_2+0$. This is less than the magnetizing field H. Hence, if no other effect occurred, the specimen would be diamagnetic.

To explain paramagnetism it is assumed that a magnetizing field will turn the planes of the electron orbits of some substances toward the perpendicular to the direction of the magnetizing field. If many electron orbits are turned till their planes are nearly normal to the magnetizing field, the induction will be large and in the direction opposite to that produced by the change in the speed of the electrons.

The resultant induction of the specimen would be due to the difference between the effect of changing the speeds of the electrons in their orbits and the effect of turning the planes of the electron orbits. If the first effect predominates, the specimen is diamagnetic. If the second effect predominates, the specimen is paramagnetic.

CHAPTER XXII

ELECTROMAGNETIC WAVES

370. Interaction of Changing Electric and Magnetic Fields of Force.—While a vertically downward electric field between A and B, Fig. 406, is diminishing, a magnetic field is produced in the horizontal plane and in the counterclockwise direction as one looks downward. In fact, a magnetic field of force is produced wherever there is a changing electric field of force (Art. 290). Maxwell showed that the direction of the magnetic field produced by a diminishing electric field is the same as that of the magnetic field about an electric current flowing in the direction opposite to the electric field; and that the direction of the magnetic field produced by an increasing electric field is the same as the direction of that about an electric current flowing in the same direction as the electric field.

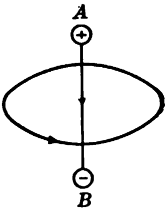

FIG. 406.

Henry and Faraday showed (Art. 333) that an electric current is produced in a conductor situated in a region where there is a changing magnetic field. Maxwell showed that, whether there be a conductor or not, an electric field of force is produced wherever there is a changing magnetic field. The lines of force of the electric field produced by a changing magnetic field in a dielectric are closed curves enclosing the magnetic field. In Fig. 407 the light lines represent the lines of force of the electric field produced by an increasing magnetic field represented by the heavy line. If the magnetic lines are in horizontal planes, the electric lines of force are closed curves in vertical planes. When the above represented magnetic field diminishes in value,

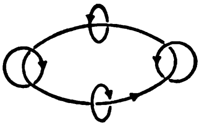

FIG. 407.

the electric field will be in the direction opposite to that represented in the figure.

In general, a changing electric field or a changing magnetic field produces a field of the other kind. As the field produced gains energy, the field which produces it loses energy.

371. Propagation of an Electromagnetic Disturbance through a Dielectric.—Suppose that a potential difference between two vertical rods *A* and *B*, Fig. 408, be suddenly diminished. During the diminution of electric field there is produced a magnetic field in the horizontal plane in the direction indicated in Fig. 408. While this magnetic field is increasing in value, it is encircled by an electric field having lines of force in vertical planes as indicated in Fig. 409. This increasing electric field produces an encircling magnetic field, Fig. 410, in the same direction as that produced by the decreasing electric field in Fig. 408. As the last formed magnetic field increases in value, it is encircled by another electric field, and so on. As these magnetic and electric fields are formed one after another, an electromagnetic pulse or disturbance is propagated through the dielectric medium.

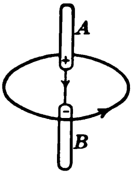

FIG. 408.

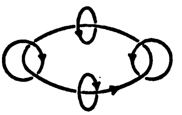

FIG. 409.

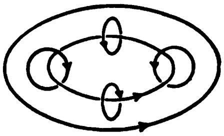

FIG. 410.

At the moment when the original electric field becomes zero, Fig. 408, the encircling magnetic field also becomes zero. The diminution of this magnetic field produces an encircling electric field in the direction opposite that shown in Fig. 409. This increasing electric field produces, in turn, a magnetic field in the direction opposite that shown in Fig. 410. It thus appears that the first expanding electromagnetic pulse is followed by another in which the magnetic and the electric fields are in directions opposite those in the first pulse.

372. Electromagnetic Radiation.—When the potential difference between two neighboring conductors becomes sufficiently great, an electric discharge occurs. The discharge does not cease when the two conductors become of equal potential. In fact, charge continues to move till the conductor which formerly was at lower potential becomes of higher potential than the other. This over-discharge is due to inductance or electric inertia. Another discharge now occurs in the opposite direction. This is followed by another in the original direction, and so on. The discharges in opposite directions constitute a series of electric oscillations. The system of conductors in which the oscillations occur is called an electric oscillator.

Each electric oscillation gives rise to an electromagnetic pulse which is propagated through the surrounding dielectric as described in the foregoing Articles. The series of electromagnetic pulses constitutes a train of electromagnetic waves. Electromagnetic waves traverse empty space with the speed of light.

The wave-length of the waves emitted by a linear oscillator, such as is considered in the preceding Articles, is about 2.5 times the combined length of the two rods. The frequency of the electric oscillations of any oscillator is increased when either the inductance or the capacitance is decreased. For an oscillator of small inductance and capacitance, the frequency is many millions per second. The amount of energy radiated per period by a linear oscillator is proportional to the square of the capacitance, the square of the change in voltage, and the frequency of oscillation.

On account of the resistance of the oscillator and the amount of energy radiated at every oscillation, the energy of each oscillation is much less than that of the preceding oscillation of the series. This fact is described by the statement that the oscillations are strongly " damped."

The component electric and magnetic fields of an electromagnetic wave train produced by the oscillatory discharge of a vertical linear oscillator are represented in Fig. 411. The magnetic lines of force due to the various oscillations are horizontal circles. The lines of the associated electric field are closed loops in vertical planes.

At the oscillator, when the electric field is maximum the magnetic field is zero. Thus, at the oscillator the two fields are 90° apart in phase. But at points farther from the oscillator than one-fourth of the wave-length of the wave, the electric and magnetic components are in phase, as represented in Fig. 412.

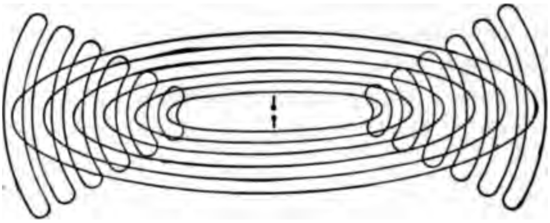

Fig. 411.

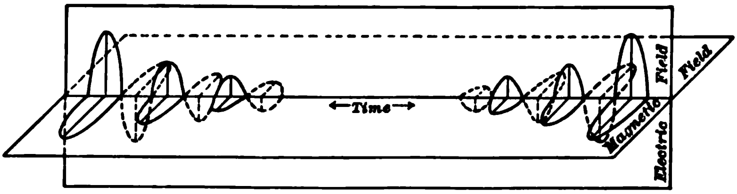

Fig. 412.

373. The Hertz Experiments.—If a magnetic field be cut by a conductor, an electromotive force will be induced in the conductor. If the magnetic field be oscillatory, the induced electromotive force will be oscillatory. If the conductor be divided by a narrow gap, each part will be a seat of oscillating electromotive force, and each side of the gap will rapidly alternate between high and low potential. When one side of the air-gap is at high potential, the other is at low potential. If the potential difference be sufficiently great, there will be a discharge across the gap. The conductor may be at rest and be cut by the magnetic field that travels with electromagnetic waves. Such a conductor then becomes our simplest detector of electromagnetic waves. The sensitivity of any detector is greatly increased by adjusting its capacitance and inductance till the natural frequency of electric oscillations upon it equals the frequency of the incident electromagnetic waves. This process of adjustment is called "tuning" the detector to resonance with the waves. A tuned detector is sometimes called an electric resonator.

The principal pioneer work in electromagnetic radiation was done by Heinrich Hertz in 1888. He found that electromagnetic waves are transmitted by nonconducting materials and reflected by conducting substances. To concentrate the electromagnetic energy he placed the linear oscillator in the focal line of a parabolic mirror made by bending a sheet of metal into the form of a parabola. He also used linear resonators provided with similar parabolic reflectors.

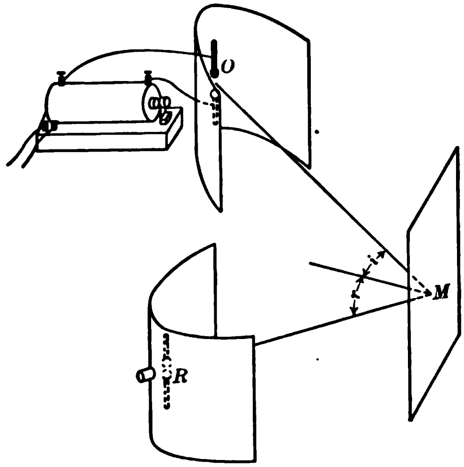

FIG. 413.

Fig. 413 represents a Hertz linear oscillator O provided with a parabolic mirror and induction coil, a linear resonator R provided with a parabolic mirror and a microscope for observing the spark-gap, and a plane mirror M. With this arrangement Hertz found that electromagnetic waves obey the ordinary laws of reflection (Art. 158).

If a wave be reflected back upon itself, a train of standing waves will be produced (Art. 154). Hertz placed a plane mirror normal to the axis of the parabolic mirror of an oscillator Fig. 414. On moving a resonator R back and forth between the oscillator and the plane mirror, he found places separated by equal distances at which the resonator gave long sparks, and intermediate places at which no sparks occurred. The places where sparks of maximum length occur are antinodes for both the electric and the magnetic waves, and the places where no sparks occur are nodes for both systems of waves (compare Art. 188). By this method the wave-length of the electromagnetic waves radiated by the oscillator can be measured. The frequency of the waves can be computed from the capacitance and inductance of the oscillator. Knowing the wave-length λ and the frequency n, the velocity can be computed by means of the relation, (110),

FIG. 414.

$$v = n\lambda.$$

By this method, the velocity of electromagnetic waves of all wave-lengths is found to equal that of light, 3×10^{10} cm. per sec.

Electromagnetic waves of lengths from one-fourth of an inch to several feet have been measured directly. Wave-lengths of many miles can be determined from the known velocity of electromagnetic waves and the frequency of a receiver in resonance with the wave. In making the determination, the capacitance and inductance of the receiver are adjusted till the latter is in resonance with the wave, and the frequency is then computed.

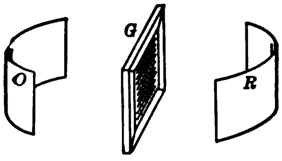

FIG. 415.

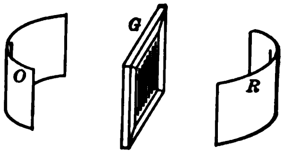

FIG. 416.

By means of large prisms of nonconducting materials, together with an oscillator and a resonator provided with parabolic mirrors, Hertz found that electromagnetic waves obey the laws of refraction, Art. 165.

Hertz placed in the path of electromagnetic waves a grating formed of parallel wires stretched across a plane frame. When the wires were perpendicular to the linear oscillator, Fig. 415, i.e., parallel to the plane of the magnetic field, the wave was freely transmitted. But on turning the grating 90° so that the wires were parallel to the linear oscillator, Fig. 416, i.e., perpendicular to the plane of the magnetic field, no wave was transmitted. This result shows that the train of magnetic waves is plane polarized. It follows that the vibrations of electromagnetic waves are transverse to the direction of propagation.

374. Radio or Wireless Telegraphy.—Hertz's experiments immediately suggested the possibility of transmitting signals by means of electromagnetic radiation. But in order that the radio method may be used over such great distances as those used in ordinary telegraphy it is necessary to have a much more powerful oscillator and a much more sensitive detector than used by Hertz. The power of the oscillator is made greater by increasing the potential differences and the capacitance. The energy dissipated in transmission on account of scattering by reflection and refraction is made smaller by increasing the wave-length.*

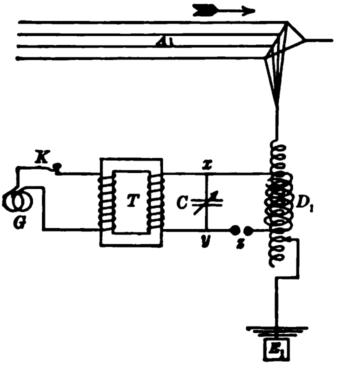

FIG. 417.

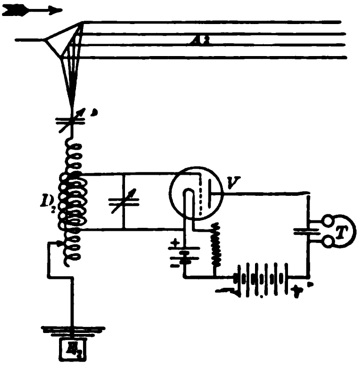

FIG. 418.

A common form of transmitting circuit used in radio telegraphy is indicated in Fig. 417. An alternating current generator *G* is in series with a key *K* and the primary winding of a step-up transformer T. The secondary winding is in series with a spark gap g and the primary of an air core transformer D_1. One end of the secondary winding of the air core transformer is joined to the earth and the other end is joined to an aerial or "antenna" consisting of a group of wires A_1 stretched above the earth. The electromagnetic waves radiated into space start from the secondary winding of the air core transformer and the attached antenna.

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* Waves used in radio telegraphy vary from 200 meters to 20,000 meters in length.

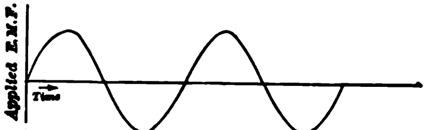

FIG. 419.

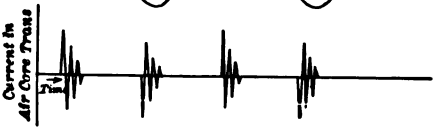

FIG. 420.

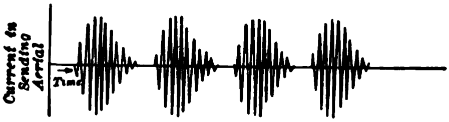

FIG. 421.

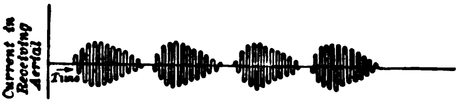

FIG. 422.

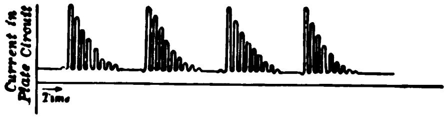

FIG. 423.

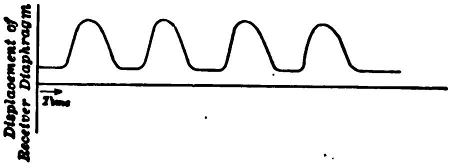

FIG. 424.

When the key is closed, the alternator impresses on xy a varying potential difference as represented by Fig. 419. The length of the air gap g is adjusted so that a discharge will occur when the potential difference at its terminals is somewhat less than the maximum developed by the transformer. This discharge produces in the circuit xyz a group of oscillations as represented in Fig. 420. These oscillations induce in the antenna another group of electromagnetic oscillations. If the natural frequency of oscillation in the antenna is the same as that in the spark-gap circuit, the amplitude of the potential variation in the antenna will be very great. The variation of current in the antenna is represented in Fig. 421. The groups of electromagnetic oscillations in the antenna send out groups of electromagnetic waves which traverse space with the velocity of light. The number of groups of waves per second is twice the number of cycles per second of the alternating generator *G*. The frequency of oscillation in each group depends upon the capacitance and inductance of the transmitting circuit and is usually more than 40,000 per second.

If the electromagnetic waves sweep across a distant antenna, A_2, Fig. 418, this antenna becomes the seat of an oscillating electromotive force of the same frequency as that of the waves, Fig. 422. The amplitude of the current induced by the oscillating electromotive force is greatest when the receiving antenna is in resonance (sometimes called "in syntony") with the electromagnetic waves. Even when the receiving antenna is tuned to the incoming waves the oscillations are so weak that special devices must be used for their detection. All of these devices make use of a telephone receiver. A telephone diaphragm has so much inertia that it does not respond to the rapid oscillations in a wave group. Nor would a sound sensation be produced if the ear drum were set into vibration with such a high frequency. But the frequency of the groups, which is about 1000 per sec., is within the range of a telephone and of the human ear. So that, if the waves of each group could be rectified, that is, rendered nearly unidirectional, the telephone diaphragm would receive about 1000 impulses per second during the time that the transmitting key *K*, Fig. 417, is closed. The most commonly employed device for rectifying the waves in each group is the three-electrode vacuum tube (Art. 365).

Fig. 418 shows one type of receiving circuit. In this figure, D_2 is an air core transformer, *V* is a three-electrode vacuum tube, and *T* is a telephone receiver. The variations in the receiving antenna current induce electromotive forces of the same frequency in the grid circuit. The changing charge on the grid affects the strength of the current from the plate to the electron-emitting filament. When there are no incoming waves, the grid is charged negatively by the electrons emitted by the heated filament, and the plate current has a certain value. When there are electromagnetic oscillations in the antenna, the negative charge on the grid becomes alternately increased and decreased with the frequency of the oscillations. When the negative charge on the grid diminishes, the plate current increases; when the negative grid charge increases, the plate current diminishes. The resulting rectified plate current is represented in Fig. 423.

The telephone diaphragm will be attracted by the first impulse. The frequency of the current oscillation is so much higher than that of the telephone diaphragm that the diaphragm will be attracted, in turn, by each of the succeeding strong impulses of the wave group before the diaphragm has reached the end of its swing. In fact, the diaphragm will be deflected but once for each wave group, as indicated in Fig. 424. Thus, the receiving telephone will vibrate with the frequency of the wave groups, that is, with double the frequency of the alternator at the sending station.

375. The Radio Direction Finder.—A vertical transmitting antenna emits electromagnetic waves of the same intensity in all horizontal directions. A vertical receiving antenna has the same sensitivity for detecting waves approaching it in all horizontal directions. The "inverted L" antenna, Figs. 417 and 418, has a certain degree of directive tendency. It emits waves of greatest intensity in the direction of the arrow in Fig. 417, and is most sensitive for receiving waves approaching it in the direction of the arrow in Fig. 418. There is another type of antenna called the "loop aerial" which has a much greater directive tendency. This ordinarily consists of a rectangular coil of about 12 turns of wire, about 6 feet on a side, mounted on a vertical brass column capable of rotation, as indicated in Fig. 425. The ends of the coil are attached to two insulated copper rings, *R*, on which press copper brushes connected to a sensitive detector system.

When the coil is rotated by means of a handle *H*, the intensity of the received signals varies, being greatest when the plane of the windings of the loop aerial passes through the transmitting station, and being nearly zero when the plane of the windings is perpendicular to the line from the transmitting station. A pointer *P* attached to the supporting column moves over a divided circle *S* fastened to a fixed table.

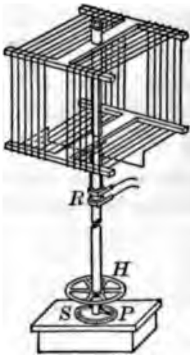

FIG. 425.

An important application of the loop aerial is in the determination of the position of vessels at sea. The coasts of the United States and the coasts of several of the European countries are dotted with stations devoted to this duty. Each station is equipped with a radio outfit consisting of a loop aerial and a detector system of high sensitivity. Such an outfit is called a "direction finder" or "radio compass."

Suppose that a ship is a few hundred miles from New York and the commanding officer wishes to know its latitude and longitude. He calls New York in the regular commercial manner and signals "what is my position?" The New York supervising operator signals to the ship "send the signal *MO* for two minutes," and simultaneously orders the neighboring radio compass operators to find the directions from their stations of the ship which is sending *MO* signals. At the end of the two minutes the directions of the ship from the various neighboring shore stations are telegraphed to the New York supervisor. The latter telegraphs them to the ship. The ship's officer then plots the bearings on a chart as in Fig. 426. The point of intersection of the various lines gives the position of the ship. The time from the instant when the supervising radio compass operator was called, to the instant when the ship receives the various bearings, seldom exceeds five minutes.

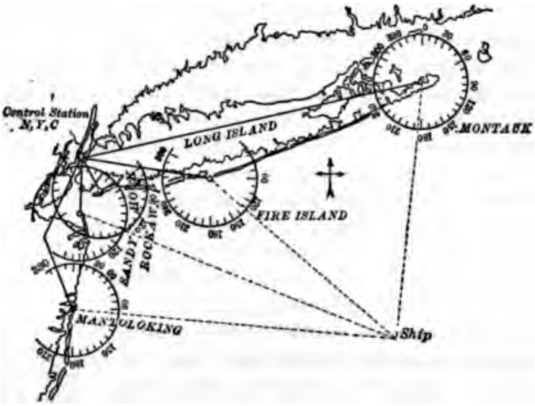

FIG. 426.

LIGHT

CHAPTER XXIII

THE NATURE OF LIGHT

376. Light is a Wave Motion.—That which is capable of effecting the sensation of sight is called *light*. That part of physics which deals with the phenomena and laws of light is called *optics*.

At the opening of the nineteenth century Thomas Young performed an experiment that is of fundamental importance in the theory of optics. This experiment can be repeated in the following manner. A narrow slit S, in a diaphragm A, is illumined by light from the sun or some other source L. In the diaphragm B there are two narrow slits, S_1 and S_2, close together, parallel to S, and equally distant from it. Light which traverses the slits S_1 and S_2 is received on the screen C.

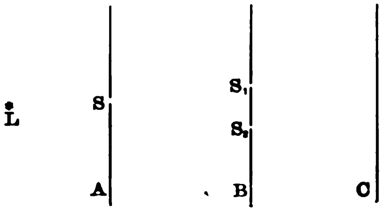

FIG. 427.

When the apparatus is arranged as indicated, it is found that the illumination of the screen C is not uniform or continuous, but that it consists of a series of narrow bright bands alternating with narrow dark streaks, with their lengths parallel to the slits in the diaphragms. It is observed that if either S_1 or S_2 is covered, the other remaining open, the illumination of the screen becomes continuous, that is, without bands. When both slits, S_1 and S_2, are open, the intensity of the illumination at a bright band is more than double the intensity at the same place when but a single slit is open. But in the space between two adjacent bright bands there is little or no illumination when light comes from both slits, whereas there is considerable illumination when light comes from a single slit.

The fact that the illumination at a given point due to light from two sources can be of nearly zero value, whereas when light comes from but one of the sources the illumination at the same point is great, indicates that light is a phenomenon capable of interference effects. So far as we know, interference effects can be produced only by wave motions. Therefore we conclude that light is a wave motion.

Fig. 428 is a reproduction of a photograph of a set of interference fringes obtained by using a photographic plate for the screen C, Fig. 427, and at B a diaphragm provided with two slits, one of which is partly closed by a narrow tongue as shown in Fig. 429. It will be noted that in the photograph, the regions illumined by light from two slits are crossed by bright and dark fringes, whereas the region illumined by light from a single slit contains no fringes.

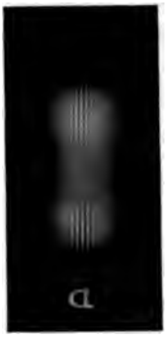

Fig. 428.

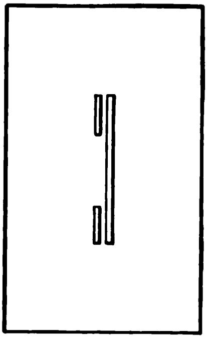

Fig. 429.

377. The Determination of the Wave-length of Light.—In Fig. 430, let L represent a point source that emits light waves of uniform length. Light from this source, after traversing the three slits of Young's apparatus, strikes the screen XX_2. At a point X, light from the slits S_1 and S_2 arrives in the same phase. Consequently, at this point there is reinforcement. At some point Y_1, where the distance S_1Y_1 differs by one-half wave-length from the distance S_2Y_1, light from the two slits arrives in opposite phases. There will here be destructive interference. At some point X_1, so situated that its distance from one slit is two half wave-lengths less than its distance from the other slit, there will again be reinforcement, and consequently a bright band. In the same way it is seen that there will be other dark bands on the screen at places Y_2, Y_3, etc., such that

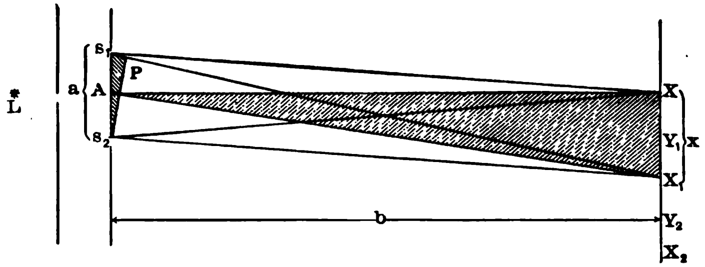

FIG. 430.

$$S_1Y_2 - S_2Y_2 = \tfrac{3}{2}\lambda$$

$$S_1Y_3 - S_2Y_3 = \tfrac{5}{2}\lambda, \text{ etc.,}$$

and that there will be other bright bands at places X_2, X_3, etc. such that

$$S_1X_2 - S_2X_2 = \tfrac{4}{2}\lambda$$

$$S_1X_3 - S_2X_3 = \tfrac{6}{2}\lambda, \text{ etc.,}$$

where λ represents the wave-length of the given disturbance.

This experiment affords a method for determining roughly the wave-length of the light that illumines the slits. The rationale of the method is as follows:

From X_1 lay off on X_1S_1 a distance X_1P equal to X_1S_2, and draw S_2P. Then, since $PX_1 = S_2X_1$, and since the angle PX_1S_2 is very small, S_2P is very nearly perpendicular to the lines S_1X_1, AX_1 and S_2X_1.

Since AX is perpendicular to S_1S_2, and S_2P is very nearly perpendicular to AX_1, the angles S_1S_2P and XAX_1 are very nearly equal. Moreover, AXX_1 is a right angle, and S_1PS_2 very nearly a right angle. Consequently, the triangles S_1PS_2 and X_1XA are very nearly similar.

It follows that

$$\frac{S_1P}{S_1S_2} \doteq \frac{X_1X}{AX_1}. \qquad \ldots\ldots\ldots \qquad (220)$$

Since X_1 is at the middle of the first bright band, and since PX_1 is equal to S_2X_1, S_1P is one wave-length. Letting λ denote the wave-length of the light, and substituting in (220) the symbols represented in Fig. 430, we have

$$\frac{\lambda}{a} \doteq \frac{x}{\sqrt{(b^2+x^2)}}.$$

Or, since x is very small compared with b,

$$\frac{\lambda}{a} \doteq \frac{x}{b}.$$

Consequently, to the above degree of approximation, the wave-length of the light emitted by the source is

$$\lambda \doteq \frac{ax}{b}. \qquad \ldots\ldots\ldots \qquad (221)$$

It is found that the wave-length of light from a given source depends upon the medium through which the light is traveling. For example, the wave-length of light in water is about two-thirds of the wave-length in air.

The length of a light wave is very small. The diameter of an average human hair is about one four-hundredth of an inch. The average length of light waves in air is about one one-hundredth of the diameter of a human hair.

378. Velocity of Light.—Consider light proceeding along the path SM, Fig. 431. If a mirror M be placed normal to SM, the incident light will be reflected back along the path MS and will enter an eye placed behind S. Let a toothed wheel having teeth and spaces of equal width be placed with the axle parallel to the direction of the light. When the wheel is at rest, light traversing the space between two teeth will return through the same space. Suppose that the wheel can be rotated so fast that while light goes from A to M and back, a tooth will have advanced just enough to intercept the returning light. If the time occupied by a tooth in moving into the position occupied by the adjacent space be denoted by t, then the speed of the light is

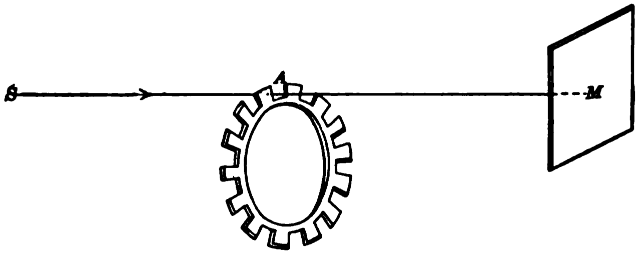

FIG. 431.

$$v=\frac{2(AM)}{t}.$$

If the angular speed of the wheel be doubled, the reflected light will pass through the next space. If the angular speed be trebled, the reflected light will be intercepted by the second tooth.

This first terrestrial method successfully used for the determination of the velocity of light was employed by Fizeau in 1849. In Fizeau's experiment, the wheel had 720 teeth and 720 spaces, all of the same width. The distance between the wheel and the mirror was 8633 meters. The first eclipse of the light occurred when the wheel was making 12.6 revolutions per second. Hence the time occupied by a tooth in moving its own width was

$$t=\frac{1}{2(720)(12.6)}\text{ sec.}$$

This would give for the speed of light

$$v\left(=\frac{2(AM)}{t}\right)=2(8633)\,[2(720)(12.6)]$$

$$\doteqdot 309{,}000{,}000\text{ meters per second}$$

$$\doteqdot 192{,}000\text{ miles per second.}$$

More recent determinations of the speed of light *in vacuum* give 300,574,000 meters per second, or 186,700 miles per second. The speed depends upon the medium being traversed, being greatest in vacuum. (In water, the speed of light is about two-thirds the speed in vacuum.)

There are certain distant stars, Algol for example, that change greatly in brilliancy within a very short time. During these changes in brilliancy the color remains constant. Now if red light traveled faster than the violet, then when the star suddenly increased in brilliancy the star would first appear red. As no change of color appears, we conclude that in empty space the speed of light of all wave-lengths is the same.

379. The Luminiferous Ether.—For the propagation of waves a medium is necessary. Since light is transmitted through interplanetary space and through the most nearly perfect vacuum that can be produced, we therefore believe that ordinary matter is not necessary for its propagation. This compels us to conclude that there is a medium other than matter by which light is propagated. This medium is not perceived by our senses, but the above facts convince us of its actual existence. This medium is called the *luminiferous ether* (i.e., light-bearing spirit), or briefly, *the ether*. The student should be warned against confusing the luminiferous ether with the various volatile liquids that in Chemistry are called ethers.

Since ether is the medium by which light is propagated, and light is transmitted by ordinary matter, we conclude that the ether permeates all space by filling the interstices between the molecules and atoms of matter.

Young's experiment shows that light is propagated by waves in the ether. It does not tell, however, whether these waves are due to a periodic to-and-fro motion of particles of the ether, or whether these waves are due to a periodic change of some one of the properties of ether. By means of waves, energy can be transferred from one region to another not only by a vibration of particles of the intervening medium, but also by means of a periodic electric or magnetic disturbance handed on successively from one portion of the medium to another.

380. Color and Wave-length.—Light waves of different wave-lengths produce different color sensations. It is found that the wave-length of the light that we call yellow is about 0.000059 cm. An object emitting or reflecting light of this particular color would be said to be of the color corresponding to wave-length 0.000059 cm. Light consisting of waves all of which have the same wave-length is called *monochromatic* or *homogeneous* light.

The human eye is sensitive to waves of lengths from 0.000033 cm. to 0.000081 cm. Waves of the former length produce the sensation called violet, and waves of the latter the sensation called red. All of the other colors have wave-lengths between these limits. Beyond these limits there are waves of other lengths to which the human eye does not respond. By means of photography waves have been observed that are as short as 0.00001 cm., and also waves as long as 0.0001 cm. The waves shorter than the violet are called ultra violet, and the waves longer than the red are called infra red. X-rays are now believed to consist of ether waves which are vastly shorter than any visible wave. The waves used in wireless telegraphy are vastly longer than any visible waves.

If the two halves of the first slit in Young's experiment be illumined by monochromatic lights of different wave-lengths, Fig. 432, there will be formed on the screen C a system of bands for each color. By adjusting the positions of the two sources, the two sets of bands can be caused to overlap as indicated in the figure. If instead of two separate monochromatic sources, we were to use a single light source which emits two sets of waves of the same wave-lengths as the two monochromatic sources, we would get a double set of bands just like the overlapping bands of the previous experiment. If sunlight be employed, the screen will be crossed by a series of rainbow-colored bands. This indicates that sunlight is a mixture of lights of many colors.

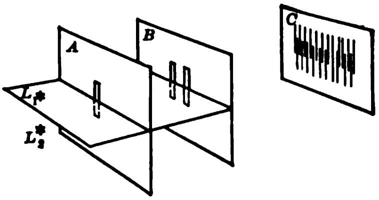

FIG. 432.

381. Fluorescence and Phosphorescence.—Many substances will absorb radiance and transform it into waves of different wave-length. For example, solutions of quinine sulphate, æsculine and of certain of the analine dyes are brightly luminous while exposed to invisible radiance of frequencies higher than the violet. Glass and many other solids are brightly luminous while exposed to X-rays. As the phenomenon of the absorption of invisible radiance and its transformation into visible waves was first observed with fluorspar, the phenomenon was named *fluorescence.* The characteristic "bloom" of mineral oils is an example of fluorescence. A cardboard screen covered with crystals of some fluorescent material, as calcium tungstate, will brightly fluoresce while exposed to X-rays. Such screens are much used in the X-ray examination of broken bones and the location of foreign bodies in the flesh.

Some bodies will continue luminous for some time after the exciting cause has been removed. Impure sulphide of calcium will remain luminous for such a long time after the exciting light source has been removed that it is used for painting match safes, keyholes and other articles which otherwise would be difficult to find in the dark. As this phenomenon was first observed in barium sulphide which at that date was popularly called Bolognese phosphorus, it was named *phosphorescence.*

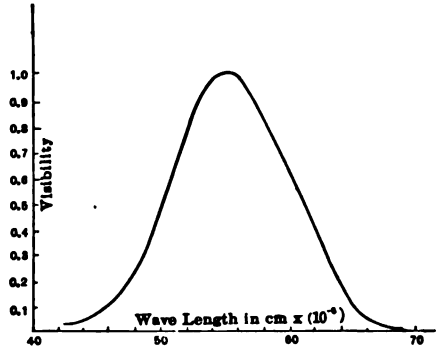

FIG. 433.

382. Color and Visibility.—For light of certain colors the eye is less sensitive than for light of other colors. In order that deep blue light (wave-length 0.000046 cm.) and deep red light (wave-length 0.000066 cm.) may produce equally intense sensations of brightness, the energy of the two waves must be equal. But to produce an equally intense sensation of brightness with yellow-green light (wave-length 0.000054 cm.), the energy of the latter need be but one-tenth as great as the energy of either of the other two colors. This is expressed by the statement that the visibility of the yellow-green light is ten times as great as that of either the deep blue or the deep red light of the specified wave-lengths. The relative visibility of light of different wave-lengths is represented graphically in Fig. 433. The product of the energy and visibility of light of any particular wave-length is called the *luminosity* of the light of that wave-length.

On account of the low visibility and low range of blue, American vessels traversing waters in which enemies are suspected are lighted at night only by blue lamps.

383. Light Waves are Transverse.—Soon after Young's interference experiment, Malus made an observation which showed that the vibrations constituting light waves are transverse to the direction of their propagation through the medium.

The observation of Malus can be repeated in the following manner: Each of the mirrors *A* and *B*, Fig. 434, is capable of rotation about a horizontal axis, and the mirror *B* is in addition capable of rotation about a vertical axis. If sunlight be reflected from the mirror *A* to the mirror *B*, it is found that when the horizontal axes of the mirrors are parallel (as in Fig. 434), light is copiously reflected from *B*, whereas when the horizontal axes of the mirrors are at right angles (as in Fig. 435), a much smaller amount of light is reflected from *B*. If the mirrors are so arranged that their horizontal axes are at right angles to one another, and if, in addition, the plane of each mirror makes an angle of about 33° to the direction of the path of the light incident upon it, the light reflected from *B* will be a minimum. If now the upper mirror be rotated about its vertical axis, the light reflected from *B* will increase, becoming a maximum when the horizontal axes of the mirrors are parallel. If the rotation be continued, the light reflected from *B* will diminish, becoming again almost zero when the horizontal axes are again at right angles to one another, and will again increase and become a maximum when the axes are parallel. Thus in two positions, 180° apart, the light reflected from *B* is almost zero;

while at two intermediate positions the light reflected from *B* is a maximum.

Fresnel pointed out that these phenomena show that light waves cannot be longitudinal. If the vibrations constituting light were longitudinal they would meet a mirror in exactly the same way whether the disturbance had or had not been previously reflected from another mirror; but if the vibrations were transverse to the direction of propagation of the disturbance no such symmetry would exist. For instance, imagine two transverse wave motions *a* and *b* to be incident on a mirror *M*, Fig. 436, perpendicular to the plane of the paper. Let the parallel lines across *a* indicate that in this wave the vibrations are parallel to the plane of the paper, and let the dots on *b* indicate that in this wave the vibrations are perpendicular to the plane of the paper. These two transverse wave motions meet the mirror in different ways, and we should not expect that they would be similarly reflected.

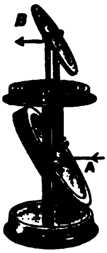

FIG. 434.

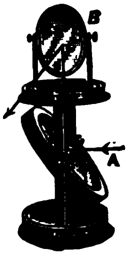

FIG. 435.

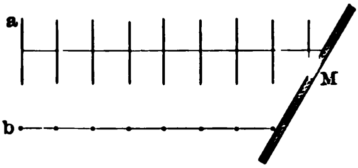

FIG. 436.

Reflection would occur as observed by Malus if light waves consist of vibrations that are successively in a great many directions transverse to the line of propagation, and if, in addition, the vibrations that are reflected consist of those that are nearly parallel to the surface of the mirror. Thus in Fig. 434, if only those vibrations of the light incident on *A* that are nearly parallel to the surface of the mirror are reflected, then the vibrations constituting the light between *A* and *B* are for the most part parallel to the horizontal axis of *A*. And since the surface of *B* is parallel to this direction, the light that is reflected from *A* will also be reflected from *B*. But when the mirror *B* is turned as in Fig. 435, its plane has no line parallel to the plane in which occurs the greater part of the vibrations constituting the light between *A* and *B*, and consequently there is little light reflected from *B*.

It thus appears that a mirror can serve as a polarizer of light waves. The degree of polarization depends upon the angle between the direction of the incident light and the plane of the mirror.

The direction along which light is traveling is called the *ray*. The direction along which the incident light is traveling is called the incident ray, and that along which the reflected light is traveling is called the reflected ray. The plane containing the incident ray, the reflected ray and the normal to the reflecting surface, at the point of incidence, is called the *plane of incidence*. That particular plane of incidence in which light is most copiously reflected is called *the plane of polarization*. With the apparatus arranged as in Figs. 434 and 435, the plane of the paper coincides with the plane of polarization of the light that has been reflected from the lower mirror. According to the generally accepted theory of light, the vibrations of plane polarized light are perpendicular to the plane of polarization.

CHAPTER XXIV

THE PROPAGATION OF LIGHT

§ 1. *Light Quantities*

384. Measurement of Solid Angles.—A plane angle ϕ is measured by the ratio of the length x of the arc of any circle drawn with O as a center, to the length of the radius of this circle. Thus, Fig. 437,

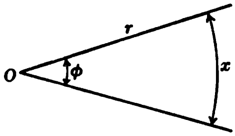

FIG. 437.

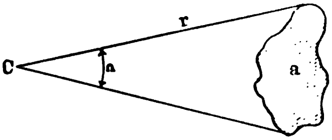

FIG. 438.

$$\phi = \frac{x}{r} \text{ radians.}$$

A solid angle Ω may be measured in an analogous manner. With C as a center, Fig. 438, construct a sphere of radius r. The elements of the pyramidal faces enclosing the solid angle Ω will cut out of the sphere a surface of area a. Whatever the magnitude of the radius r, the ratio of the area of the surface a to the square of the radius r is a constant quantity. Consequently, the measure of the solid angle Ω is taken as

$$\Omega = \frac{a}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

For example, since the area of the sphere is $4\pi r^2$, a sphere subtends at its center a solid angle of 4π units. This unit is called the *space radian*, or *steradian*.

385. Luminous Flux.—The total visible energy emitted by a source per second is called the total *luminous flux* from the source. The luminous flux of any given wave-length is measured by the product of the total energy emitted per second and the sensibility of the eye for the radiance of the given wave-length. Or, in symbols,

$$F_\lambda = K_\lambda \Phi_\lambda, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

where F_λ represents the luminous flux of wave-length λ, Φ_λ represents the total radiance of that wave-length emitted per second, and K_λ is the retinal stimulus coefficient or visibility of light of that wave-length. If F represent the total luminous flux of all wave-lengths, we have

$$F = \Sigma F_\lambda.$$

The primary luminous standard is a lamp devised by Hefner. This lamp, which burns amyl acetate, is shown at *l*, Fig. 440. Before the Hefner standard lamp was devised, candles were commonly used in Great Britain, France and America as luminous standards. The standard British candle was one that burned 120 grains of spermaceti per hour. On account of the lack of uniformity of even the most carefully made candles, candles are now seldom used in actual photometric measurements. But as actual candles were employed for a long time, in these countries light quantities are still usually expressed in terms of candles.

The unit of luminous flux is the luminous flux emitted in one space radian by a standard lamp, and is called the lumen. If the standard light source is at *C*, Fig. 438, then the luminous flux in the solid angle Ω is one lumen when this angle is one space radian. Since there are 4π space radians in a sphere, the total luminous flux from a standard source is 4π lumens.

Since the luminous flux from a Hefner lamp is not the same as that from a standard candle, we have a Hefner-lumen and a candle-lumen. By international agreement, ten-ninths of a Hefner-lumen are taken to equal one candle-lumen. The candle-lumen is the unit of luminous flux employed in English-speaking countries. In cases where there is no danger of confusion the unit is called simply the "lumen." Following this practice we shall henceforth use the word lumen in the sense of candle-lumen. To give a notion of the magnitude of the unit of luminous flux, one might note that the tungsten filament lamps now used in domestic lighting emit a luminous flux of about four lumens per watt of electric energy supplied.

386. Luminous Intensity.—That property of a source of emitting luminous flux is called *luminous intensity.* The luminous intensity in any given direction of a point source is measured by the luminous flux from that source in the given direction per unit solid angle. Thus, if the total luminous flux from the source be F, the luminous intensity I has the value,

$$I=\frac{F}{4\pi}. \qquad \qquad (224)$$

It follows from this equation that the luminous intensity of a point source will be unity when it emits a luminous flux $F=4\pi$ lumens. This is the light flux from a standard unit source. The units of luminous intensity are called the hefner and the international candle. A luminous intensity of ten-ninths of a hefner equals one international candle.

The average candle-power measured in all directions from a source is called the *mean spherical candle-power* of the source. It equals the total luminous flux in lumens, divided by 4π. The average candle-power in the horizontal plane passing through the source is called the *mean horizontal candle-power* of the source.

387. Brightness.—The luminous flux density emitted by a surface, that is, the flux emitted per unit of emissive area, is called the *luminous radiation* from the surface. Luminous radiation is expressed in lumens per square centimeter. The ratio of the luminous intensity of a surface, to the projection of the area on a plane normal to the line of sight is called the *brightness* or *intrinsic brilliancy* of the surface.

A disk of plaster of paris appears to be of nearly the same brightness from whatever angle it is viewed. From whatever direction it is viewed, the apparent area of the surface is proportional to the cosine of the angle between the line of sight and the normal to the surface. Hence the surface emits light in every direction with an intensity very nearly proportional to the cosine of the angle of emission. A surface that emits light proportional to the cosine of the angle of emission is said to be perfectly diffusing, or to emit according to Lambert's cosine law.

The unit of brightness is the brightness of a perfectly diffusing surface radiating or reflecting one lumen per sq. cm. of projected area, and is called the *lambert.* The lambert is so great that for most purposes, the millilambert (0.001 lambert) is a more convenient unit. Brightness is also expressed in candles per square centimeter and in candles per square inch.

The human eye ceases to function at brightnesses below about 0.0000007 millilambert, and above about 50 lamberts. At the upper limit the eye is so dazzled that vision is impossible. For this reason we have police regulations limiting the brightness of automobile headlights. The dazzling effect of a search light beam was much used during the recent World War against enemy aviators flying at night.

Brightnesses of various light sources expressed in lamberts, are roughly as follows: crater of the arc lamp, 40,000; Nernst lamp glower, 1500; tungsten filament, 500; carbon filament, 250; acetylene flame, 25; kerosene flame, 5; gas flame, 2.

388. Illumination.—The luminous flux incident on unit area of a surface is called the *illumination* of the surface. Thus, the illumination E of a surface of area S which intercepts a luminous flux F is

$$E=\frac{F}{S}. \qquad (225)$$

From this equation are derived the units of illumination, one lumen per square meter, called the *lux;* one lumen per square centimeter, called the *phot;* and the lumen per square foot.

At a distance r from a point source of intensity I, the illumination has the value, (225) and (224),

$$E\left[=\frac{F}{S}\right]=\frac{4\pi I}{4\pi r^2}=\frac{I}{r^2}. \qquad (226)$$

From this equation we see that at unit distance from a point source of unit intensity, the illumination is unity. Consequently, the lumen per square meter (or lux) is also called the *meter-candle* or the *candle-meter*, and the lumen per square foot is also called the *foot-candle* or the *candle-foot*.

The illumination on a study table or drafting board should be from 4 to 8 foot-candles, whereas on the floors of corridors an illumination of 1 foot-candle is sufficient. For shop work requiring observation of machine operation the general illumination should be about 4 foot-candles. But for work requiring discrimination of small details as in watch making, wood engraving and fine tool work, the illumination of the article should be from 8 to 12 foot-candles. For sewing on black cloth the illumination must be at least double that required for sewing on white cloth.

389. Illumination at Different Distances from a Point Source.—Consider a luminous point of intensity I situated in a transparent isotropic medium. From (226) the illuminations at distances r_1 and r_2 will be

$$E_1 = \frac{I}{r_1^2} \text{ and } E_2 = \frac{I}{r_2^2}.$$

Whence,

$$\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}. \qquad (227)$$

Therefore, in the case of light emitted from a luminous point in a transparent isotropic medium, the illumination of a surface at any distance is inversely proportional to the square of that distance from the source. If the luminous source is an extended surface, the front of the emitted wave will not be spherical and the above "inverse square" law will not apply.

390. Photometry.—The art of comparing luminous intensities is called *photometry*. Two beams of light of the same color and different light-flux densities will produce retinal sensations of the same kind, but different in magnitude. But, even for the same color, twice the light flux density does not mean twice as bright a light.

Although the eye cannot accurately compare illuminations of different color (Art. 382), or of different magnitude even though they are of the same color, still the eye can judge of the *equality* of illuminations of the *same color* with a satisfactory degree of precision. This fact is the basis of a simple method of comparing the luminous intensities of two small sources.

Consider two point sources emitting light of the same color uniformly in all directions. Let a small white screen A, Fig. 439, be placed between the two sources and normal to the line connecting them. Let the two sources be of luminous intensities I_1 and I_2, and let the distances of the screen from the two sources be r_1 and r_2, respectively. Then the illuminations on the two sides of the screen due to the two point sources are, (226),

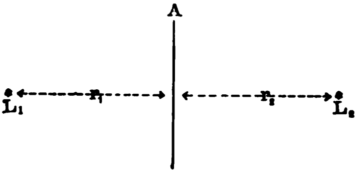

FIG. 439.

$$E_1 = \frac{I_1}{r_1^2} \text{ and } E_2 = \frac{I_2}{r_2^2}.$$

If the screen be moved back and forth until the two sides are equally illumined, that is, until $E_1 = E_2$, then

$$\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}.$$

Whence,

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}. \quad \quad (228)$$

Although this equation is strictly true only for point sources, it holds very well for sources which are small compared with their distance from the screen. The equation assumes that all the light which reaches the screen comes directly from the source. If there are surfaces which reflect light onto the screen the equation does not apply. In the case of a search light, for instance, where the source is backed by a reflector, the equation does not apply at all.

In speaking of the intensity of a search light, the term "equivalent candle-power" is often used. Thus, the expression 100,000 equivalent candle-power in a certain direction means that the source is producing illumination in that direction equal to what would be produced on the average in all directions by a point source of intensity 100,000 candles.

In Fig. 440 is represented the actual apparatus for the comparison of luminous intensities. In this case the intensity of the gas flame L is to be compared with the intensity of the flame of the Hefner lamp l. M is a gas meter, R is a pressure regulator, and m is a manometer. The screen is in the box P. The photometer screen is moved back and forth along the scale S till the two sides of the screen are equally illumined. The distances of the screen from the two light sources are then observed. The luminous intensity of the gas flame can now be computed by means of (228).

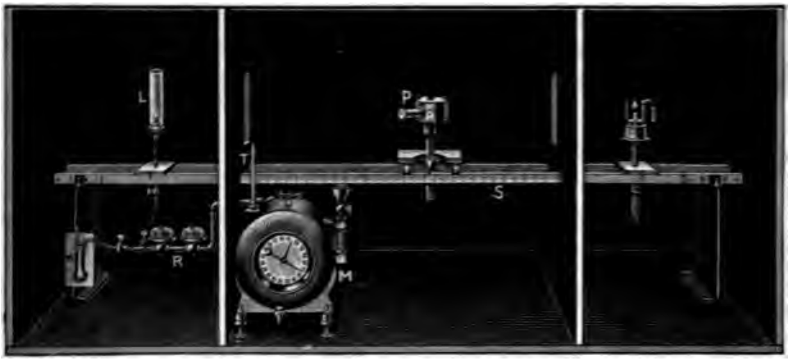

Fig. 440.

LIGHT UNITS AND THEIR DEFINING EQUATIONS

Name of Quantity.	Name of Unit.	Defining Equation.
Luminous flux	Lumen	$F = \Sigma K_\lambda \Phi_\lambda$
Luminous intensity	Candle	$I = \frac{F}{4\pi}$
Illumination	Foot-Candle *	$E = \frac{F}{S} = \frac{I}{r^2}$
Brightness	Lambert †	$b = \frac{I}{S \cos \theta}$

* Other units of illumination are the phot and the lux; 1 phot = 929 foot-candles; 1 lux = 0.093 foot-candle; 1 foot-candle = 0.001076 phot = 10.76 lux.

† Other units of brightness are the candle per square inch and the candle per square centimeter; 1 candle per square inch = 0.4868 lambert; 1 candle per square centimeter = 3.1416 lamberts; 1 lambert = 2.054 candles per square inch = 0.3183 candle per square centimeter.

SOLVED PROBLEMS

Problem.—A floor area of 500 sq. ft. is to be given a mean illumination of 4 foot-candles by means of lamps which have a luminous flux of 250 lumens each. Find the number of lamps required.

Solution.—From (225) the flux required $F = ES$, or
lumens per lamp × number of lamps = foot-candles × square feet

$$250n = 4 \times 500$$

or, the required number of lamps,

$$n = 8.$$

Problem.—The floor, walls and ceiling of a room 20 ft. × 15 ft. × 10 ft. having a mean coefficient of absorption of 0.3 are to be given an illumination of 4 foot-candles. The lamps to be used have a mean spherical candle-power of 20. Find the number of lamps required.

Solution.—Part of the illumination at any point is due to light reflected from the other walls. The luminous flux which the lamps must supply to any area equals the flux which that area absorbs. There must be supplied a flux

$$F = 0.3\ (20 \times 15 \times 10)4 = 3600 \text{ lumens.}$$

Each lamp gives a flux of $4\pi 20 = 251$ lumens.

Consequently, the required number of lamps is

$$\frac{3600}{251} = 14.$$

§ 2. *Reflection and Simple Refraction of Light*

391. Reflection of Light.—Light being a wave motion, the laws of reflection of light are those already considered in the chapter on Wave Motion. Of these laws, the ones of most frequent application are:

(*a*) The angle of reflection equals the angle of incidence:

(*b*) The reflected ray, the incident ray and the normal to the mirror at the point of incidence, lie in the same plane:

(*c*) In the case of reflection from a plane mirror, the image is as far behind the mirror as the object is in front, and is on the line through the object normal to the mirror.

For an image to be formed, the mirror does not need to extend far enough to be intersected by the line from the object to the mirror. The production of an image does not depend upon the presence of an eye in the proper position to view it. In Fig. 441, the image of *A* is at *A′*. To see this image, the eye must be in front of the mirror *M* and somewhere within the angle *YA′Z*.

After reflection from a vertical plane mirror, the top and the bottom of the image are seen in their correct positions relative to one another, but the right-hand side and the left-hand side appear reversed, Fig. 442. Or, stated more briefly, the image appears erect and perverted.

An optical figure resembling an object formed by light from the object is called a real *image*. If a piece of white paper be placed at the place where a real image is formed, a picture of the object will appear on the piece of paper. In Fig. 441 light does not converge at the point *A′*, and in Fig. 442 light does not converge at *A′B′*, but after reflection from the mirrors the light appears to have come from these places. These places are called virtual images. No picture of the object will appear on a piece of paper placed at a virtual image. Only virtual images are formed by light reflected from plane mirrors.

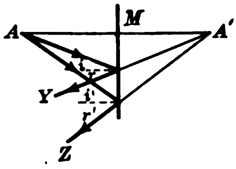

FIG. 441.

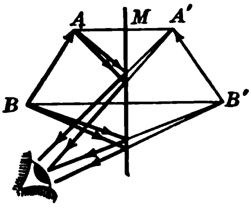

FIG. 442.

392. Multiple Reflections.—After two, or any even number of reflections from plane mirrors, the right and left of an image appear in their correct positions.

If an image be in front of a reflecting surface, another image will be formed just as though it were an object. In Fig. 443, M_1 and M_2 represent two plane mirrors which are parallel and which face each other. Between these surfaces is an object *A*. An image A_1 of the object is formed behind the first mirror and another image *A′* behind the second mirror. As A_1 is in front of the mirror M_2 there is formed an image A_2 of A_1. As *A′* is in front of the mirror

M_1, there is formed an image A'' of A'. Proceeding in the same manner we find that an infinite number of images of the one object would be formed. The successive images are of gradually diminishing brightness. The number of images that can be seen depends upon the position of the observer and upon the brightness of the images. A person at O beyond the mirrors can see all sufficiently bright images that are included between two lines extending from the eye through the nearer edges of the mirrors.

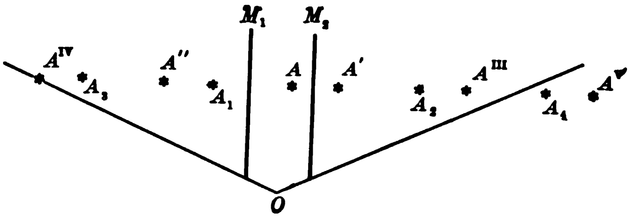

FIG. 443.

If an object be placed in the angle between two reflecting surfaces M_1 and M_2, a series of images will be formed, arranged symmetrically about the line of contact between the two mirrors, as indicated in Fig. 444. An image behind both reflecting surfaces will produce no image. If the angle between the mirrors be a submultiple of 180°, it can be shown that the two images behind both reflecting surfaces (A_3 and A''', Fig. 444), will be superposed; and that the object plus the number of separate images equals 360° divided by the angle between the mirrors.

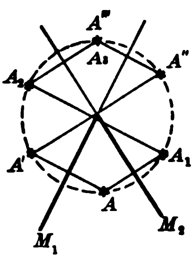

FIG. 444.

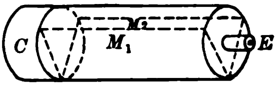

FIG. 445.

The property of inclined mirrors to form a series of images arranged symmetrically about a point is the basis of the kaleidoscope. This instrument consists of a tube containing two plane mirrors inclined at an angle which is a submultiple of 180°, Fig. 445. At one end of the tube is a cell C having the inner end of clear glass and the outer end of ground glass. Within the cell are a number of bits of colored glass of various

shapes. On placing the eye at *E* and looking along the axis of the tube, one sees the bits of colored glass and their multiple images arranged in a beautiful pattern of circular symmetry. On rotating the cell, the bits of glass fall into a different arrangement and another pattern is produced.

393. Reflection and Absorption of Light at an Opaque Surface.—If light be incident on a rough opaque surface, the incident wave will be broken up and scattered in all directions. If, however, the distance between the elevations of the surface be less than a quarter wave-length of light, that is, less than about 0.000005 inch, the incident wave will be reflected with very little scattering. Such a surface is said to be *polished*. A body which has a polished surface capable of reflecting light without scattering is called an *optical mirror.*

Of the light incident on a body, a part is reflected, a part may be transmitted, and the remainder is absorbed. The fraction of the incident light that is reflected from a polished surface depends upon the wave-length of the light, the angle of incidence, the material of which the body is made and upon the medium in front of it. Other conditions being the same, when light is traveling in air, more light is reflected from glass than from water; and if it is traveling in water, a still smaller part is reflected from glass. When light of wave-length 0.00006 cm., goes from air to polished surfaces of different materials at perpendicular incidence, the fraction reflected is about as follows: copper, 0.72; gold, 0.84; steel, 0.55; mercury backed by glass (German mirror plate), 0.70; silver, 0.93; silver backed by glass (French mirror plate), 0.88.

When the surface is unpolished, the reflection is diffuse. The fraction of the total incident light that is reflected by some familiar unpolished substances is about as follows: white paper, 0.70; snow, 0.78; black paper, 0.05; black velvet, 0.004.

394. Selective Reflection and Absorption.—Most substances absorb light waves of different frequencies to an unequal degree. A substance that absorbs waves of all frequencies except of that which produces the sensation called red will reflect and transmit only waves of that frequency. Such a substance is said to be red or to have a red hue. This phenomenon of selective absorption and reflection of light is the most common cause of the color of objects.

No substance reflects light of but one frequency. That is, no substance is monochromatic or of a simple color. A substance that reflects completely light of all wave-lengths is called white. A substance that reflects incompletely, but to an equal extent, all light incident upon it, is said to be gray. A substance that absorbs all light incident upon it, reflecting none and transmitting none, is said to be black.

A body appears black unless it receives light of the frequencies which it reflects. A red body will appear black unless it receives light of some frequency which produces the sensation of red. The color of a body that absorbs all incident light except that of a few frequencies will be intense. A color free of admixture with white is said to be *saturated* or to be of *high chroma.*

The color of a piece of velvet is much more intense than that of a piece of satin of the same material and dye. Much white light is reflected from the smooth satin surface. This white dilutes the color or reduces the chroma. Light incident on velvet is reflected several times in the "pile" of the surface before emergence. With each reflection there is selective absorption. Consequently the emergent light is nearly free of white. The color is saturated or of high chroma.

395. Refraction of Light.—In going from one medium into another in which the speed is different, light is refracted according to the laws already considered (Art. 165). It has been shown that in the case of any two isotropic media:

(*a*) the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant;

(*b*) this constant quantity equals the relative index of refraction of the two media;

(*c*) the refracted ray, the incident ray, and the normal to the refracting surface at the point of incidence, lie in the same plane.

The index of refraction of a substance depends upon the temperature and is different for light of different wave-lengths.

The indices of refraction of a few familiar substances for light of wave-length 0.000058 cm. are about as follows: Ice, 1.3; water, 1.33; crown glass, 1.5 to 1.6; flint glass, 1.6 to 1.9; Canada balsam, 1.5; carbon bisulphate, 1.6; diamond, 2.4.

The apparent trembling of objects seen through a stream of air rising from the heated ground or the top of a stove is due to the fact that the refractive index of the moving heated air is less than that of the cooler surrounding air. The twinkling of stars is probably due to similar inequalities in the refractive index of the moving air between the stars and the observer.

The air has a greater refractive index than the ether. For this reason the refractive index of the atmosphere gradually decreases from the earth upward. Consequently light from a heavenly body entering the earth's atmosphere in any direction except along a radius of the earth is gradually bent out of its course toward a radius of the earth. Due to this fact a heavenly body is visible while still about a half degree below the horizon. The sun rises earlier and sets later than it would if this refractive effect did not occur.

396. The Fraction of the Incident Light that is Reflected.— The fraction of the incident light that is reflected at a polished surface depends upon the angle of incidence and upon the relative refractive indices of the two substances bounding the reflecting surface.

The fraction of the light incident at various angles which is reflected from a polished surface of glass of relative refractive index 1.55 is given in the table below:

i	Reflected.	i	Reflected.
0°	4.65%	70°	18.00%
20°	4.68	75°	26.19
40°	5.26	80°	39.54
50°	6.50	85°	61.77
60°	9.73	90°	100.00

The amount of light reflected by a substance increases when the difference between the refractive index of the substance and that of the surrounding medium increases. The table on the top of the next page gives the fraction of the light incident normally, in air, that is reflected by substances of various refractive indices.

If a piece of clear glass be immersed in a clear liquid of the same refractive index, the glass will be invisible. The refractive index of glass is different for light of different colors. Consequently a piece of glass immersed in a liquid may be invisible when illumined by light of one color and be visible when illumined by light of a different color.

μ	Reflected.	μ	Reflected.
1.0	0.0%	2.0	11.1%
1.2	0.8	2.2	14.1
1.4	2.8	2.4	17.0
1.6	5.3	2.6	19.8
1.8	8.2	2.8	22.5

A wet spot of water color is brighter than the same spot after the water has dried out. This is because there is a greater difference between the refractive index of air and the wet pigment than that between air and the dry pigment. The brightness of a pigment is heightened even more by mixture with oil or varnish. The plumage of certain individual birds is brighter than that of others of the same species. This is strikingly exhibited by flamingoes. Some individuals are of a much brighter red than others. The difference is probably due to the larger amount of oil in the feathers of the redder birds.

White blotting paper is so highly porous that it has an equivalent reflecting surface much greater than the superficial area of the sheet. Its intense whiteness is due to the large amount of light that is diffusely reflected. If the pores be filled with oil of about the same refractive index as the paper, the equivalent reflecting surface is diminished and the whiteness is reduced.

397. Total Reflection.—Consider the passage of light from a medium in which the speed is less to another in which the speed is greater. To fix the ideas, let S, Fig. 446, be a point source of light in water. At the surface separating the water from the air above, there will be both reflection and refraction. Light arriving at a point D will there be partly reflected, and the remainder will be transmitted into the second medium. At any point on the interface separating the air and water the angle of reflection will equal the angle of incidence. And if the index of refraction of water relative to air be μ, then, for any angle of incidence i the angle of refraction R will be given by the equation

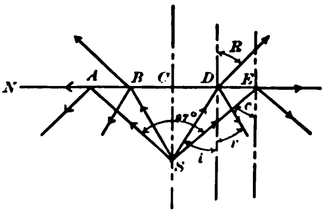

FIG. 446.

$$\frac{1}{\mu}=\frac{\sin i}{\sin R}.$$

Since μ is greater than unity, this equation shows that R is greater than i. At some point of the surface, E, the angle of incidence will be such that $R=90°$. Beyond E light will be reflected but there will be no light transmitted into the second medium. The value of i for which $R=90°$, that is, the smallest value of the angle of incidence at which light will be totally reflected, is called the *critical angle* for the two media. Denoting this angle by c, we have for the case where $R=90°$, $\sin R=1$, and

$$\frac{1}{\mu}=\frac{\sin c}{1}.$$

Whence, the critical angle of incidence is

$$c=\sin^{-1}\frac{1}{\mu}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

Since the refractive index of a substance is different for light of different wave-lengths, this equation shows that the critical angle of incidence depends upon the wave-length of the incident light.

Consider an eye in water at E, Fig. 447. Let c be the critical angle of incidence for water with respect to air. Now light from any point above the water will enter the eye at E, but this light must traverse the circular area between A and B. Thus, to an eye at E the surface of the water appears to be an opaque reflecting ceiling pierced by a round window immediately overhead. Objects nearly overhead will appear little distorted, but objects near the horizon, much distorted.

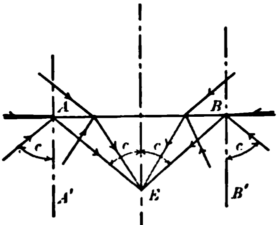

FIG. 447.

Taking the index of refraction of water to be 1.33 we obtain from (229)

$$c=\sin^{-1}\frac{1}{1.33}=\sin^{-1} 0.75=48° \; 30'.$$

Whence $\qquad AEB=97°.$

Fig. 448 shows the view that would be seen by an eye in water directed upward. It was taken by a camera submerged in water.

Transparent jewels are so cut that as much as possible of the light entering the top shall be reflected at the lower faces. The refractive index for a diamond is greater, and consequently the critical angle is smaller, than for any other jewel. It follows that total reflection will occur at smaller angles of incidence from the lower faces of a diamond than from any other jewel. This is the reason that a properly cut diamond, Fig. 449, is so very brilliant.

The color of a fine powder is much fainter than that of the unpulverized substance, and the color of a foam is much fainter than that of the liquid of which it is formed. The powder and the foam are white if the undivided substance is not too deeply colored. This effect is due to two causes. Incident white light is diffusely reflected copiously from the surface of the small particles and bubbles. Light entering either the fine particles or the small bubbles travels such a short distance before being reflected out of the material that only little absorption of light can occur.

Fig. 448.

398. Reflecting Prisms.—Reflection occurs whenever light is incident on the surface separating two media in which the speed of light is different. Usually a part of the energy of the incident wave is transmitted by the second medium. But when light travels from a medium in which the speed is less to a medium in which it is greater, there will be no transmitted wave if the angle of incidence at the interface is greater than the critical angle for the given media. In many optical instruments application is made of this fact for changing the direction of a beam of light without sensibly changing its intensity.

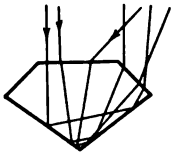

Fig. 449.

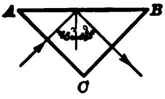

Fig. 450.

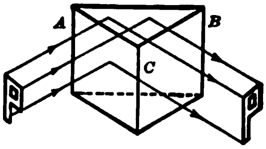

Fig. 451.

Figs. 450 and 451 represent a right-angled prism of glass. If the glass be of refractive index 1.5, the critical angle in air is 42°, (229). Consequently, if light traverses normally the face *AC* of a right-angled prism of such glass, it will be totally reflected at the face *AB* and will emerge normal to the face *BC*.

If it be desired to have the image inverted, a second reflection will be necessary. The two required reflections may be produced by substituting for the hypotenuse face two surfaces inclined to one another like the two sides of a gable roof, Fig. 452. This so-called Amici totally reflecting prism is used in certain periscopes and panoramic gun sights.

In Fig. 453, *ACB* represents an isosceles glass prism of such a vertex angle *C* that light incident on the face *AC* parallel to the base will be refracted and strike the base at an angle greater than the critical angle of incidence. After total reflection at the base, the light will proceed to the face *BC* and emerge parallel to the incident direction. The diagram shows that on emergence the relative positions of the rays *x* and *y* are reversed. This device is commonly used to invert an image. When so used, it is called an "erecting prism." It should be remarked that the face *AB* acts simply as a plane mirror and produces the same effect as any other plane mirror. The prism is only a convenient device for causing the light to strike the mirror at the proper angle and for causing the emergent light to be parallel to the entrant light.

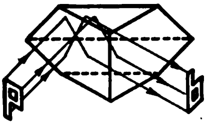

FIG. 452.

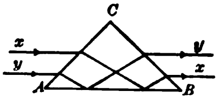

FIG. 453.

399. The Change of Wave-length on Refraction.—The speed of propagation of any wave motion is, (110),

$$v = n\lambda,$$

where n represents the number of vibrations per second, and λ represents the distance traveled during the time of one vibration. Since the frequency is constant, the wave-length must change whenever the speed changes.

The amount of change in the wave-length of light on passing from a medium in which the speed is v_1 to a medium in which the speed is v_2 is easily determined. Denote the wave-lengths of the light in the two media by λ_1 and λ_2, respectively. Then

$$v_1 = n\lambda_1 \text{ and } v_2 = n\lambda_2.$$

Whence (113, p. 212),

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \mu,$$

where μ is the index of refraction of the second medium relative to the first for light of the given frequency.

Consequently,

$$\lambda_2=\frac{\lambda_1}{\mu}. \qquad (230)$$

400. Dispersion.—It is found that although light of all frequencies has the same velocity in the ether, light of different frequencies traverses transparent matter with different speeds. This fact is described by the statement that the index of refraction of a substance is different for light of different frequencies. Since the amount that light is deviated from its course in going from one medium to another depends upon the relative speeds in the two media, it follows that light of different frequencies will be refracted by different amounts in going from one transparent medium to another. Thus, suppose the luminous point S, Fig. 454, is a source of waves having the frequency that produces the sensation of red, and also of waves that produce the sensation of blue. Let a small portion of the wave front pass through a hole in the diaphragm D and traverse the glass prism P. It is found that a screen placed so as to receive the light after emerging from the prism will exhibit two spots of light, one red and one blue. Whatever may be the mixture of light emitted by the source, the unequal refraction of light of different frequencies effected by traversing the prism will separate the composite light into its constituent colors. The separation into its components of light consisting of a mixture of different frequencies is called *dispersion*.

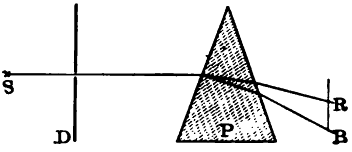

FIG. 454.

If one of two prisms of equal angle deviates light through twice the angle that the other does, it might be supposed that it would also disperse it twice as much. This is not, however, found in general to be true. Two prisms of different substances placed as in Fig. 455 may give a resultant deviation of zero and a resultant dispersion of finite value.

Again, one substance may have a higher index of refraction than another and have a smaller dispersion for any two given colors. Thus, if a prism, Fig. 456, of the first substance be placed adjacent to a prism of the second substance, and of such an angle as exactly to neutralize the dispersion produced by the first prism, the light emerging from the system will be deviated from its original direction, but will be undispersed.

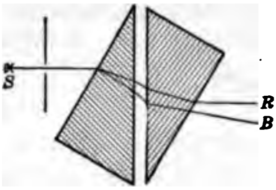

Fig. 455.

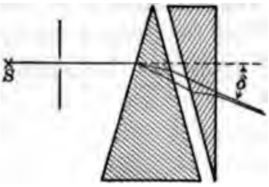

Fig. 456.

Questions

1. If sunlight travels in straight lines and if at the earth the sun's rays are parallel, how is daylight disseminated through a room so that it reaches every part? How does a room with only a north window get any light?

2. A dust-free room is closed light tight, except for a small round hole in a curtain at one side of the room. Suppose a perfect mirror interposed so as to reflect the light to the ceiling. What does one now see? How would the presence of dust in the room affect the experiment?

3. What form should be given a mirror which is to be used behind an arc lamp for a locomotive head-light? Show clearly with diagrams, and give physical reasons.

4. Does a man above the surface of the water appear, to a fish below it, farther from or nearer to the surface than he actually is? Show by use of Huyghens' construction.

5. When a man stands with his back to the window and looks into a lens, he sees two images of the window. Explain how they are produced.

6. Over deserts and over the ocean in still hot weather, when the air is composed of layers of different densities, travelers sometimes see in the sky inverted images of distant objects (i.e., mirages). Show that this is a phenomenon of total reflection.

7. A fish is below the surface of the water. A man shoots at the place where the fish appears to be, holding the gun at an angle of 45° with the surface of the water. Does the bullet pass above or below the fish? Explain.

8. In what direction must a fish look to see the setting sun?

9. To a person looking at a thick block of glass from above, a picture pasted on the bottom of the block appears as though it were inside the glass. Explain fully.

10. If a diver should look out from below the surface of a very clear still lake, how would the appearance of the shore both above and below the water line differ from its appearance if the lake could be suddenly dried up?

11. Objects seen across the top of a red hot stove appear unsteady and indistinct. Explain.

CHAPTER XXV

LENSES AND LENS SYSTEMS

§ 1. *The Cardinal Points*

401. The Spherical Lens.—(Reread Arts. 163 and 164.) If a diaphragm having a small aperture be placed between a screen and a group of luminous points, Fig. 457, there will appear on the screen a small spot of light for each point source. These small spots will be arranged in a figure like the group of luminous point sources. The figure on the screen resembles the group of object points. An optical figure resembling an object, formed by light from the object is called a *real image*. The space around the small image spots is dark on account of destructive interference. A more detailed discussion of image formation is given in Chapter XXVII.

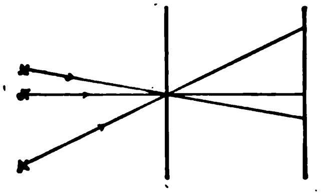

FIG. 457.

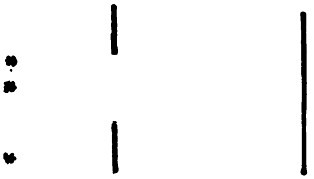

FIG. 458.

If the aperture in the diaphragm be large, Fig. 458, there will appear on the screen a large diffuse spot of light that does not resemble the group of object points. This spot is bright but is too diffuse to be an image. When the aperture is small, the image is sharp but dim. It would be highly desirable to have the brightness obtainable with a large aperture together with the sharpness of image obtainable with a small aperture. A lens enables us to obtain this result.

Consider a single luminous point *S* in front of a large aperture *az* and screen *XX'*. It is required that the light traveling along the various paths *Sa*, *Sb*, *Sc*, etc., shall arrive at some point *i* in the same phase.

The ray *Sa* can be bent into the ray *ai* by means of a prism of proper angle, Fig. 460. The other rays from the object point *S* can be caused to converge at *i* by means of other prisms of proper angle.

Light traveling along the unequal paths *Sai*, *Sbi*, etc., will arrive at *i* in different phases unless a proper retardation be introduced in the shorter paths. The proper amount of retardation can be produced by an adjustment of the thicknesses of the various prisms. If the angles and thicknesses of the various prisms have been properly selected, most of the light which comes from a point source and traverses the system will arrive in the same phase at a single point. As the number of prisms is indefinitely increased, the angles between the successive prisms disappear and the surface becomes smoothly curved. It can be shown that the required surface is not far from spherical.

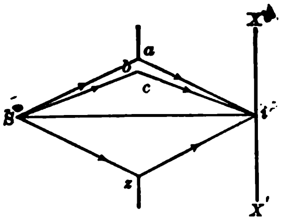

FIG. 459.

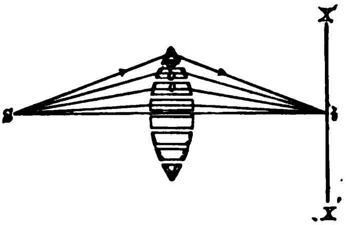

FIG. 460.

As spherical surfaces are economically ground, the surfaces of lenses are usually spherical. In Fig. 460, the lens approximating to the system of prisms is indicated by dotted lines. Light traversing the edges of the lens converges to a point nearer the lens than light traversing the central part of the lens. The correction of this fault is considered in a later Article.

A lens which renders a light wave more convergent is called a *converging* lens. A lens which renders light more divergent is called a *diverging* lens. There are lens systems which are convergent for light traversing them in one direction and divergent for light traversing them in the opposite direction. A converging lens or lens system is said to be *positive,* and a diverging lens or lens system is said to be *negative.* A convex glass lens in air converges light. A convex lens of air in water may diverge light.

402. Principal Points.—For purposes of graphical representation it is often convenient to represent the rays or paths along which light travels, instead of the fronts of the advancing wave. In isotropic media, rays are normal to the wave fronts.

Throughout the present section of this chapter we shall consider the effect of a lens bounded by spherical surfaces upon light that traverses the lens near the center and in a direction nearly parallel to the principal axis. To make the diagrams clearer, however, the rays in the diagrams illustrating these Articles will not be confined to this narrow region.

In Fig. 461 the ray starting from *A* parallel to the principal axis of the lens will be *ABCF.* Produce *AB* and *FC* till they intersect at *D.* Through *D* draw a line *DP* perpendicular to the principal axis. In so far as the portions of the ray outside of the lens are concerned, the light along the ray *AB* proceeds as though it had traversed the straight line *AD* and had then been bent into the path *DF.* Similarly, in so far as the portions of the ray outside the lens are concerned, light proceeding from the right to the left along the ray *A'B'* emerges from the lens as though it had traversed the straight line *A'D'* and had there been bent into the path *D'F'.* Through *D'* draw a line *D'P'* perpendicular to the principal axis.

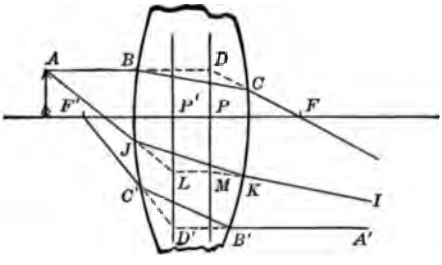

FIG. 461.

The importance of the planes *DP* and *D'P'* lies in the fact that a ray such as *AJ,* which is not parallel to the principal axis, leaves the lens as if it had followed a straight path AJL to the plane $D'P'$, had then traveled parallel to the principal axis to the plane DP, and had then followed another straight path MKI. That is, there are two planes parallel to the principal axis of a lens or lens system, which possess the property that the prolongation of any incident ray meets the first, and the prolongation of the corresponding emergent ray meets the second, in points equally distant from the principal axis. They are called the "principal planes" of the lens or lens system.

The points P and P' where the principal planes are cut by the principal axis are called "principal points." There is a pair of principal points for light of each color. In the case of most lenses, however, the principal points for all colors are so nearly coincident that in the present discussion the departure will be neglected.

The distance from the emergent principal point to the corresponding principal focus, real or virtual, is called the *principal focal length* of the lens or lens system. In the case of the converging lens represented in Fig. 461 the principal focal length is PF when the light travels from left to right, and is $P'F'$ when the light travels from right to left. If the lens had been a diverging lens, the principal foci F and F' would have been virtual and the principal focal lengths would have been $P'F$ and PF', respectively. It can be shown, though the proof will not be here given, that if the media on the two sides of a lens are the same, then the two principal focal distances are equal.

The distance between the image and the nearer lens surface is called the *back focus* of the lens. In the case of a system of lenses or a single thick lens, the principal focal distance may be considerably different than the back focus.

403. Parallel Rays Meet in the Principal Focal Plane.—Let P, P' represent the principal points, and F, F' the principal foci of a positive lens. Consider light from the point A, of a small object perpendicular to the principal axis, which traverses the path AD parallel to the principal axis, and also light which traverses some other path AK. After emergence from the lens, the light will proceed along the paths dA_1 and kA_1.

Also consider light from some point C such that $AC = KP'$. That is, the rays AK and CP' are parallel. If the image is not to be distorted we must have

$$\frac{AC}{AB}\left(=\frac{kP}{dP}\right)=\frac{A_1C_1}{A_1B_1},$$

or,

$$\frac{kP}{A_1C_1}=\frac{dP}{A_1B_1}. \quad \ldots \ldots \quad (231)$$

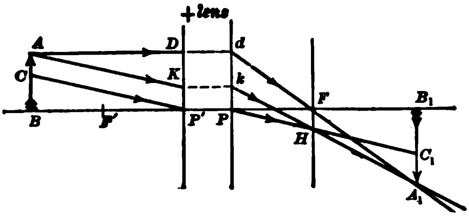

FIG. 462.

From the similar triangles kHP and A_1HC_1,

$$\frac{kP}{A_1C_1}=\frac{PH}{HC_1},$$

and from the triangles dPF and FB_1A_1,

$$\frac{dP}{A_1B_1}=\frac{PF}{FB_1}.$$

Therefore, (231) becomes

$$\frac{PH}{HC_1}=\frac{PF}{FB_1}.$$

From the proposition, "if a straight line divides two sides of a triangle proportionately, it is parallel to the third side," it follows that FH is parallel to B_1C_1. If the object is at right angles to the principal axis, the image is at right angles to the principal axis. Consequently, FH is at right angles to the principal axis.

The plane through the principal focus normal to the principal axis is called the *principal focal plane.* It has now been shown that if the image is undistorted, incident parallel rays (AK and CP'

in the diagram), after refraction by a lens, meet at a point in the principal focal plane. If the light be supposed to originate at a point H in the principal focal plane we will have the converse proposition: Two rays from a point in the principal focal plane of a lens will after emergence be parallel.

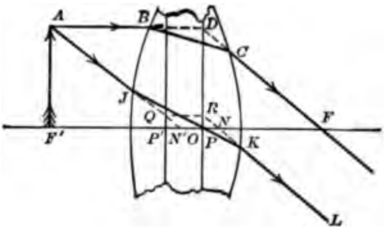

FIG. 463.

404. Nodal Points. — Consider a positive lens with principal points situated at P and P', and principal foci at F and F', Fig. 463. Light parallel to the principal axis and incident at B will after emergence pass through the principal focus F. From A, on the focal plane through F', let light pass parallel to CF to the point J. From the property of rays from a point in the focal plane just proven, the emergent ray KL is parallel to CF.

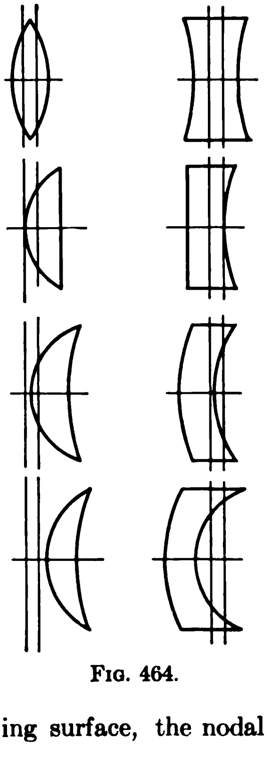

FIG. 464.

Produce AJ and LK. The intersections, N' and N, of these lines with the principal axis have the property that incident light directed toward the nearer one will on emergence proceed in the parallel line that passes through the other. The two points, N and N', having the property that if incident light is directed toward one, the emergent light will proceed in a parallel direction from the second, are called *nodal points*.

The positions of the nodal points of lenses of several shapes when made of glass of index of refraction 1.5, are given in Fig. 464. It will be noted that, depending upon the curvature of the bounding surface, the nodal points may be within or outside the lens.

For a lens bounded by surfaces of the same radii of curvature, the nodal points divide the distance between the two poles into thirds. The nodal points of a sphere coincide at the center. A lens with sides of different radii of curvature, struck from the same center, has a single nodal point. It is possible to construct a lens of glass such that the nodal points are at infinity. This lens has no principal focal length.

The principle underlying the ordinary method of locating experimentally the nodal points of a lens or lens combination will now be considered. Let a narrow beam of parallel light be incident on the pole of a lens having nodal points N, N', Fig. 465. A narrow beam of light incident at the pole of a lens is called a *centric pencil*. After traversing the lens, light incident in the direction AN' will emerge in the direction NB. If the lens be rotated through an angle θ about an axis through N, light incident in the direction AN'_1 will emerge in the same direction as before. That is, if a centric pencil be incident on a lens, the direction of the emergent light will be unaffected by a rotation of the lens about a line which is perpendicular to the principal axis and passes through the nodal point of emergence.

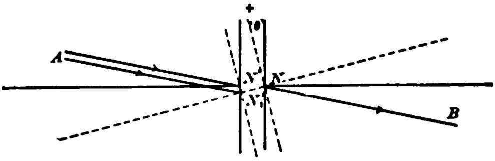

FIG. 465.

The fact that if a lens or lens system be rotated about a line through the nodal point of emergence, the image will remain stationary is utilized in the "panoramic" camera. In this camera the lens system is turned about such an axis while the film either remains stationary or is moved by clockwork in the opposite direction. In order that each part of the image may be equally sharp, the film is bent into the arc of a circle having a radius equal to the focal length of the lens. By this device, the various parts of an object extending through any angle can be successively brought to sharp focus in proper relation on a long sensitive film.

The principal points, the nodal points and the principal foci constitute a system called the *cardinal points* or *Gauss points* of a lens. Knowing the cardinal points of a given lens or lens system, the emergent ray corresponding to any assigned incident ray can be constructed.

405. Equivalent Points.—It will now be shown that in the usual case in which the media on the two sides of the lens are the same, the nodal points coincide with the principal points. From the construction of Fig. 463, the triangles $QP'N'$ and RPN are equal, and the triangles $AF'N'$ and DPF are equal.

Whence,

$$P'N'(=PN)=PF-NF, \quad . \quad . \quad . \quad . \quad . \quad (232)$$

and

$$F'P'+P'N'=PN+NF.$$

Or since $P'N'=PN$

$$F'P'=NF.$$

Substituting in (232),

$$P'N'=PN=PF-F'P'.$$

When the media on the two sides of the lens are the same, the two principal focal lengths are equal; that is, $P'F'=PF$. Whence, in this case,

$$P'N'=PN=0.$$

Consequently, when the media on the two sides of the lens are the same, the nodal points coincide with the principal points. The term *equivalent points* is used to denote the superimposed principal and nodal points of a lens or lens system. The two planes through the equivalent points normal to the principal axis are called the *equivalent planes* of the lens or lens system.

406. Summary of the Properties of Equivalent Points and Planes.—The properties of equivalent points and equivalent planes which are of greatest utility in the study of lenses are as follows:

1. An incident ray, parallel to the principal axis of a lens, will emerge as though it had proceeded to the second equivalent plane

and had there been changed in direction so as to pass through a principal focus.

2. An incident ray, directed toward any point on the first equivalent plane, will emerge from the lens as though it came from a point at an equal distance from the principal axis on the second equivalent plane.

3. An incident ray, directed toward the first equivalent point, will emerge from the lens as a parallel ray from the second equivalent point.

4. Incident parallel rays meet after emergence in a principal focal plane.

5. When a centric pencil is incident on a lens, the lens can be rotated about a line perpendicular to the principal axis through the equivalent point of emergence without motion of the image being produced.

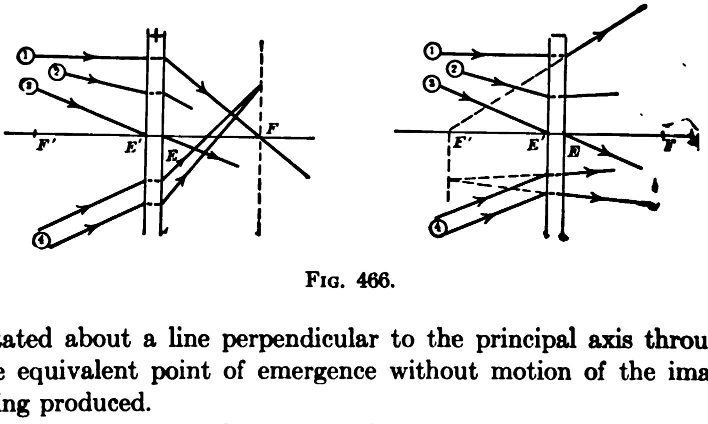

FIG. 466.

The first four of these properties are used in determining the paths of light through lenses. The fifth is used in locating experimentally the point from which to measure principal focal lengths.

§ 2. *The Position of the Image*

407. The Location of the Image of an Object.—In the following diagrams, the object will be represented by a heavy arrow, and its real image by a light arrow. After transmission by a lens, light, without forming a real image, may diverge as though it came from a second object similar to the actual object. The region from which the light appears to diverge is called a *virtual image*. A virtual image will be represented by a light dotted arrow. An image in space used as an object for another lens is called a *virtual* or *aerial object*, and will be represented by a heavy dotted arrow. The position of the image of an object situated at various distances from a lens will now be determined; first graphically, and then analytically. Only lenses will be considered that are bounded on the two sides by the same medium.

For uniformity of representation, the light will always be represented as traveling from the left to the right.

(*a*) *Converging Lens. Object Farther from the Lens than the Principal Focus.*—In Fig. 467, let F, F' and E, E' represent the principal foci and the equivalent points, respectively, of a converging lens. These four points being given, the actual curvature of the surfaces and the index of refraction of the glass may be dismissed from consideration.

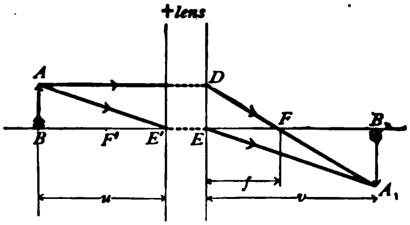

FIG. 467.

From the end A of the object draw two lines,—one parallel to the principal axis as far as the second equivalent plane, and another through the first equivalent point E'. Then, in accordance with Property 1, Art. 406, draw a line from D through the principal focus F. And in accordance with Property 3, draw a line from E parallel to AE'. The image of the point A will be on each of these lines and consequently will be at their intersection A_1. In the same manner, the position of the image of any other point of the object can be found. The image A_1B_1 is real and inverted.

An analytical expression for the position of the image will now be found. In the figure, the triangles ABE' and A_1B_1E are similar, and also the triangles DEF and A_1B_1F. Whence,

$$\frac{E'B}{EB_1}=\frac{AB}{A_1B_1}\left(=\frac{DE}{A_1B_1}\right)=\frac{EF}{FB_1}.$$

It is customary to represent the principal focal distance EF

by the symbol f, the distance $E'B$ of an object from the first equivalent plane by u, and the distance EB_1 of the image from the second equivalent plane by v. Using this notation, the above equation becomes

$$\frac{u}{v}=\frac{f}{v-f}$$

or,
$$uv-uf=vf.$$

Dividing by uvf,

$$\frac{1}{v}=-\frac{1}{u}+\frac{1}{f} \quad \ldots\ldots\ldots \quad (233)$$

(*b*) *Converging Lens. Object Nearer the Lens than the Principal Focus.*—From the end A of the object, Fig. 468, draw two lines,—one parallel to the principal axis as far as the second equivalent plane, and another through the first equivalent point E'. Then, Property 1, Art. 406, draw a line from D through the principal focus F. And in accordance with Property 3, draw a line from E parallel to AE'. The image of the point A will be on the intersection of these two lines, or of these lines produced. Whence, A_1 is the image of A. In the same manner, the position of the image of any other point of the object can be found. The image A_1B_1 is virtual and erect. That is, no image is actually formed, but the light emerging from the lens proceeds as though it came from an object similar to AB but situated at A_1B_1.

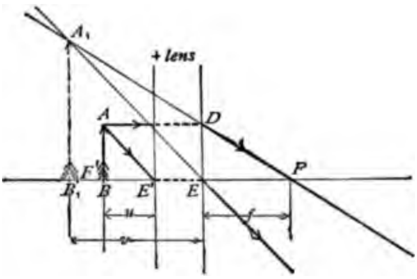

FIG. 468.

An analytical expression for the position of the image may be determined as in the preceding case. From the construction of the figure, the triangles ABE' and A_1B_1E are similar, and the triangles DEF and A_1B_1F are similar. Consequently,

$$\frac{E'B}{EB_1}=\frac{AB}{A_1B_1}\left(=\frac{DE}{A_1B_1}\right)=\frac{EF}{B_1F}.$$

Using the customary notation, as indicated in the figure, this becomes

$$\frac{u}{v}=\frac{f}{v+f}$$

Whence,

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (234)$$

(*c*) *Converging Lens. Aerial Object to the Right of the Lens.*—Before striking a lens or mirror, light from a point source is divergent. After reflection from a mirror or refraction by a lens, light proceeds along rays that may be divergent, parallel or convergent. We will now consider the effect of a convex lens on convergent light.

Let light diverging from a point A, not shown in the figure, and rendered convergent by a lens not shown, proceed toward the point A_1, Fig. 469. Let a converging lens having equivalent points E', E, and principal foci F', F, be interposed in the path of the light. The ray GE' incident at the first equivalent point will emerge from the lens in the parallel direction EE'_1 from the second equivalent point (Property 3, Art. 406). An incident ray NO, parallel to the principal axis, will on emergence be bent into the line DD_1 passing through the principal focus F (Property 1, Art. 406). The intersection of EE'_1 and DD_1 is the position of the image of the point A_1 when the given lens is used.

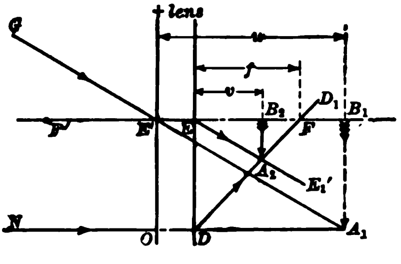

FIG. 469.

Proceeding as above, we find the relation between the aerial object distance u, the actual image distance v, and the principal focal distance f, to be

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (235)$$

(*d*) *Diverging Lens. Real Object.*—Here again, the two triangles ABE' and A_1B_1E, Fig. 470, are similar, and the triangles DEF' and A_1B_1F' are also similar. Whence,

$$\frac{E'B}{EB_1}=\frac{AB}{A_1B_1}\left(=\frac{DE}{A_1B_1}\right)=\frac{EF'}{F'B_1}$$

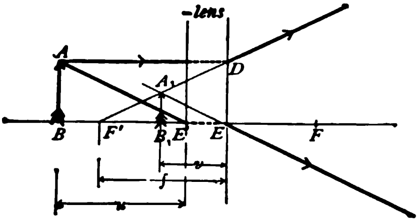

FIG. 470.

From the diagram, it will be noted that in the present case, the principal focal length, that is, the distance between the emergent equivalent point and the principal focus, is EF'.

Using the customary notation, we obtain

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

(*e*) *Diverging Lens. Aerial Object to the Right of the Lens and Outside the Principal Focal Distance.*—Consider the effect of a diverging lens on light already rendered convergent by a preceding lens. Without the diverging lens, Fig. 471, light following the paths GE' and ND would converge at the point A_1. On the introduction of the diverging lens having equivalent points E', E, and principal foci F', F, the emergent light will diverge as though it had come from the point A_2.

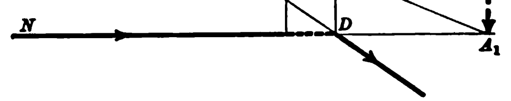

FIG. 471.

The triangles A_1B_1E' and A_2B_2E are similar, and also the triangles DEF' and A_2B_2F'. Whence, proceeding as in the other cases, and remembering that since this is a diverging lens, the principal focal length is EF', we obtain

$$\frac{1}{v}=-\frac{1}{u}+\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (237)$$

(*f*) *Diverging Lens. Aerial Object to the Right of the Lens and Within the Principal Focal Distance.* Without the diverging lens, Fig. 472, light following the paths GE' and ND would converge to the point A_1. On the introduction of the diverging lens having equivalent points E', E, and principal foci F', F, the emergent light will converge at the point A_2.

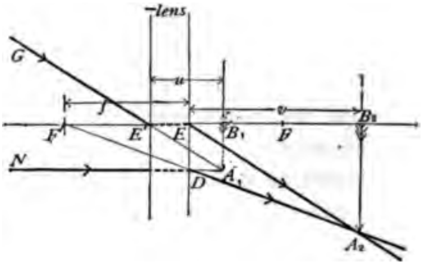

FIG. 472.

The triangles A_1B_1E' and A_2B_2E are similar, and also the triangles DEF' and A_2B_2F'. Whence, proceeding as in the other cases, and remembering that since this is a diverging lens, the principal focal length is EF', we obtain

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

It should be noted that when but one lens is employed a real image is always inverted.[1] By means of a second lens this image may be again inverted, that is, caused to be erect.

Before we can use Equations (233–238) in a numerical problem, we must know whether the given problem comes under Case *a*, *b*, *c*, *d*, *e* or *f*. For example, let it be required to find the principal focal length of a positive lens such that when $u=5$ in., $v=15$ in.

---

[1] This statement applies to all actual lenses. But it may be mentioned that if the distance between the convex spherical surfaces be sufficiently great, the real image will be erect. However, such a thick piece of glass would usually not be called a lens but would be called a rod with convex ends.

If $u>f$, we have Case *a*, and $f=3\frac{3}{4}$ in. Whereas, if $u<f$, we have Case *b*, and $f=7\frac{1}{2}$ in.

408. The Standard Lens Formula.—An inspection of Equations (233–238) shows that the relation between u, v and f for the various cases is the same, except as to the signs of these quantities. This suggests the possibility of adopting a convention such that by giving proper signs to the quantities in a single equation, one equation could be made to apply to all cases.

It should be noted that the object distance u is the distance from the *incident* equivalent plane to the object, whether real or aerial; the image distance v is the distance from the *emergent* equivalent plane to the image, whether real or virtual; the principal focal distance f is the distance from the *emergent* equivalent plane to the focus, whether real or virtual, formed by incident parallel rays.

Measuring u, v and f from the corresponding equivalent planes, let us consider these quantities as positive when they extend in the direction light is being propagated, and negative when they extend in the opposite direction. With this convention, by giving the proper signs to the quantities u, v and f, the equation

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f} \qquad \text{(239)}$$

will apply to either a converging or diverging lens in any assigned position relative to a real object, a real image, or an aerial object.

409. Magnification.—When light from a luminous object traverses a converging lens and forms a real image, the ratio of the length of the image to that of the object is called the *linear magnification* of the object, or the *linear magnifying power* of the lens.

Now the normal eye can be accommodated to rays that are either parallel or slightly divergent. Vision is most easy when the rays to the eye from a given point are parallel; that is, when the point is at a great distance. Vision is most distinct when the point is about 10 in. (25 cm.) from the eye. Observers usually focalize a microscope for most distinct vision. When the virtual image of the object seen in a microscope is 10 in. from the eye, the ratio of the size of this virtual image to the size of the object is taken to be the linear magnification produced by the microscope.

The ratio of the angle subtended at the eye by the image, to the angle subtended at the eye by the object, is called the *angular magnification* of the object, or the *angular magnifying power* of the lens or lens system. The term "linear magnifying power" is not applied to telescopes.

The proper magnification is determined by the limits of comfortable vision for the details of the image that are to be observed. The magnification produced depends upon the curvatures and material of the lens. Experience has shown that in reading type of the size used in this book, most people hold the book at a distance of about 25 cm. from the eyes. When the page is held at this distance, the angle at the eyes subtended by the individual letters has an average value of about 0.006 radian. From this fact we conclude that for most comfortable vision the size of the details should be such as to subtend an angle at the eye of about 0.006 radian. However, for short spaces of time we can study details, without undue strain, that are from one-tenth to ten times this size.

SOLVED PROBLEM

Problem.—A certain optical system consists of two positive lenses having principal focal lengths in the ratio 3 : 1. The distance between the equivalent points of each lens being given, and the distance between the adjacent equivalent points of the two lenses being one-half the sum of the focal lengths of the two lenses, locate graphically the equivalent points and the principal foci of

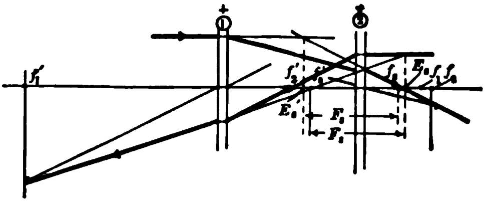

Fig. 473.

the system. On the diagram indicate the principal focal length of the system. From the diagram show whether the system is positive or negative.

SOLUTION.—In the figure, quantities with the subscript "1" refer to the lens marked "1," and quantities with the subscript "2" refer to the lens marked "2." Quantities with the subscript "s" refer to the lens system taken as a unit.

An inspection of the diagram shows that for this particular lens system, (*a*), one equivalent point lies between the lenses at a distance from the short-focus lens equal to about one-third the principal focal length of that lens; (*b*), the other equivalent point of the system lies beyond the short-focus lens and not far from the principal focus of the long-focus lens; (*c*), for light traversing the combination in one direction the system produces convergence, whereas for light in the opposite direction the system produces divergence; that is, the combination may be either a positive or negative system, depending upon the direction of passage of the light.

§ 3. *The Aberrations of Lenses and Lens Systems*

410. The Spherical Aberrations.—Spherical surfaces being the only ones that can be easily ground with considerable precision, lenses are almost always bounded by such surfaces. Only such lenses are considered in the present section. In Art. 401, it has been shown that a spherical wave, after traversing a spherical lens of wide aperture, will, in general, emerge with a front that is not spherical. When a spherical wave is incident on a lens, the departure of the form of the emergent wave front from a spherical form is called *spherical aberration.*

By filling with smoke or dust particles the region through which light passes, it is possible to render visible the path of light. By this device, photographs were taken of a beam of parallel light incident on a plano-convex lens, when the plane face was toward the light source, Fig. 474, and also when the convex face was toward the light source, Fig. 475. In the first case the lack of a point focus, that is, the departure of the emergent wave from the spherical form, is very marked. Light traversing the lens in this direction suffers great spherical aberration. In Fig. 475, the light converges to a focus of somewhat smaller dimensions. That is, the form of the emergent wave front is more nearly spherical. Consequently, in this case, the spherical aberration is less.

If a screen perpendicular to the principal axis of the lens be placed in the narrow part of the emergent beam of light, one will observe a small bright spot in the center of a larger disk of light. At a certain position the diameter of the disk will be a minimum. The bright disk of smallest diameter is called the *circle of least confusion.*

The positions of the principal points of a lens with spherical aberration depend upon the inclination of the incident light to the prinicpal axis, and also upon the distance between the point of incidence and the pole of the lens. There is no definite principal focus of a lens having spherical aberration. However, it is customary to assume the principal focus to be situated at the circle of least confusion, produced by an incident plane wave proceeding toward the center of the lens in the direction of the principal axis. The principal focal length of such a lens is usually taken to be the distance from this circle of least confusion to the emergent equivalent point for axial rays.

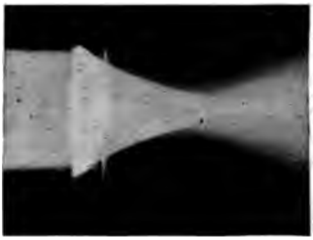

FIG. 474.

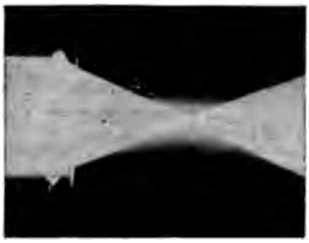

FIG. 475.

Due to spherical aberration, the image of a point source will never be a point, but will be a nebulous spot. The image of an extended object will, in general, be indistinct, curved and distorted.

Five different spherical aberrations are distinguished as follows: axial spherical aberration, astigmatism, curvature of field, distortion and coma.

411. Axial or Longitudinal Spherical Aberration.—A narrow cone of light proceeding from a point, or advancing toward a point, is called a light *pencil.* After refraction by a lens, or reflection from a mirror, a pencil may be cylindrical. A pencil parallel to the principal axis of a lens is called an axial pencil. An axial pencil coinciding with the prinicpal axis is called a direct axial pencil; an axial pencil incident at a point remote from the pole is called an eccentric pencil; a pencil incident on the pole of a lens is called a *centric* pencil; a centric pencil oblique to the principal axis is called an *oblique centric* pencil. A wide pencil is called a *beam.*

In Art. 401 it is shown that light parallel to the principal axis and incident near the edge of a converging spherical lens will come to a focus nearer the lens than will light incident near the center. In the case of a diverging lens a similar statement holds: The virtual focus for light parallel to the principal axis and incident near the edge is nearer the lens than is the virtual focus for light incident near the center. The longitudinal distribution of the focus of an axial beam is called *axial* or *longitudinal aberration.*

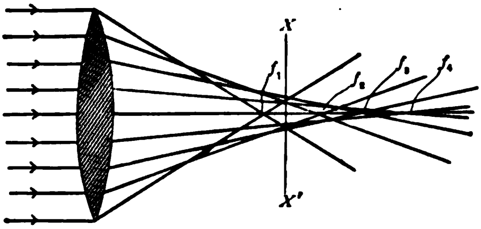

FIG. 476.

The axial aberration for the lens illustrated in the figure is measured by the distance (f_1f_4) between the focus for the marginal pencils and the focus for the centric pencil. With a convex lens, f_4 is to the right of f_1, whereas with a concave lens, f_4 is to the left of f_1. The axial aberration of a converging lens is taken to be positive. Then that of a diverging lens is negative.

Associated with the axial spreading of the focus is a transverse spreading. Due to this cause, the image of a point source will have an appreciable magnitude.

The axial and transverse aberration of a converging lens can be counterbalanced to a considerable extent by the addition of a diverging lens of greater focal length.

412. Astigmatism.—In Fig. 477 is shown the effect of a converging lens of large aperture on a plane wave incident at a great angle to the principal axis of the lens. In this case the emergent wave, instead of converging to a point or to a small circle of least confusion, converges to two "focal lines." In the figure, one of the focal lines shows as a short bright line parallel to the side of the page. The other focal line is normal to the plane of the page at the place where the beam appears most narrow. When a wave which on incidence is spherical, converges after emergence to two focal lines, the phenomenon of departing from a point focus is called *astigmatism* (i.e., without a point). The distance between the two focal lines is called the *astigmatic difference.*

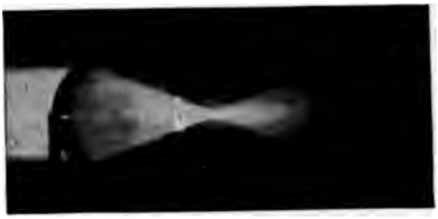

Fig. 477.

The two focal lines are at right angles to one another. The one nearer the lens is normal to, and the one farther from the lens is in, the plane containing the object point and the principal axis of the lens.

The difference between the effects of a spherical lens on axial beams and on oblique beams can be illustrated by means of a string model such as is represented in Fig. 478. Rays diverging from a point O on the principal axis of the lens L are converged to a point focus F. Rays diverging from a point B at a considerable distance from the principal axis of the lens converge to two line foci VV' and HH'.

Imagine a diagram on a blackboard, in which there are long white lines in various directions, to be placed normal to the axis of a converging spherical lens. Let a white screen be placed in the focal plane of the lens. The image formed on this screen will have the following characteristics: the image of all lines directed toward the intersection of the principal axis of the lens and the plane of the diagram will be sharp to the edge of the field. This is due to the fact that although the images of all points near the edge of the object field will be lines, these image lines are radial, and for any radial line of the object the image lines will be superposed. The images of all lines of the object perpendicular to the radial ones just considered will be broad and blurred except near the center of the image field. This is due to the fact that the image of each point of one of these object lines is a line perpendicular to the object line. The length of the image line of a point near the axis of the lens is small, but is appreciable for points farther from the axis.

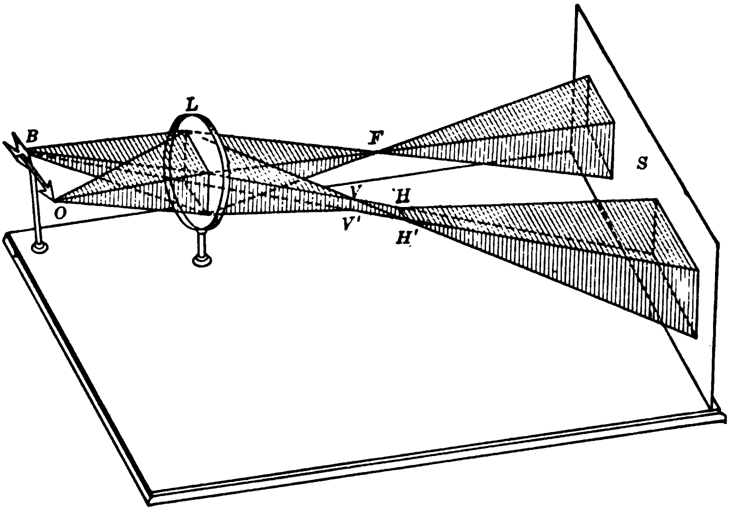

Fig. 478.

If the receiving screen be brought nearer the lens than the principal focus, a position will be reached such that the images of the ends of the radial lines of the object will be broad and nebulous, and the images of lines perpendicular to them will be distinct.

If with the receiving screen in the focal plane of the lens we use a diagram consisting of black lines on a white background, the images of the radial lines will be distinct and of the same width throughout, whereas images of the lines perpendicular to them will be narrower at the edge of the image field than at the center. (Why?) They will be nebulous at the center and sharp at the edge of the field. (Why?)

By means of simple lenses of different thicknesses and refractive indices, bounded by spherical surfaces of different curvatures, a compound lens can be constructed that will produce negligible astigmatism.

Lenses are sometimes used which are bounded by surfaces that are cylinders with parallel axes. After traversing a converging cylindrical lens, a wave which on incidence was plane will converge to a single focal line. This line focus is the principal focus of the lens. After refraction by a converging cylindrical lens, light from a point source will produce a line focus that may be either real or virtual depending upon the distance between the source and the lens. A compound lens consisting of a spherical and a cylindrical component produces astigmatism in all pencils whether they are incident axially or obliquely, centrically or excentrically.

413. Curvature of Field.—A pencil of homogeneous light *AB*, advancing toward the principal axis and incident at any point of a lens, will be brought to a focus at a point nearer the lens than will a pencil *CB*, parallel to the principal axis and incident at the same point of the lens. In other words, the focal length of a lens for the pencil *AB* is less than the focal length for light advancing parallel to the principal axis and incident on the same point of the lens. The greater the inclination of *AB* to the principal axis, the shorter will be the focal length. For this reason the image of an object plane normal to the principal axis will be curved. The lack of planeness of the image of a plane surface normal to the principal axis of a lens is called *curvature of field.*

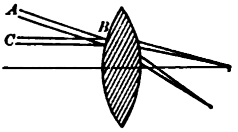

Fig. 479.

In Figs. 480 and 481 is represented a converging lens provided with a diaphragm or "stop" in which there is a small opening opposite the center of the lens. In Fig. 480 the object *AB* is farther from the lens than the principal focus, whereas in Fig. 481 the object is nearer the lens than the principal focus. In both cases the object is so large that it subtends a large angle at the center of the lens.

An inspection of Fig. 467 shows that when an object is beyond the principal focus of a convex lens, the shorter the focal length of the lens, the nearer to the lens will be the image. Then, since the focal lengths of a lens are smaller for oblique than for axial pencils, the image will be curved, with the concavity toward the lens, Fig. 480. The image of a plane normal to the principal axis will be saucer shaped.

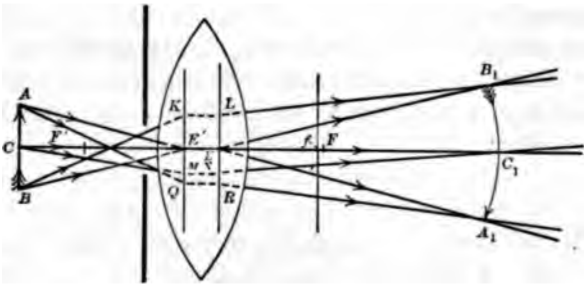

Fig. 480.

An inspection of Fig. 468 shows that when an object is between a convex lens and the principal focus, the shorter the focal length, the farther from the lens will be the image. Then, since the focal lengths of a lens are smaller for oblique than for axial pencils, the image (virtual, in this case) will be curved, with the concavity away from the lens, Fig. 481.

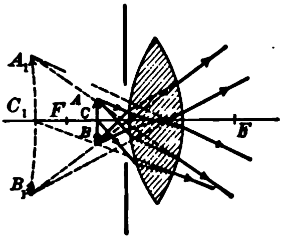

Fig. 481.

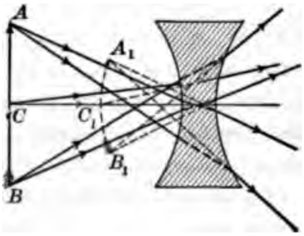

Fig. 482.

An inspection of Fig. 470 shows that the shorter the focal length of a concave lens, the nearer to the lens will be the image. Then, since the focal lengths of a lens are smaller for oblique than for axial pencils, the image will be curved, with the concavity toward the lens, Fig. 482.

By adding a convex lens to a concave lens, a compound converging lens can be produced that will give zero curvature of field.

Although for lenses having spherical aberration there is a pair of principal planes for pencils of each different obliquity to the principal axis, the variation in the position of the principal planes is too small to be represented in the diagrams of this and the succeeding Articles.

414. Distortion.—A nonuniform magnification of an image is called *distortion*. Consider oblique pencils which come from points of a distant object and are incident on a simple converging lens. In Fig. 483 there is a diaphragm between the object and the lens, and in Fig. 484 there is a diaphragm between the lens and the image. Draw through the first equivalent point a ray parallel to the incident pencil. The ray from the second equivalent point will intersect the principal focal plane at A'. But since a pencil incident near the edge of a simple lens is refracted more than a pencil in a parallel direction incident near the pole of the lens, the pencils in the above figures will converge, not at a point A', but at some point a'. Thus, with a diaphragm between the object and the lens, Fig. 483, the edges of the image of an extended object will be less magnified than the central region. When the diaphragm is between the lens and the image, Fig. 484, the edges of the image of an extended object will be more magnified than the central region. All lines of an extended object that do not pass through the principal axis of the lens will appear curved in the image.

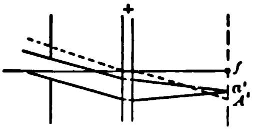

FIG. 483.

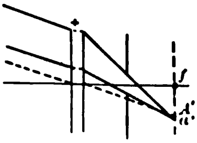

FIG. 484.

The entire image field will be curved. The image of a plane will not be plane, but will be spherical. For this reason the image formed on a plane screen will not be sharp throughout its extent. If the screen be placed at such a position that the central portion of the image is sharp, then the peripheral portions will be nebulous.

It should be remarked that although for the purpose of simplifying the construction of the diagrams in this Article the object has been considered to be at a great distance from the lens, there would be distortion of the image and curvature of field whatever the distance.

Figs. 485 and 486 are from photographs of images of an object consisting of a set of parallel straight lines crossed at right angles by another set of parallel straight lines. With the diaphragm between the object and a converging lens, Fig. 485, was produced; with the diaphragm between the lens and the image, Fig. 486, was produced.

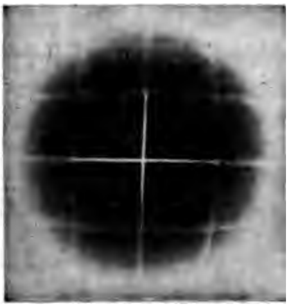

Fig. 485.

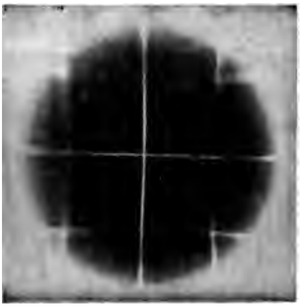

Fig. 486.

415. Coma.—Even though a lens were corrected for astigmatism, rays AB and AB', Fig. 487, equally inclined to the line AE', would converge to a point a, while rays AC and AC', equally inclined to the line AE', would converge to a point a'.

Light that has traversed the central portion of the lens comes to a point focus at A', but the light that has traversed a concentric zone of the lens comes to a ring focus. The diameter of the ring focus depends upon the diameter of the particular zone of the lens traversed by the light. The ring foci due to light that has traversed the different zones of the lens are not concentric, but are spread out into a line as shown in Fig. 488.

If the lens were also corrected for curvature of field, various ring foci would be in one plane. But even though the lens were corrected for astigmatism and curvature of field, the image of a point source at a distance from the principal axis would be a nebulous comet-shaped volume instead of a sharp point. The aberration that exists when light from a luminous point incident obliquely on a lens of large aperture forms a volume image instead of a point image is called *coma*.

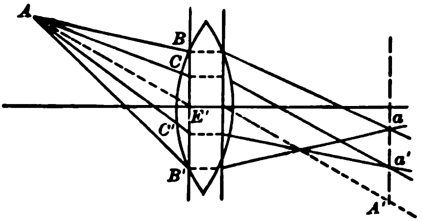

FIG. 487.

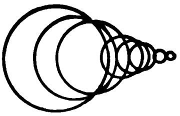

FIG. 488.

The outline of the volume image can be studied by placing a white receiving screen in the image and normal to the axis of the pencil. In Fig. 489 is shown the appearance on a screen placed at aa', Fig. 487. The point of the bright pear-shaped spot corresponds to the position A', and the middle of the opposite end to the position a. The upper side was out of the plane of Fig. 487, and toward the reader. The cross-section (perpendicular to the principal axis of the lens) of the volume image at the end of the image farthest from the lens is a broad, nebulous nearly straight

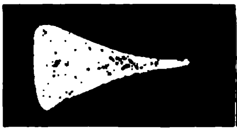

FIG. 489.

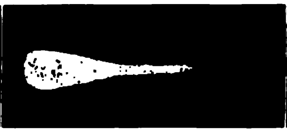

FIG. 490.

FIG. 491.

line, Fig. 491. The cross-section at a mid position is shown in Fig. 490.

416. Methods of Reducing the Effects of Spherical Aberration. —In the Articles immediately preceding, it has been shown that the spherical aberration of a simple lens depends upon the distance from the center of the lens to the point of incidence, the radii of curvature of the lens surfaces and the inclination of the incident ray to the principal axis. A spherical wave incident on a lens of wide aperture bounded by spherical surfaces will not be spherical on emergence.

A glass lens bounded by continuous faces can be made that has negligible longitudinal aberration, but such lenses are too expensive for ordinary use. A lens of negligible longitudinal spherical aberration can also be constructed that has one face that is not continuous but is divided into concentric zones, each zone having a spherical surface of the proper curvature. Such a lens is called a Fresnel lens. The small pressed glass Fresnel lenses used for search lights, head lights and semaphore lights, Fig. 492, consist of a single piece of glass. In the large Fresnel lenses used for lighthouses, each zone is made up of several separate pieces of glass.

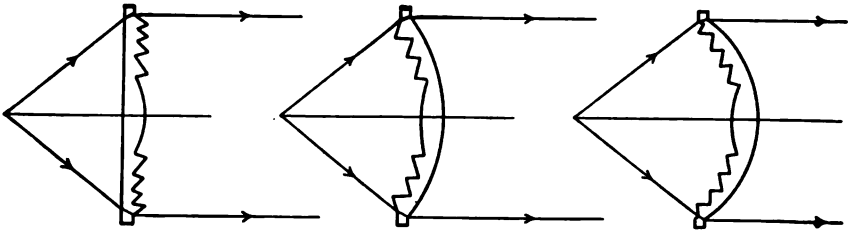

FIG. 492.

All of the spherical aberrations can be reduced by diminishing the effective aperture of the lens by stopping out either the marginal part or the central part. In cheap cameras this method is commonly employed. The method has the fault that by cutting off a large amount of light the brightness of the image is considerably diminished. A camera in which this method is used requires a long exposure.

A lens which considerably refracts light has a large spherical aberration, whereas one that refracts to but a small extent has a small spherical aberration. The magnitude of spherical aberration increases rapidly with increase of refraction. To produce small spherical aberration the shape of the lens must be such that at neither face shall there be a large deviation. For a given total deviation, the spherical aberration will be a minimum when the deviations at the two surfaces of the lens are equal,—that is, when the angles of emergence and incidence are equal. Fig. 493 shows light parallel to the principal axis and incident on the plane surface of a plano-convex lens. In Fig. 494 the light is incident on the convex surface of the same lens. In the second case the spherical aberration is less than in the first case.

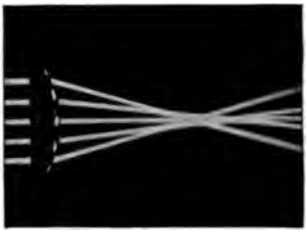
Fig. 493.

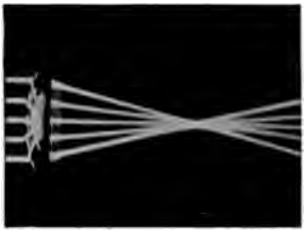
Fig. 494.

Fig. 495 shows a double convex lens having the two surfaces of radii of curvature in the ratio of 6 to 1, and made of glass of refractive index 1.5. In this case the angle of emergence equals the angle of incidence, and the spherical aberration is the smallest possible for a single lens. A lens having surfaces of such curvatures that the spherical aberration is minimum is called a "crossed lens."

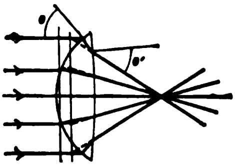
Fig. 495.

When moderate refraction is required, spherical aberration can be reduced to zero by combining a lens of negative spherical aberration with one of positive.

When a large refraction is required, spherical aberration can be made as small as desired by combining several lenses of such shapes that the refraction produced at each surface shall be small. Other things being constant, spherical aberration will be least when the whole bending of a ray is divided equally amongst the lens surfaces. For a system of two given lenses and for incident axial rays, it can be shown that each lens will produce the same deviation when the lenses are separated by a distance equal to the difference between their principal focal lengths. Consequently a system of two lenses of principal focal lengths f_1 and f_2 gives minimum spherical aberration when the distance x between the adjacent equivalent planes is

$$x = f_1 - f_2.$$

417. Chromatic Aberration.—It has been shown (Art. 380) that sunlight consists of a mixture of a great number of different colors. The same fact is more easily shown by means of a prism. Since light of different wave-lengths is refracted by different amounts, a lens will not bring the components of white or other unhomogeneous light to a single focus even if there be no spherical aberration. There will be a separate focus for each color. The failure of a lens to form a single focus for the various components of unhomogeneous light is called *chromatic aberration.*

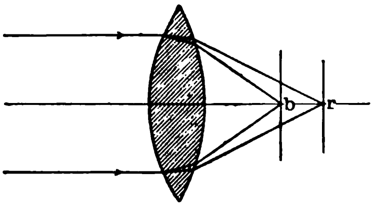

Fig. 496.

When white light from an object traverses a simple converging lens, Fig. 496, there will be formed a series of colored images of different sizes at different distances from the lens,—the blue image being nearest the lens and consequently smallest, while the red will be the farthest from the lens and consequently the largest. The addition of a diverging lens would cause all these images to retreat from the lens, the blue being more affected than the red. If the diverging lens have greater dispersive power and smaller refractive power than the converging lens, the images of some two different colors may be brought into coincidence. By cementing together two lenses having focal lengths in the same ratio as the dispersive powers of the materials of which they are composed, we shall have a compound lens that will cause the images of any two predetermined colors to be coincident and also of the same size. Thus, in Fig. 497, if the converging crown glass lens alone were used the red light would be focalized at *R*, the blue would be focalized at *B*, and light of other colors at intermediate points. The addition of a diverging lens made of glass having greater dispersive power than crown glass will cause light of two colors,—say red and blue, to come to a common focus *F*. A lens cannot be made that will bring waves of all lengths to the same focus. But by causing the brightest two colors to focalize at a single point the colored border about the central bright spot is practically eliminated. A compound lens that focalizes at a common point light of two wave-lengths is said to be *achromatic* for the two selected wave-lengths.

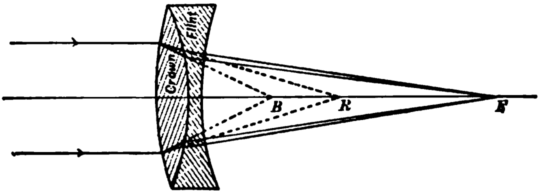

FIG. 497.

By means of a lens of three components of different focal lengths and dispersive powers, it is possible to cause images of three predetermined colors to coincide. Such a lens is said to be "apochromatic." An apochromatic lens used in three-color photography is illustrated in Fig. 498. This system consists of two compound lenses each of three elements.

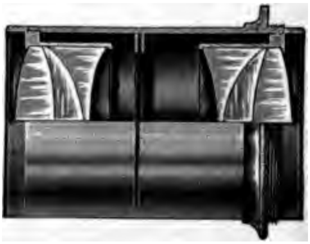

FIG. 498.

Partial achromatism can be obtained by the use of two lenses of the same kind of glass. By having the two lenses separated by the proper distance, light of various wave-lengths from a luminous point may be caused to emerge in rays that are parallel but not coincident, Fig. 499. Light of each color from an object will form a separate image. These separate images will not be in the same place and they will not be of the same size. But when the lenses are separated by a certain distance, all of the images will subtend the same angle at an eye placed in front of the combination. The sensation produced will be practically the same as though the images were coincident, that is, as though the systems were achromatic. It can be shown, though the proof will not here be given, that the condition for this partial achromatism is that the distance between the component lenses shall be

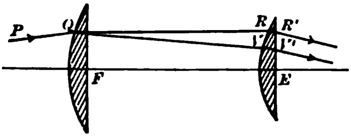

FIG. 499.

$$x=\tfrac{1}{2}(f_1+f_2).$$

CHAPTER XXVI

SOME OPTICAL INSTRUMENTS

§ 1. *The Human Eye*

418. Structure and Function of the Eye.—Optically, the human eye consists of an aperture *P* (called the pupil) in a diaphragm *I* (called the iris), a lens *L*, and a screen *R* (called the retina). These parts are enclosed in a nearly spherical opaque envelope *S* provided with a curved transparent round window *C*. The cornea, *C*, and also the transparent gelatinous liquids which fill the space between the cornea and the lens, and the space between the lens and the retina, have an index of refraction nearly equal to that of water. The lens has an index of refraction of about 1.44.

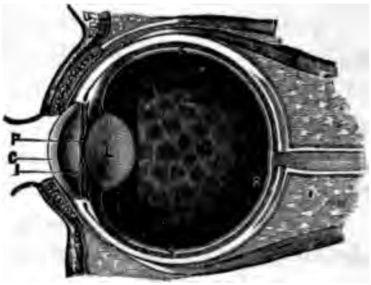

Fig. 500.

The two principal points of the human eye are between the cornea and the lens, and the two nodal points are within the lens. The front principal focal length is about 1.5 cm., and the rear principal focal length is about 2.0 cm. The front principal focus lies about 1.28 cm. in front of the cornea.

The image of an object in front of the eye is formed on the retina. The adjustment of the human eye so as to focalize on the retina images of objects at different distances is effected principally by an involuntary alteration in the curvature of the lens. In amphibia and snakes, the lens is moved back and forth.

One sees a point source with least effort when the rays from the point to the eye are nearly parallel. The point appears to be most distinct, however, when distant from the eye about 10 in. (25 cm.). In using a telescope, the instrument is focalized for most easy vision; that is, the distance between the lenses is adjusted till the emergent rays are parallel. In using a microscope however, most observers focalize the instrument for most distinct vision, that is, till the rays entering the eye appear to come from an object about 10 in. distant.

Different parts of the retina are unequally sensitive. The most sensitive part is a small spot of a diameter that subtends at the principal point of emergence of the lens an angle of less than one degree. The remainder of the retina is so much less sensitive than this spot that a person always involuntarily moves the eye into such a position that the retinal image is formed on this area.

The eye has chromatic and all the spherical aberrations to a high degree. As an optical instrument it is poor. But the powers of automatic adjustment of the focal length of the lens and of the diameter of the pupil make it an admirable sense organ.

Since the eye comprises a single lens, and real images are formed on the retina, these images are inverted.

419. Ocular Defects and their Correction.—Light from a distant object, after traversing a normal eye at rest, will form an image on the retina. In order that the image of a near object may be on the retina the lens of the eye must be made more converging. This is accomplished by the contraction of a muscle about the edge of the lens. The process of changing the focal length of the lens of the eye is called *accommodation.*

The accommodative power of the eye decreases with old age. Loss of accommodative power is called *presbyopia.* Good images of distant objects are produced on the retina of a presbyopic eye. But since the focus of light from an object near such an eye will be behind the retina, it follows that the image on the retina of a near object will be indistinct. To produce a distinct image of a near object the focus must be brought forward to the retina. This is accomplished by spectacles which effect the proper convergence of the light entering the eye. Since the presbyopic eye is nearly devoid of accommodative power, spectacles of different focal length would be required if equally distinct images were required of objects at different distances from the eye.

An eye which when at rest converges light from a distant point to a focus in front of the retina is said to be *myopic.* Since sharp images are produced on the retina of a myopic eye by light from near objects, myopia is popularly described by the name *near-sightedness.* Since the convexity of the lens of the eye cannot be diminished, there is no internal means of neutralizing myopia. Myopia is corrected by spectacles having diverging lenses of such curvature that parallel light after traversing the spectacle lens and the eye is brought to a focus on the retina. Fig. 501 is a photograph of a certain object as it would appear to a person who is somewhat myopic. Fig. 502 is a photograph of the same object as it would appear to the same person through the proper diverging lens.

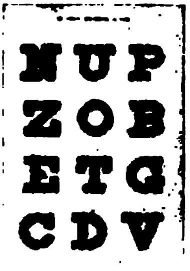

Fig. 501.

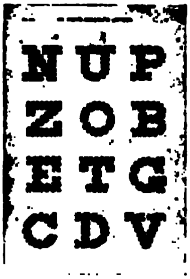

Fig. 502.

An eye which when at rest converges light from a distant point to a focus behind the retina is said to be *hypermetropic* or *far-sighted.* By accommodation, such an eye will focalize light from distant objects on the retina. But to bring light from a near object to a focus on the retina without too great eye strain, converging spectacle lenses must be employed. Fig. 503 is a photograph of a certain object as it would appear to a person who is somewhat hypermetropic. Fig. 504 is a photograph of the same object as it would appear to the same person through the proper converging

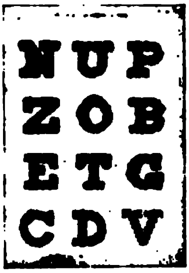

Fig. 503.

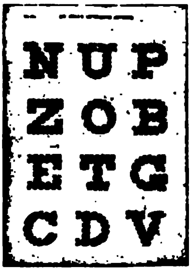

Fig. 504.

lens. If the images of distant objects are focalized on the retina without accommodation when spectacles are used, then by the aid of accommodation objects nearer the eye can be seen.

The faces of both the cornea and the lens of a normal eye are spherical surfaces. In many eyes, however, the curvature of either the cornea or the lens is not the same in different meridians. With these eyes, the image of a point source is a line and not a point. That is, these eyes are astigmatic. The astigmatism is said to be "regular" when the meridians of greatest and of least curvature are at right angles to one another. An eye that is regularly astigmatic cannot focalize at the same time lines of an object that are at right angles to one another. Regular astigmatism may be corrected either by a positive cylindrical lens that will increase the refraction along the meridian which has the least curvature, or by a negative cylindrical lens that will diminish the refraction along the meridian of greatest curvature. Fig. 505 is a photograph of a certain object as it would appear to a person who is somewhat astigmatic. It will be noticed that horizontal black lines are narrow and nebulous as described on p. 547. Fig. 506 is a photograph of the same object as it would appear to the same person through the proper cylindrical lens with the axis of the cylindrical focus horizontal.

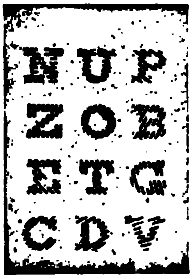

FIG. 505.

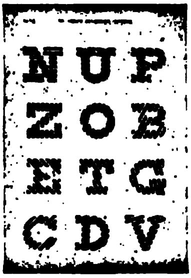

FIG. 506.

If the meridians of greatest and of least curvature are not approximately at right angles, or if there is an irregularity in the curvature along some one meridian, the astigmatism is said to be "irregular." Usually this form of astigmatism cannot be corrected by lenses.

When an astigmatic eye is also either near-sighted or far-sighted, one face of the spectacle lens is a cylindrical surface and the other face is a spherical surface.

Sometimes when the lens of the eye is excessively convex or opaque, it is removed. It is then necessary to use spectacles of convex lenses. As such an eye is incapable of any accommodation, spectacles of a different convexity will be required for near and for distant vision.

420. Visual Acuity.—Two point sources cannot be distinguished as separate points if the angle at the eye between lines from the points is less than about one minute. For the shape of an object to be discerned, the object must subtend at the eye an angle of not less than five minutes. For testing vision, test cards are used which have lines of letters of such sizes that at various distances, the letters subtend at the eye an angle of $5'$ and the thickness of the strokes of the letters subtends an angle of $1'$. For instance, a common form of test card has nine lines of letters, the letters of the top line of such a size that if the card were 200 ft. from an observer, the angle subtended at the eye by the height of a letter would be $5'$. The letters of the remaining lines are of such heights that each would subtend the same angle at distances of 100 ft., 70 ft., 50 ft., 40 ft., 30 ft., 20 ft., 15 ft., and 10 ft. respectively.

The ratio of the maximum distance at which an observer can read a line of letters, to the maximum distance at which a normal eye can read the same letters, is called the observer's *visual acuity*. Visual acuity is easily measured by means of a test card as above described at a known fixed distance from the observer. Beginning at the top of the card, the observer reads the letters of each line until a line is reached which cannot be read. Then, the observer's visual acuity equals the ratio of his distance from the card, to the distance at which a normal eye could read the last line which the observer could read. For example, if the observer when 20 ft. from the test card can read only so far as the line which a normal eye can read at 50 ft., the observer's visual acuity is $\frac{20}{50}$.

421. The Numbering of Spectacle Lenses.—Since a convex lens of small aperture converges a plane wave to a principal focus, the lens changes the curvature of the wave from zero to $1/f$, where f denotes the principal focal length of the lens. Similarly, a concave lens of small aperture imprints on the wave traversing it a curvature of $1/f$. And, in general, we see from (239) that the effect of a lens is to imprint on a wave traversing it a curvature of $1/f$. The cause of this change in curvature is the greater retardation of the speed of light where the glass is thick than where it is thin. Since this retardation is independent of the curvature of the incident wave, it follows that if the curvature of the wave remains constant while traversing the lens, the curvature of the emergent wave always differs from the curvature of the entrant wave by the constant amount $1/f$.

Spectacle lenses are so thin that no appreciable error is made in assuming that the " power " of a spectacle lens to alter the curvature of a wave front equals the reciprocal of the principal focal length. It is customary to denote the " power " of a spectacle lens by the reciprocal of the principal focal length expressed in meters. A spectacle lens of one meter focal length is said to have a " power " of one *diopter*. A lens of two meters focal length has a " power " of 0.5 diopter. One of 0.25 meter focal length has a " power " of four diopters.

In the case of an astigmatic eye, the positions of the meridians of greatest and least curvature are described in terms of the angles they make with the horizontal. Angles are measured counterclockwise from the right-hand end of the horizontal line through the eye, as seen by a person looking toward the eye.

The notation used to describe spectacle lenses will be illustrated in the following oculists' prescriptions:

O. D.+0.75 *D. sph.*
O. S.+2.00 *D. sph.*

This means, " for the right eye (oculus dexter) a positive spherical lens of 0.75 diopter, and for the left eye (oculus sinister) a positive spherical lens of 2.00 diopters."

Again, *O. D.*+3.50 *D. cy. ax.* 45°
O. S.+2.25 *D. cy. ax.* 135°

reads, " for the right eye a positive cylindrical lens of 3.50 diopters with the axis at 45° from the horizontal, and for the left eye a positive cylindrical lens of 2.25 diopters with the axis at 135° from the horizontal."

Again, *O. D.* −0.50 *D. sph.* +1.25 *D. cy. ax.* 90°
O. S. +0.75 *D. sph.* −1.00 *D. cy. ax.* 165°

means, "for the right eye a lens that is equivalent to a negative spherical lens of 0.50 diopter in combination with a positive cylindrical lens of 1.25 diopters with the axis vertical, and for the left eye a lens that is equivalent to a positive spherical lens of 0.75 diopter in combination with a negative cylindrical lens of 1.00 diopter with the axis at 165° from the horizontal."

§ 2. *The Camera*

422. The Photographic Camera.—The camera consists of a box having in one end an aperture covered with a converging lens, and in the opposite end a device for holding a screen sensitive to light. The distance between the lens and the sensitive plate is so adjusted that after traversing the aperture and lens the light from an object in front of the camera produces an image on the screen. In making this adjustment a plate of ground glass is often used. When the camera is "focused," that is, when the distance between the lens and the ground glass is so adjusted that the image on the ground glass plate is sharp, the lens is covered and a sensitive photographic plate is substituted for the ground glass. This plate is now "exposed" by uncovering the lens for the proper interval of time, and then "developed" and "fixed" by immersion in certain solutions.

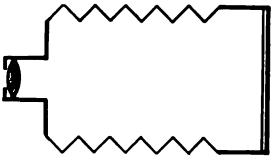

FIG. 507.

423. The Photographic Objective.—Most luminous bodies emit waves of different wave-lengths. The wave-length that produces the sensation called yellow affects most strongly the sensation of sight, whereas that which produces the sensation called violet affects most strongly the usual photographic plate. When the image has been focalized by the eye for the yellow waves, it is desirable that it should be in focus for the violet waves. Consequently, a camera lens should have the same focal length for yellow and for violet; that is, should be without chromatic aberration for waves of these two frequencies.

For landscape photography, a single crown-flint glass compound lens is commonly used. Though the various spherical aberrations are present, they will not be obtrusive if the object does not contain long straight lines. These aberrations can be greatly reduced by diminishing the lens aperture by means of a "stop." The use of a small aperture is, however, attended by the necessity of a longer exposure.

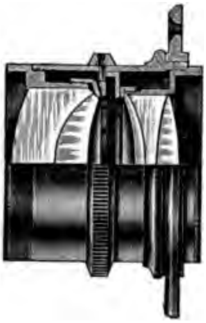

Fig. 508.

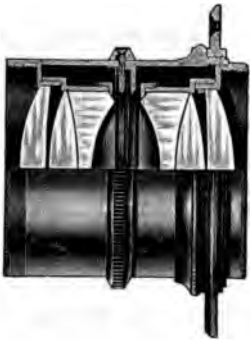

Fig. 509.

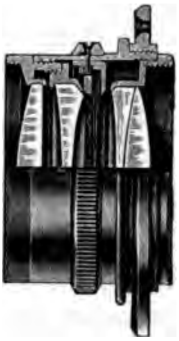

Fig. 510.

For a short exposure the stop must be large. With a large stop the image will be distorted and unequally sharp in different parts unless the photographic objective is corrected for spherical aberration. The various spherical aberrations can be reduced to any desired degree by the use of two compound lenses separated by a distance. If each compound lens is corrected for chromatic aberration, a system can be constructed that will be corrected for both chromatic and spherical aberration. Three modern corrected photographic objectives are illustrated in Figs. 508, 509 and 510.

Various names are given to photographic objectives, depending upon the particular property which is to be emphasized. The term "achromatic" is applied to a lens in which chromatic aberration has been corrected for two colors, and the term "apochromatic " to a lens corrected for three colors. The terms " rectilinear " and " orthoscopic " signify freedom from distortion. A lens free from astigmatism is called a " stigmat " (i.e., a point), an " anastigmat " (i.e., back again to a point), an " orthostigmat " (i.e., same point), a " verastigmat " (i.e., true point), etc.

A lens system nearly free of both chromatic and spherical aberration is termed " aplanatic." A lens that includes on the photographic plate a wide angle of view is called a " wide angle " lens.

424. The Teleobjective.—The size of the real image of a given object is proportional to the distance between the image and the emergent equivalent point of the lens. With a converging lens or lens system at a fixed distance from the object, this distance increases with increase in the principal focal length of the lens or lens system.

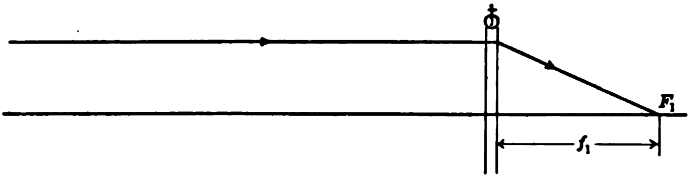

FIG. 511.

Fig. 511 represents parallel light from a distant point incident on a positive lens of principal focal length f_1. Suppose that f_1 is 10 in. The image will be nearly 10 in. from the back face of the lens; that is, the back focus of the lens is nearly 10 in.

By replacing this lens by one of four times the principal focal length, the image will be nearly four times as large, and the back focus will be increased in the same proportion. A bellows extension of 40 in. is possible, but not convenient.

If a negative lens is placed between the lens in Fig. 511 and the ground glass, the light from the distant point A will proceed as indicated in Fig. 512. It will be observed that the emergent equivalent plane of the combination E_s is at a considerable distance from either lens. The principal focal length f_s is now much greater than the back focus of the system. In fact, by using a negative lens of one-third the principal focal length of the positive lens, and a distance between the lenses of 7.66 in., the principal focal length of the system will be 40 in., and the back focus 10 in.

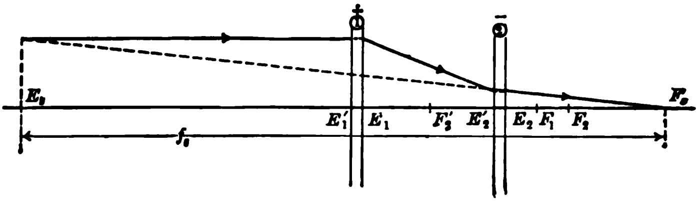

FIG. 512.

By means of this device the " back focus " can be kept small and still the image of the distant object may be large. By altering the distance between the two lenses, the principal focal length of the system can be altered (and consequently the size of the image), within wide limits. This property is applied in the construction of telephotographic objectives for taking pictures of distant objects. A modern telephotographic objective is illustrated in Fig. 513. This consists of a corrected objective, shown to the left, and a diverging lens combination, shown to the right.

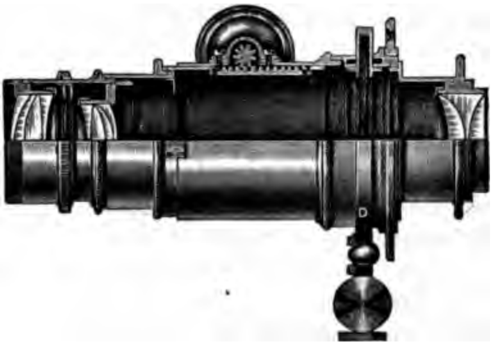

FIG. 513.

By using the converging component of the teleobjective alone, a photographic negative could be taken and an enlarged copy of this negative could be made. But even though the final picture be of the same size as the picture taken at once by means of the teleobjective, it would be decidedly inferior to the latter. In enlarging a picture, the grain of the negative as well as the minute irregularities of texture become more and more obtrusive as the magnification is increased. So that though the resolving power of the teleobjective is no greater than that of the converging component, a sharper picture of a distant object can be produced by its use.

425. Depth of Field.—Light from two points at different distances from a lens will form images that are at different distances from the lens. That is, sharp images of points at different distances from a lens will not be formed on a plane perpendicular to the principal axis of the lens. But in most photographic work the object consists of parts at different distances from the lens, and the plate receiving the image is plane. This condition can be met only by a sacrifice of sharpness. But for most purposes maximum sharpness is not required.

The image of a luminous point at A_1, Fig. 514, will be formed at A'_1. On a screen placed at A'_1 there will appear a small and distinct dot of light. On moving the screen either to the right or to the left, the small dot will fade into a larger circle of light, bounded by the cone of rays converging at A'_1. This circle is called the "circle of confusion."

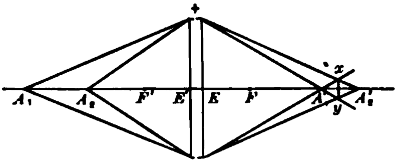

FIG. 514.

If the diameter of the circle of confusion due to each point of an extended object be not greater than about 0.01 in., the image of the object will be sufficiently sharp for most purposes. Hence, to produce a satisfactory picture, the screen may be out of the focal plane by such a distance that the diameter of the circle of confusion due to any object point does not exceed about 0.01 in.

If the luminous object point be moved to A_2, the image will move to A'_2. If the distance xy be not more than about 0.01 in.

the image formed on a screen at xy, of objects situated anywhere between A_1 and A_2, will be satisfactorily sharp. The quality which renders sufficiently sharp the images of objects situated at different distances from the lens is called *depth of field*, or *depth of focus*. It depends upon the principal focal length and upon the aperture of the lens. With a short focus lens the distance $A'_1A'_2$ corresponds to a greater range of object distance A_1A_2 than with a long focus lens. Again, since with a small aperture the distance $A'_1A'_2$ corresponding to a fixed diameter of circle of confusion xy is greater than for the same lens with a wider aperture, it follows that with a small aperture a satisfactory image will be formed of objects extending through a greater range of distances from the lens than if the same lens were used with a larger aperture.

Any lens will give depth of field if used with a small aperture. Great speed and great depth of field cannot be obtained at the same time.

426. The Fixed Focus Camera.—In the preceding Article it has been shown that by means of a stopped lens of short focal length a satisfactory image can be obtained of a landscape or other object that extends through a considerable range along the axis of the lens. This quality is utilized in the design of "universal" or "fixed focus" cameras; that is, in cameras in which the back focus is essentially the same for objects extending from a few feet from the lens to infinity.

In Fig. 515, A' is the image of the luminous point A, and xy is the diameter of the blurred image of a luminous point at infinite distance from the lens. If xy be not more than about 0.01 in., then the images of all points from A to infinity will be satisfactory. We will now find the distance AE', beyond which all points are in satisfactory focus on the screen through A'. This distance is called the "hyperfocal distance."

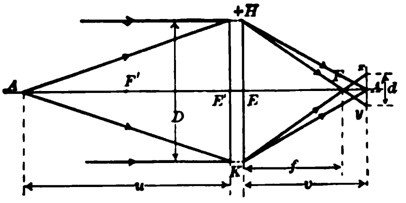

FIG. 515.

Since the triangles HKF and xyF are similar,

$$\frac{HK}{xy}=\frac{EF}{FA'},$$

or, using the notation indicated in the figure,

$$\frac{D}{d}=\frac{f}{v-f},$$

and, (233)

$$\frac{1}{v}=-\frac{1}{u}+\frac{1}{f}.$$

We shall now combine these two equations by eliminating v, and, solving for u, find the distance beyond which all points are in satisfactory focus. From the above equations,

$$Dv-Df=df \quad \text{or} \quad v=\frac{f(d+D)}{D},$$

$$uf=-vf+vu \quad \text{or} \quad v=\frac{uf}{u-f},$$

$$\therefore \quad \frac{uf}{u-f}=\frac{f(d+D)}{D}.$$

Clearing of fractions and solving for u, we find the distance beyond which all points are in satisfactory focus to be

$$u=\frac{(d+D)f}{d}. \qquad \qquad (240)$$

In this equation, d is the maximum allowable diameter of the blurred image of an object point at infinity, D is the diameter of the aperture of the lens, and f is the principal focal length of the lens.

427. Brightness of an Image.—If a point source of light is distant u from a lens having an aperture of diameter D, there will be incident upon the lens a quantity of light which varies directly with the area of the aperture and inversely with the square of the distance between the lens and the source. Consequently, the quantity of light in the image of a point source is

$$k'\frac{D^2}{u^2},$$

in which k' is a constant of proportionality.

If the source be an object of uniform intrinsic brightness and diameter x, the quantity of light incident upon the lens is

$$\frac{kD^2x^2}{u^2},$$

where k depends upon the intrinsic brightness of the object. Since this light is spread over an image of diameter y, the brightness of the image of the extended object is

$$\frac{kD^2x^2}{u^2y^2}.$$

Now for pencils making small angles with the principal axis of a corrected lens, Fig. 516, the ratio of the size of the object to the size of the image equals the ratio of the object distance to the image distance.

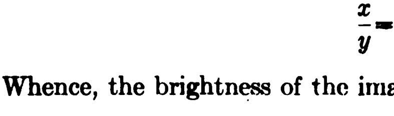

FIG. 516.

That is,

$$\frac{x}{y}=\frac{u}{v}.$$

Whence, the brightness of the image of an extended source is

$$\left(=\frac{kD^2x^2}{u^2y^2}\right)=\frac{kD^2u^2}{u^2v^2}=k\left(\frac{D}{v}\right)^2. \quad \ldots \ldots \quad (241)$$

If the object be at a great distance from the lens, the image will be near the principal focus, and v nearly equal to f. Under this condition, the brightness of the image of an extended object

$$\doteq k\left(\frac{D}{f}\right)^2. \quad \ldots \ldots \ldots \quad (242)$$

428. Diameter of Stop and Duration of Exposure.—The quantity which expresses the brightness of the image (242) is used in the computation of the duration of the exposure required with stops of different diameters. In taking a photograph, a stop of such a diameter is selected that the image will show the required detail and depth of field. After the stop has been selected, the required duration of exposure must be determined. The proper duration of exposure varies inversely with the brightness of the image, and (242) shows that this is measured by the ratio of the square of the diameter of the stop employed to the square of the principal focal length of the lens. It is customary to have for each photographic objective a series of stops marked with numbers which express the relative durations of exposure required when the various stops are used. If it be desired to construct a series of stops with which the exposures shall be in the ratio 1 : 2, 1 : 4, 1 : 8, etc., that is, with which the brightness of the image shall be in the ratio $1 : \frac{1}{2}$, $1 : \frac{1}{4}$, $1 : \frac{1}{8}$, etc., the procedure will be as follows.

We shall assume that the diameter of the largest stop which can be used is one-fourth the principal focal length of the lens. For this stop, $\frac{D_1}{f}=\frac{1}{4}$. Consequently, the brightness of the image, which is measured by $\frac{D_1{}^2}{f^2}$, is represented by $\frac{1}{16}$. For the next stop which is to give an image half as bright, $\frac{D_2{}^2}{f^2}$ must equal $\frac{1}{32}$. That is, $\frac{D_2}{f}=\frac{1}{5.65}$, or, $D_2=\frac{f}{5.65}$.

Proceeding in like manner, the table on the following page was constructed.

Most makers have adopted the series of diameters given in the table, and mark the individual stops with the numbers given in the first column. Few lenses can be used with such a large aperture as the first one in the table. But in using several lenses it is a great convenience to have the numbering of the stops start with a diameter, which, for each lens, is the same fraction of the focal length. In this case, though the larger apertures in the table are unavailable for use, the stops that can be used are num-

Diameter of Stop.	$\frac{D}{f}$.	$\left(\frac{D}{f}\right)^2$.	Relative Exposure.
$\frac{f}{4}$	$\frac{1}{4}$	$\frac{1}{16}$	1
$\frac{f}{5.65}$	$\frac{1}{5.65}$	$\frac{1}{32}$	2
$\frac{f}{8}$	$\frac{1}{8}$	$\frac{1}{64}$	4
$\frac{f}{11.3}$	$\frac{1}{11.3}$	$\frac{1}{128}$	8
$\frac{f}{16}$	$\frac{1}{16}$	$\frac{1}{256}$	16
$\frac{f}{22.6}$	$\frac{1}{22.6}$	$\frac{1}{512}$	32
$\frac{f}{32}$	$\frac{1}{32}$	$\frac{1}{1024}$	64
$\frac{f}{45.2}$	$\frac{1}{45.2}$	$\frac{1}{2048}$	128
$\frac{f}{64}$	$\frac{1}{64}$	$\frac{1}{4096}$	256

bered as in the first column. It should be remarked, however, that some makers construct series that start with apertures other than $\frac{1}{4}f$. The numbering of a series of stops in terms of diameters expressed as a fraction of the focal length of the lens is called the *f system* of numbering. Thus, the symbol "$f : 32$" represents a stop of a diameter one thirty-second of the principal focal length of the lens.

Stops are also numbered so as to indicate the relative exposure required, that is, according to the numbers given in the last column of the table. This is called the "uniform system" of numbering. Thus the symbol "*u. s.* 64" represents a stop that requires 64 times the duration of exposure that would be required with the first stop of the series.

§ 3. *The Telescope and the Microscope*

429. Galileo's Telescope.—This instrument consists of a converging lens, a diverging lens, and means for altering their distance apart. The converging lens is directed toward the object under observation and is called the *objective;* the diverging lens is toward the eye and is called the *ocular.* In Fig. 517 if the ocular were not present, light from the object AB would form an inverted real image at A_1B_1. If, however, a diverging lens (2) be introduced between this image and the objective (1), the light pencils from points of the object after traversing the ocular will converge less. If the focal length of the negative ocular be sufficiently short, the emergent pencils will diverge from one another. That is, the image will now be virtual. By producing back the emergent rays of the diagram it will be seen that the image is erect.

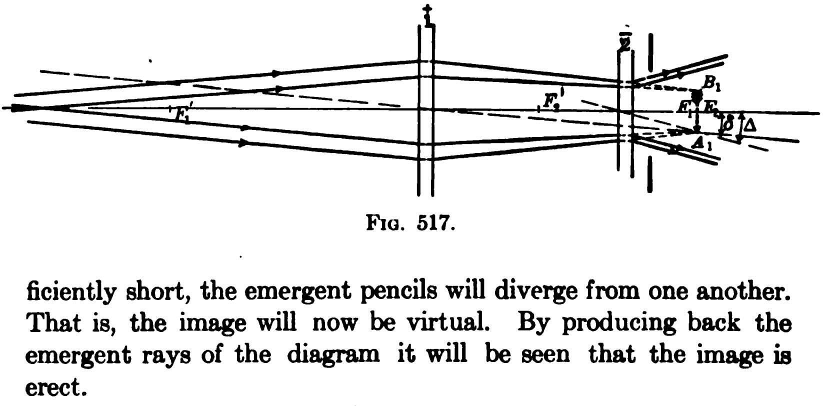

FIG. 517.

For most easy vision, the pencils emerging from the ocular should be cylindrical. This result is accomplished by making the distance from the ocular to the aerial object A_1B_1 equal to the principal focal length of the ocular. If the object be at a great distance, the aerial object A_1B_1 will be at the principal focus of the objective. The introduction of the negative ocular causes the image to be virtual. If the object be at infinity, the virtual image will also be at infinity. The virtual image is erect. The length of the Galilean telescope equals the difference of the principal focal lengths of the objective and ocular.

Assuming that the lenses are without aberration, the magnifying power of the Galilean telescope can be readily obtained. From definition, the angular magnifying power of an optical system is the ratio of the angles subtended at the eye by the image and by the object. In the case of a telescope, the distance from the object to the eye is so nearly equal to the distance from the object to the objective that it is customary to call the magnifying power of a telescope the ratio of the angle subtended at the eye by the image to the angle subtended at the objective by the object. Thus, representing the principal focal lengths of the objective and the ocular by f_1 and f_2, respectively, the magnifying power of the Galilean telescope focalized for most easy vision on a distant object is

$$M=\frac{\Delta}{\delta}=\frac{\tan^{-1}\dfrac{F_2A_1}{f_2}}{\tan^{-1}\dfrac{F_2A_1}{f_1}}\doteqdot\frac{\dfrac{F_2A_1}{f_2}}{\dfrac{F_2A_1}{f_1}}=\frac{f_1}{f_2}.$$

The ordinary opera glass, field glass, and marine glass consist of two telescopes of this type, one for each eye. Since Galileo's telescope gives erect images, is compact and cheap, it is well

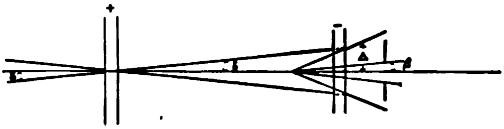

FIG. 518.

suited to such use. The serious disadvantage of this instrument is its very small field of view. This may be seen from Fig. 518. Since the cylindrical pencils emerging from the ocular diverge from each other, the pupil of an eye in front of the ocular will exclude all the light except that included within the small angle β. That is, light from any point outside of this angle, after emerging from the ocular, will not enter the pupil of the eye. The ocular should have a focal length not much shorter than the focal length of the eye. The greater the magnifying power of the ocular, the smaller the field of view. Since the light from different points of the object on emerging from the ocular is strongly divergent, if the eye is not placed close to the ocular the field of view will be farther diminished. It is unnecessary to have the ocular much larger in diameter than the pupil of the eye.

430. The Simple Astronomical Telescope.—In its simplest form, this telescope consists of two converging lenses mounted in opposite ends of a tube of adjustable length. The lens directed toward the object under observation is called the *objective;* the lens toward the eye is called the *ocular.* In Fig. 519, light from a distant object, after traversing the objective (1), forms an inverted real image A_1B_1. If the positive ocular (2) be placed at a distance equal to its principal focal length to the right of this image, the pencils emerging from the ocular will be cylindrical and will converge toward each other. An eye placed in front of the ocular will see an inverted image at infinity.

If the objective and ocular be without spherical aberration, the magnifying power of a simple astronomical telescope focalized for the most easy vision of a distant object will be

$$M=\frac{\Delta}{\delta}=\frac{\tan^{-1}\dfrac{F_2A_1}{f_2}}{\tan^{-1}\dfrac{F_2A_1}{f_1}}\doteqdot\frac{\dfrac{F_2A_1}{f_2}}{\dfrac{F_2A_1}{f_1}}=\frac{f_1}{f_2}.$$

That is, the magnifying power of this instrument varies directly as the principal focal length of the objective, and inversely as the principal focal length of the eyepiece. The principal focal length of the 40-inch objective of the great Yerkes telescope is 62 ft. By the use of eyepieces of various focal lengths, the magnifying power of a telescope may be changed within wide limits.

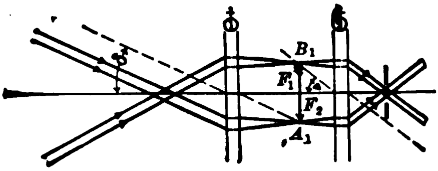

FIG. 519.

For the semi-angle Δ between the axes of the extreme pencils that enter the eye, Fig. 519, the total semi-angular field of view of the simple astronomical telescope is represented by the angle δ. This angle depends upon the diameter of the second lens, the distance between the two lenses, and the angle between the axes of the extreme pencils which, entering the eye, will produce distinct vision.

In using the telescope, the pupil of the eye should be placed in front of the ocular where the pencils emerging from the ocular cross the axis of the instrument. This place is usually indicated by a diaphragm containing a small aperture not much larger than the pupil of the eye.

431. The Use of a Telescope for Sighting.—In surveying and in a great variety of astronomical and physical determinations, angles are measured by the aid of telescopes. Suppose the angle XCY, Fig. 520, is required. If a telescope placed at C is pointed toward the object X, an image will be formed at some point in the focal plane of the objective. If the telescope be rotated about an axis through C till light from Y forms an image at the same point, the angle through which the telescope has been turned equals the angle XCY. This method requires a suitable circular scale attached to the telescope, and also a fixed point in the focal plane which can be caused to coincide in succession with the images of X and Y. The fixed point in the focal plane usually consists of the point of intersection of two very fine wires or fibers placed in the focal plane of the ocular. When the ocular is moved till the cross-wires are in the plane of the image, the eye will see the cross-wires and the image of the distant object coincident. This condition is attained by moving the eyepiece back and forth till a position is found such that when the eye is moved slightly from one side to the other there will be no displacement of the image relative to the cross-wires.

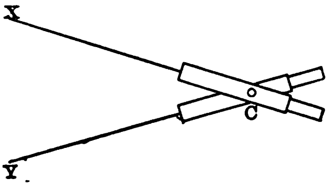

FIG. 520.

Small angular displacements of a body are frequently determined by means of the "Telescope and Scale Method." Suppose it is required to measure the deflection or angular displacement of a small magnetic needle produced by an electric current in a neighboring wire. The angle θ between the first position ns, Fig. 521, and the second position $n's'$ is the angle required. A small mirror is attached to the magnetic needle, and the image of a stationary scale $O'O''$ reflected by the moving mirror is observed with a telescope. Suppose that when the magnet is in the position ns, the point O of the scale is seen on the cross-hairs of the telescope, and that when the magnet is in the position $n's'$ the point O' is seen on the cross-wires. When the mirror is turned through the angle θ, the normal to the mirror has moved through the same angle. And since the angle of reflection equals the angle of incidence, the angle $O'CO$ equals 2θ. Denoting the distance of the scale from the mirror by L and the linear deflection OO' by x we have

$$\tan 2\theta = \frac{x}{L}.$$

From this we can easily obtain the deflection θ.

It should be kept in mind that cross-wires coincide in position with a real image. If the instrument has no real image, cross-wires are useless. For this reason, Galileo's telescope cannot be used for sighting.

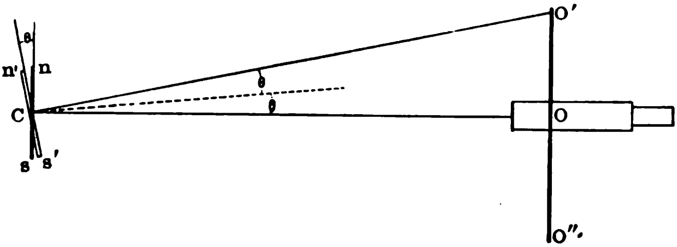

FIG. 521.

432. Telescope Objectives.—The function of the objective of an optical instrument is to collect a large amount of light emanating from the object under observation and concentrate it into a real image (as in the astronomical telescope), or into an aerial object (as in the Galilean telescope). By means of an objective of large aperture, stars can be perceived that are invisible to the naked eye. The great light-gathering power of the principal Yerkes telescope is due to the great diameter of the objective. This is an achromatic doublet 40 in. in diameter. It cost $66,000. The remainder of the telescope cost $55,000.

In the case of a telescope, the pencils from a point under observation are incident on the objective so nearly axially that the distortion produced by the objective is very small. The remaining spherical aberrations are kept low by giving the outer faces of the objective proper curvatures. The chromatic aberration is reduced by forming the objective of two lenses of different refractive indices and curvatures. An objective usually consists of a convex lens of crown glass and a concave lens of flint glass.

In order that the light incident centrally on the first component lens may be incident centrally on the second component also, the two components must be close together. In small telescopes they are usually cemented together so as to constitute a single compound lens.

433. Oculars or Eyepieces.—The purpose of the ocular or eyepiece of an optical instrument is to magnify the image formed by the light that has traversed the objective. Since the pencils incident on the ocular are oblique and excentric, the errors due to spherical aberration are much greater than they are for the objective. The defects are reduced to a minimum by employing such lenses that the bending shall be as small as possible at each surface. The bending at each refracting surface is reduced to a minimum, (*a*) by increasing the number of refracting surfaces; (*b*) by causing the bending at each surface to be the same; (*c*) by a proper selection of the curvatures of the refracting surfaces.

For most purposes, two lenses in the ocular are sufficient to make the bending at each surface small enough to reduce the spherical aberrations of the ocular to a proper amount. In the case of two lenses of given curvatures, and of principal focal lengths, f_1 and f_2, respectively, it can be shown that the condition for equal deviation of incident rays parallel to the principal axis is that the lenses shall be separated by the distance

$$x = f_1 - f_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (243)$$

For a system of two given lenses made of the same kind of glass, and of principal focal lengths f_1 and f_2, respectively, it can be shown that there will be minimum chromatic aberration when the combination is convergent, and the component lenses are separated by the distance

$$x = \tfrac{1}{2}(f_1 + f_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (244)$$

The lens of the ocular toward the eye is called the *eye lens;* the one toward the objective is called the *field lens.*

434. The Huyghens Eyepiece.—This eyepiece is a combination of two lenses of the same kind of glass designed to reduce to a minimum the effects of spherical and chromatic aberrations. If the condition of minimum spherical aberration (243) be combined with the condition of minimum chromatic aberration (244), then, for a system of two lenses made of the same kind of glass, we have

$$f_1 - f_2 = \tfrac{1}{2}(f_1 + f_2).$$

Whence, $$f_1 = 3f_2.$$

Consequently, for axial light pencils, the principal focal length of the field lens should be three times that of the eye lens, and the two lenses should be separated by a distance equal to the mean of their principal focal lengths.

This particular two-lens combination was devised by Huyghens and is called the Huyghens Eyepiece or Huyghenien Ocular. The component lenses are usually convexo-plane, that is, the convex surfaces are toward the incident light, Figs. 522 and 523.

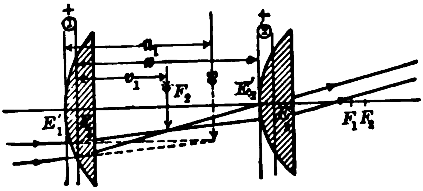

FIG. 522.

FIG. 523.

Besides this combination of two lenses of principal focal lengths in the ratio 3 to 1, Huyghens also used a combination of two lenses of principal focal lengths in the ratio of 2 : 1.

We shall now determine the distance which the field lens of a Huyghens 3 : 1 eyepiece must be from an aerial object in order that emergent rays may be parallel. In order that light emerging from the eye lens may be parallel, there must be a real image to the left at a distance equal to the principal focal length of the eye lens. And in order that a real image may be formed at this place, the field lens must be in a certain position relative to the aerial object due to the objective (objective not shown in the figure) that will now be determined.

Since for the Huyghens eyepiece of this type

$$f_1 = 3f_2,$$

and since the distance between the lenses equals the mean of their principal focal lengths, (244),

$$x\left[=\tfrac{1}{2}(f_1+f_2)\right]=\tfrac{1}{2}(3f_2+f_2)=2f_2.$$

Consequently, F_2 must be midway between the field lens and the eye lens.

Since the real image is formed at this point, the distance of this image from the field lens is

$$v_1=f_2=\frac{f_1}{3}.$$

Substituting this value in (235),

$$\frac{1}{u_1}\left(=\frac{1}{v_1}-\frac{1}{f_1}\right)=\frac{3}{f_1}-\frac{1}{f_1}=\frac{2}{f_1}.$$

Whence, the distance of the aerial object from the field lens is

$$u_1=\tfrac{1}{2}f_1.$$

Consequently, the field lens must be placed between the objective and the aerial object due to light that has traversed the objective, and at a distance from the aerial object equal to half the principal focal length of the field lens.

The aerial object will be curved and the peripheral portions less magnified than the central portion (Arts. 413 and 414). The eye lens will produce a curvature in the opposite direction. The eye lens will produce a greater magnification of the peripheral portions of the image than of the central portions. Consequently, the image seen by the eye will be nearly free of distortion and curvature. The spherical aberration can be further reduced by using for the eye lens a "crossed lens" (Art. 416), and for the field lens a convexo-concave lens having radii of curvature in the ratio 4 : 11.

If cross-wires were placed in the image within the Huyghens eyepiece, light from them would traverse but one lens. The image of the cross-wires seen by the eye would be distorted, and the greater the magnification of the eye lens, the greater the distortion. Since the final image of the object and the image of the cross-wires are unequally distorted, cross-wires are not employed in the Huyghens eyepiece except when the magnification is low.

By the graphical method used in the solution of the problem on

p. 543, it can be shown that in the case of a system of two lenses, if the principal focus of the lens upon which the light is first incident lies between the principal focus and the first equivalent point of the other lens, then the system is negative. Otherwise it is not. The Huyghens eyepiece is negative.

435. The Ramsden Eyepiece.—For measuring angles or distances, the cross-wires or scale must be placed in the real image of the object under observation, and any slight distortion produced by an observing eyepiece must affect in the same way any subsequent images of both the cross-wires and the original image of the object. Ramsden's eyepiece was designed with especial regard to these requirements. It consists of two converging lenses of equal focal length placed beyond the image formed by light that has traversed an objective lens.

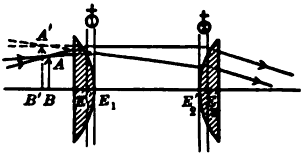

FIG. 524.

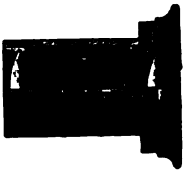

FIG. 525.

For minimum chromatic aberration of two simple converging lenses of the same material, the distance between them should be, (244), $x=\frac{1}{2}(f_1+f_2)$. Or, in the case of the Ramsden combination, the distance between the two equal components should be equal to the focal length of one of them. If this separation were made, then the field lens would be in the principal focal plane of the eye lens and the field lens would coincide with the image to be magnified. Such an arrangement would have the fault that dust or spots on the field lens would show in the field of view. To obviate this fault, the distance between the two lenses is made less than required to make the chromatic aberration a minimum. The separation is usually made two-thirds the principal focal length of one of the lenses. With simple lenses the departure from achromatism will be slight. If better achromatism is required, each lens may be composed of a flint-crown-glass combination as described in Art. 417. The defects of spherical aberration are made small by a proper selection of the radii of curvature of the lens faces.

Usually, the lenses are simple plano-convex, with the curved surfaces toward one another. Light from some point of an object, after traversing an objective not shown in the figure, will converge to a real image A, Fig. 524. After traversing the field lens (1), the light will diverge as though it came from a virtual image A'. The Ramsden eyepiece is positive.

For most easy vision, light from a point source should emerge from the eye lens in a parallel pencil. In order that light from a point source shall emerge from the eye lens in a parallel pencil, the first equivalent plane of the field lens must be distant from the image A by a definite amount which now will be determined.

With reference to the field lens, (1) Fig. 524, A is the source and A' is the image. From (234),

$$\frac{1}{BE'_1}=\frac{1}{B'E_1}+\frac{1}{f_1}. \quad \text{. . . . .} \quad (245)$$

Now from the figure,

$$B'E_1=B'E'_2-E_1E'_2.$$

But when light from a point source emerges from the eye lens in a parallel pencil,

$$B'E'_2=f_2.$$

And since the distance between the lenses,

$$E_1E'_2=\frac{2f_2}{3},$$

it follows that

$$B'E_1[=B'E'_2-E_1E'_2]=f_2-\frac{2f_2}{3}=\frac{f_2}{3}.$$

On substituting this value in (245), and remembering that $f_1=f_2$, we obtain

$$\frac{1}{BE'_1}=\frac{3}{f_1}+\frac{1}{f_1}=\frac{4}{f_1}.$$

Whence,

$$BE'_1=\frac{f_1}{4}.$$

Consequently, the field lens of a Ramsden eyepiece is placed at a distance equal to one-fourth of its own focal length beyond the image of the object formed by light that has traversed the objective.

436. The Erecting Eyepiece.—If light from an object passes through an objective and then through a simple converging eyepiece, a Huyghens eyepiece or a Ramsden eyepiece, the image which is formed is inverted. For an astronomical telescope or a microscope this leads to no inconvenience. But for a telescope used to view terrestrial objects we require an eyepiece that will give an erect image.

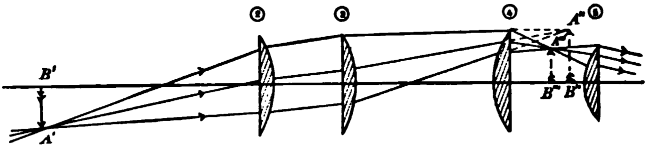

FIG. 526.

FIG. 527.

A single converging lens placed between the objective and any one of the eyepieces mentioned above will erect the image. By using two lenses, however, the refraction at any surface may be kept low, and consequently the spherical aberration (Art. 416). A common form of erecting eyepiece consists of either a Huyghens or a Ramsden eyepiece to which has been added two converging lenses of equal focal length separated by any convenient distance.

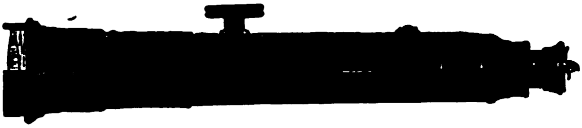

FIG. 528.

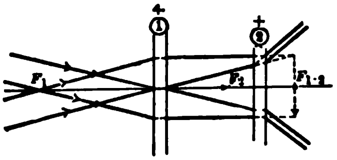

FIG. 529.—Galileo's Telescope.

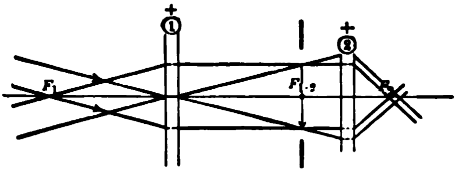

FIG. 530.—Simple Two-lens Astronomical Telescope.

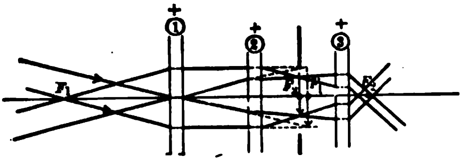

FIG. 531.—Telescope with Huyghens' Eyepiece.

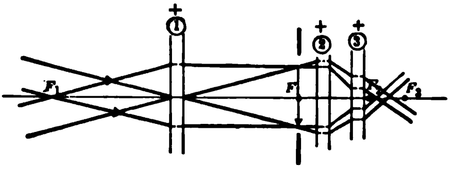

FIG. 532.—Telescope with Ramsden's Eyepiece.

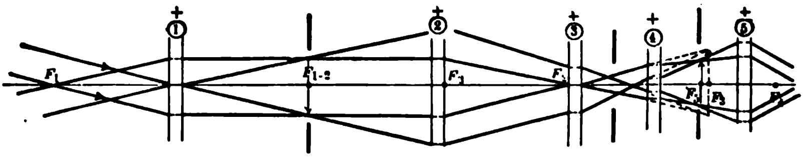

FIG. 533.—Telescope with Huyghens' Eyepiece and Erecting Lenses.

The four lenses are fitted into a tube and constitute a single unit. The arrangement of the lenses is shown in Fig. 526. $A'B'$ represents the inverted image of the object produced by light that has traversed the objective, not shown in the figure. (2) and (3) are the two lenses of equal principal focal length added to a Huyghens eyepiece (4)–(5).

If $A'B'$ is in the focal plane of lens (2), then the light which diverges from any point of $A'B'$ will be rendered parallel by (2), and after traversing (3) will again converge. If the lens (4) were not in the way there would then be formed another image $A''B''$ of the same size as $A'B'$ but right side up. But before reaching $A''B''$ the light passes through (4) and is focalized at $A'''B'''$.

An erecting eyepiece consisting of a pair of erecting lenses and a Huyghenien ocular is shown in Fig. 527. A reading telescope with an eyepiece consisting of a pair of erecting lenses added to a Ramsden ocular is shown in Fig. 528.

437. Ray Diagrams of the Ordinary Types of Telescopes.—The paths of light through telescopes with the eyepieces already described are indicated in the diagrams in the opposite page. In these diagrams, the principal focal length of the objective is the same in all cases. Each instrument is focalized for most easy vision for objects at a great distance. It is left as an exercise for the student to compute the lengths of the various telescopes in terms of the principal focal lengths of the lenses employed.

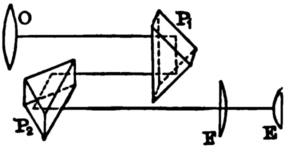

FIG. 534.

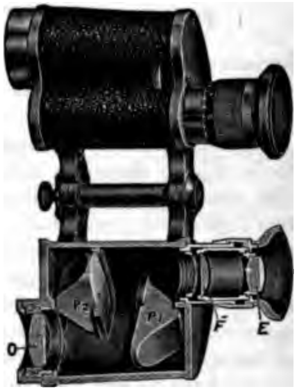

FIG. 535.

438. The Prism Binocular.—For viewing distant objects in their proper relations, the ordinary field glass, Fig. 517 has the advantage of compactness, but the serious disadvantage of a limited field of view. The astronomical and the terrestrial telescopes have a satisfactory field of view; but these instruments are too long for convenient use in the field. In the prism binocular, Fig. 534, we have the same objective and eyepiece that give the longer telescopes their wide field of view. By bending the path of light twice back upon itself, the length of the instrument is reduced to about one-third.

The reflections are produced by two totally reflecting prisms P_1 and P_2. The prism P_1 inverts the image, i.e., reverses top and bottom, leaving the right and left aspects unchanged. The prism P_2 perverts the image, i.e., interchanges the right and left sides. Thus, by means of these multiple reflections, the image has the same aspect as the object.

439. The Periscope.—The periscope is essentially a telescope bent in two right angles, with the objective end capable of rotation so that all parts of the horizon can be viewed without change of position of the observer. By means of a periscope an officer in a submarine can view objects above the surface of the sea when all of the vessel is submerged except the end of an inconspicuous tube. Also, by its aid an officer on a gun platform of a hidden battery can view distant objects on the other side of a hill or protecting shield. One form of periscope is diagrammed in Fig. 536. This consists of an objective O and eyepiece E in combination with a 90° totally reflecting prism P_1, an erecting prism P_2 and an Amici totally reflecting prism P_3. The upper end of the instrument is capable of rotation about a vertical axis so that objects at different positions on the horizon can be brought into the field of view.

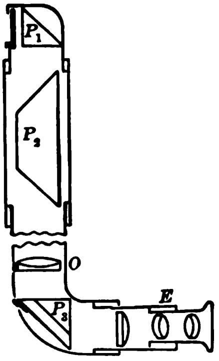

FIG. 536.

The panoramic gun sights used on artillery are similar to this form of periscope.

440. The Coincidence Range Finder.—In military operations it is necessary to be able to determine quickly the distance of objects from the observer. The usual methods for determining ranges depend upon the fact that if two angles and one side of a triangle are known, the other sides can be computed. In Fig. 537, suppose that two telescopes provided with graduated horizontal circles, and separated by a known distance BC, are pointed toward an object A. By means of the graduated circles the angles ϕ and α can be measured. Then the distance CA or BA can be computed. The precision of this so-called "triangulation method" depends upon the length of the base line and the accuracy of the angular measurements. When the observing station is permanent, as in the case of a coast fortress, the two telescopes are provided with large circular scales and are mounted on masonry foundations at the ends of a base line several rods in length.

For mobile land operations, a portable instrument depending upon the same principle has been devised in which the base line is but one meter, and which requires but one observer. In the diagrammatic representation of the principal parts of the optical system of this coincidence range finder, Fig. 538, M_1 and M_2 represent two reflectors, one at each end of a tube containing the optical system. Light from a distant object, after reflection from M_1 and M_2, traverses the objectives O_1 and O_2 respectively. By means of two additional reflectors, M'_1 and M'_2, light from the two objectives is bent so that the two images of the object are in the focal plane of a single eyepiece E. Thus, the instrument consists essentially of two telescopes having a single eyepiece.

The parts are so arranged that light incident in a direction perpendicular to the optic axis of the instrument meets the ocular reflectors at their line of intersection and forms an image in the center of the field of view of the eyepiece. In case the incident light makes an angle θ with the perpendicular to the axis of the instrument, the beam, after reflection from the end reflectors, will be inclined to the optic axis at the same angle θ, and will form an image in the focal plane of the ocular to one side of the center.

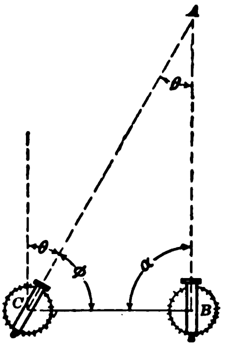

Fig. 537.

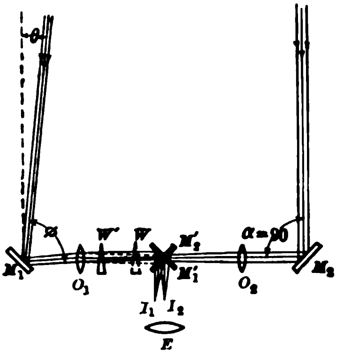

Fig. 538.

In using such an instrument, it would be directed toward the object whose range is desired. On looking through the eyepiece one finds the field of view cut by a horizontal line. In the upper half of the field of view is an image due to light that has traversed the right end of the instrument, and in the lower half is an image due to light that has traversed the left end. If the object were at an infinite distance, the angles ϕ and α would each be a right angle and the two images would be exactly in line, one above the other. But for a nearer object, one of these angles must be less than a right angle by an amount θ, and the image I_1 will be to one side of I_2. If the reflector M_1 were now rotated through an angle $\frac{1}{2}\theta$, the image I_1 would be brought into coincidence with I_2 (Art. 431). In this manner the angle θ and consequently ϕ, could be measured. The distance of the object sighted upon could then be computed.

For the range of distances for which the instrument is used, however, the variation in the angle θ is so small that the required precision in the measurement of the rotation of the reflector could not be readily obtained. For this reason, all four reflectors are fixed in position and the coincidence of the two images of the object is effected by moving a thin glass wedge, W, along the axis of the instrument. With the compensating wedge at some convenient point W, Fig. 538, the reflector M_2 can be fixed permanently in such a position that light incident perpendicularly to the axis of the instrument will traverse the instrument axially. The present position of the wedge is the setting for an object at infinity. For a nearer object, the image formed by light traversing the left side of the instrument will be at some point I_1. By moving the compensating wedge to some position W' the image I_1 can be brought into coincidence with I_2. The length between W and W' is a measure of the distance of the object from the instrument.

If the reflection at each end of the instrument were produced by a single reflecting surface, as indicated in Fig. 538, the unavoidable flexure of the containing tube due to temperature changes or mechanical strain would produce variations in the direction of the light through the instrument and would introduce considerable error in the indications. If, however, the reflection at each end were produced by two rigidly connected surfaces, then the effect of any small displacement of one surface would be compensated by an equal displacement in the opposite direction of the attached reflecting surface. The most satisfactory means for securing two rigidly connected reflecting surfaces is by the use of two silvered faces of a prism as shown at M_1 and M_2, Fig. 539.

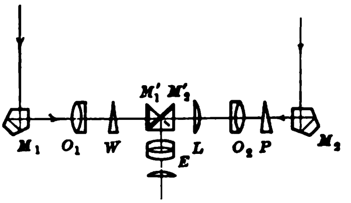

FIG. 539.

In practice, the right-angled reflections at the eyepiece are produced by two totally reflecting prisms as shown at M'_1 and M'_2 in the same figure. With one of these prisms above the other, the field of view will consist of two parts divided by a horizontal line, the upper part due to light that has traversed one objective, and the lower part due to light that has traversed the other. In general, the two halves of the image of a vertical object will not be in line. The adjustment of the range finder consists in moving the compensation wedge W till the two halves of the image of some vertical object are continuous. The distance between the object and the instrument is then indicated by a scale attached to the compensation wedge.

On account of the fact that light traversing the left objective traverses a

longer path in glass than the light traversing the right objective, the two beams of light would not focalize in the same plane. To bring these focal planes into coincidence, there is added to the right side a correction lens *L*. In addition, for the purpose of readily readjusting the optical system when it has become slightly disarranged, there is introduced a thin wedge *P* capable of rotation about the optic axis of the instrument.

441. The Compound Microscope.—In principle, the compound microscope is the same as that of an astronomical telescope having a short focus objective and a short focus ocular. The distance from the object under examination to the objective lens is but slightly greater than the focal length of the objective. The objective is so close to the object that the light entering the objective from any point of the object is strongly divergent. Excessive spherical aberration would thereby result if special precautions were not taken to correct it. This is accomplished by building up the objective of several lenses.

After traversing the objective, the light from the object forms a real image, inverted and larger than the object. This image is further magnified by means of an eyepiece. As the image is so much larger than the object, the object must be brilliantly illumined in order that the image may be sufficiently bright to be distinct. When the objective is close to the object it is so difficult to properly illumine the object from above that light is supplied usually from below.

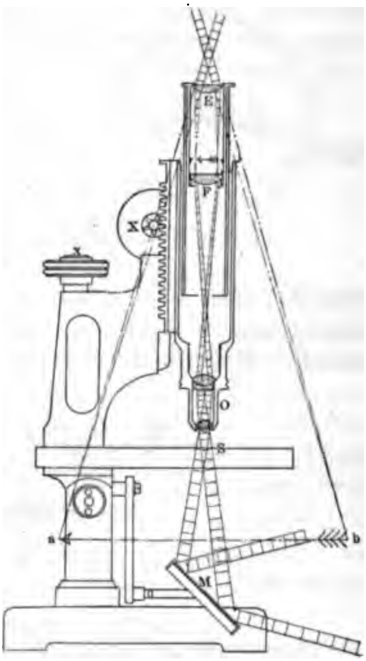

FIG. 540.

The arrangement of lenses and the path of light in a modern compound microscope are shown in Fig. 540. The object under investigation is placed on the stage *S*, and is illumined by light reflected from the mirror M. Light from the illumined object after traversing the compound objective O passes through the field lens F and forms a real image at the place shown in the figure. This image is highly magnified by means of the eye lens E. The distance from the object to the objective is adjusted by means of the pinion X and the screw x.

§ 4. *The Spectroscope and Spectrum Analysis*

442. The Prism Spectroscope.—The spectroscope is an instrument for separating the radiant energy emitted by a luminous body into waves of the various frequencies of which it is composed. The dispersing action of a prism is often employed for this purpose. In the ordinary prism spectroscope, a very narrow slit s, Figs. 541 and 542, is illumined by light from the source under investigation. The spherical waves from the various points of this slit, after being rendered plane by a lens C, are bent out of their course by a prism P, and brought to a focus F by means of a lens O. If the luminous source emits waves of a single frequency, there will be formed at F a single image of the slit. If, however, the luminous source emits waves of several different frequencies, these waves of different frequencies will be unequally refracted

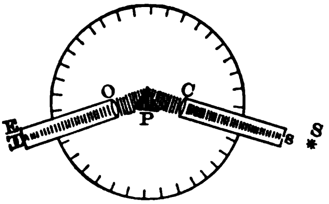

Fig. 541.

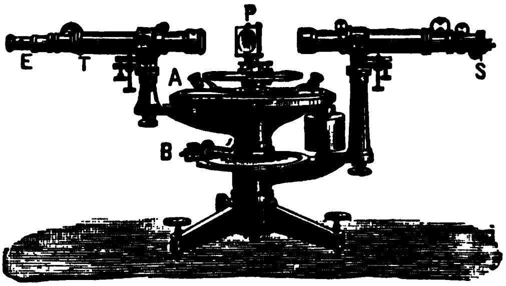

Fig. 542.

by the prism, and will form at F a separate image of the slit for each different frequency. This series of parallel images of the slit is called a *spectrum*. The spectrum is viewed by means of an eyepiece E. The tube C with the slit and the lens is called the *collimator*. The tube with the lens O and eyepiece E constitutes an ordinary reading telescope. The telescope is usually provided with a set of cross-wires.

443. The Direct-vision Spectroscope.—A combination of prisms made of glasses of different indices of refraction and different dispersive powers can be made which for light of any specified wave-length will produce dispersion without deviation (Art. 400). This fact is utilized in the production of a simple spectroscope. In Fig. 543 CCC represent three crown glass prisms, and FF represent two flint glass prisms of such angles that light traversing the combination will be dispersed, while light of a certain frequency will emerge in the same direction it entered. Such a combination is called an Amici direct-vision prism. Light from a source S after traversing a narrow slit and the compound prism is focalized by the eyepiece E. In the focal plane of this eyepiece there is formed an image of the slit for light of each frequency emitted by the source. That is, a spectrum of the source is formed in the focal plane of the eyepiece.

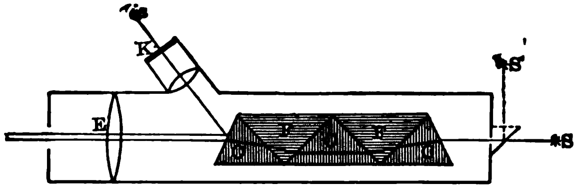

FIG. 543.

By covering one-half of the slit by a totally reflecting prism, the spectrum of a second source S' can be produced by the side of the spectrum from the other source. By this means the spectra of two bodies can be readily compared.

The instrument is often provided with a scale ruled on glass, K, so placed that light after traversing the scale and a lens, and being reflected at the air-glass surface of the last prism, forms an image of the scale beside the spectra. A particular spectrum line can then be indicated by reference to its position on this scale.

444. The Different Classes of Spectra.—When a gas free from solid particles is heated to incandescence, the spectrum consists of a series of bright-colored images of the spectroscope slit separated by dark spaces. This is called a *bright-line* spectrum. When raised to a sufficiently high temperature, any substance becomes an incandescent gas and gives a bright-line spectrum. If an incandescent gas be subjected to high pressure, the spectral lines become broader. It is probable that with sufficiently high pressure the lines would broaden till there would be no dark spaces between them.

The spectrum produced by an incandescent solid or liquid consists of a continuous ribbon of color, blue at one end, red at the other, and an unbroken series of other colors in between. Self-luminous bodies, whether solid or liquid, e.g., the incandescent particles of carbon-forming flames, the filament of an incandescent electric lamp, incandescent solid or melted iron, give *continuous* spectra. If light from an unknown source gives a continuous spectrum we infer that the luminous source is an incandescent solid or liquid, or possibly a gas under enormous pressure. Up to the present time, however, continuous spectra have not been produced in the laboratory by compressing an incandescent gas. But it is possible that at the center of a ball of gas of the size of the sun the pressure would be sufficiently great to give a continuous spectrum.

If waves emitted by an incandescent solid or liquid fall upon any nonluminous body, certain of the incident waves are reflected, others are absorbed, and the remainder are transmitted. The spectrum of the transmitted waves has the appearance of a continuous spectrum from which certain regions have been absorbed or blotted out. A spectrum of this sort is called the *absorption spectrum* of the body which produced the absorption. In the absorption spectra of solids and liquids the dark spaces are broad with nebulous edges. In the absorption spectra of gases the dark spaces consist of narrow lines with sharply defined edges.

445. Spectrum Analysis.—The bright line spectrum of an element consists of a particular grouping of lines that distinguishes that element from all others. When raised to a sufficiently high temperature any mixture or compound becomes dissociated into the elements composing it, and each component element gives its own spectrum independently of all the others. In this manner different substances can be identified by their spectra. In Fig. 544 are shown three bright-line spectra. The upper spectrum is of copper, the lower one is of zinc, and the middle one is of brass.

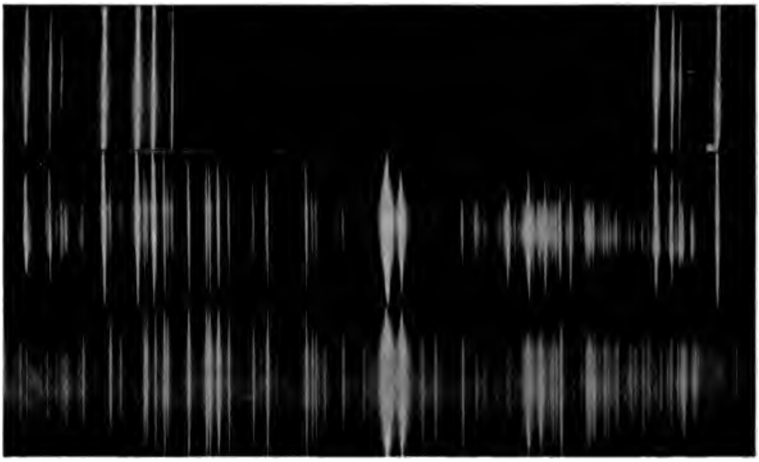

FIG. 544.

The fact that for each line in the spectra of copper and zinc there is in the spectrum of brass a similar line in the same relative position shows that brass is composed of copper and zinc. Any lines occurring in the spectrum of brass that do not occur in the spectra of copper or of zinc are due to some other element.

If the spectrum of a star is of the bright-line type, we infer that the star consists of a mass of incandescent gas. By comparing the stellar spectrum with spectra of the elements we can determine the elements composing the star.

The wonderful sensitivity of the spectroscopic method of identification of the components of a body is shown by the minuteness of the quantities of materials that can be detected. For example $3(10)^{-9}$ g. of common salt gives the characteristic spectrum of sodium; $(10)^{-10}$ g. of calcium can be detected, and $(10)^{-11}$ g. of strontium can be detected.

From theoretical considerations it can be shown that the waves absorbed by any gas are of the same frequencies as those that would be emitted if the gas were heated to incandescence. In agreement with this conclusion we find that the dark lines of the absorption spectrum of any gas occupy the same relative positions as do the bright lines in the emission spectrum of the same gas.

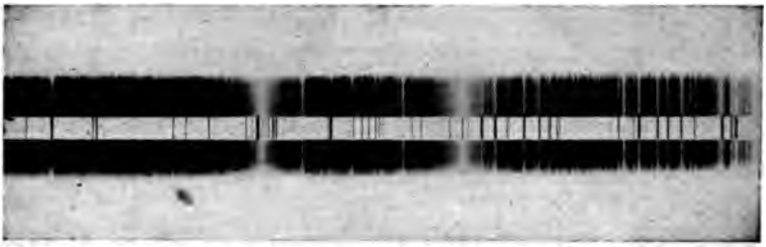

Fig. 545.

Since the spectrum of the sun is a continuous ribbon of color crossed by narrow dark lines, we infer that the sun consists of an incandescent solid or liquid nucleus (or highly compressed gaseous nucleus), surrounded by a layer of cooler gas. The dark lines of the solar spectrum permit the identification of the elements which enter into the composition of the gaseous envelope of the sun. The middle ribbon in Fig. 545 is part of the bright line spectrum of iron. The corresponding part of the spectrum of the sun is shown above and below. The coincidence of the bright lines of the iron spectrum with absorption lines of the solar spectrum proves the presence of iron vapor in the gaseous envelope of the sun. The other lines of the solar spectrum are due to elements which are also found on the earth.

Frauenhofer designated the prominent lines of the solar spectrum by letters of the alphabet. The two prominent lines in the part of the solar spectrum shown in the figure are due to calcium and are called the *H* and *K* Frauenhofer lines. The former has a wave-length $\lambda_H = 3969$ Ångström units, and the latter, $\lambda_K = 3934$ Ångström units.[1]

Since when sunlight is incident on the moon, we get from the moon the same spectrum that we do from the sun, whereas, when sunlight is not incident on the moon we get nothing, we infer that the moon is not self-luminous, but reflects sunlight.

446. Motion of Heavenly Bodies in the Line of Sight.—The spectrum, in connection with Doppler's Principle, furnishes our only scheme for determining the component velocity of a heavenly body in the line of sight. In Art. 175 it was shown that when a wave source is approaching an observer, the frequency of the waves received by the observer is increased, whereas when the body is receding from the observer the frequency is decreased. Thus when moving toward an observer, there is an apparent shortening of the wave-length of all the light received. If the body gives a line spectrum, the shortening of wave-lengths is shown by a shift of the lines toward the blue end of the spectrum. On the other hand, when the body is moving away from the observer the shift of the spectral lines will be toward the red end of the spectrum. The amount of the shift depends upon the speed of the body in the line of sight. Knowing the amount and direction of the shift, the component velocity of a star in the line of sight can be approximately determined by Doppler's Principle. Some stars are approaching, and some are receding from the earth with a speed exceeding 40 miles per second.

The astronomers of the Lowell Observatory have recently observed a shift, toward the red end of the spectrum, of the spectrum lines of the nebula *N.G.C. 584*, which indicates that this nebula is moving away from the earth with a speed of 1100 miles per second.

For a long time it was not known whether the rings around the planet Saturn are disks or whether they consist of clouds of separate small bodies.

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[1] The Ångström unit = 0.0000001 mm. = 0.0001 micron.

If they are disks, the periphery must move faster than the inside. But if the rings consist of separate bodies, the bodies toward the axis of rotation must move faster than the outer ones, else the inner bodies would fall into the inner planet.

Suppose that an image of Saturn be formed on the end of a spectrometer so that it is crossed by the slit *ST* as shown in Fig. 546. If the rings rotate in their plane, the part of a spectral line corresponding to light entering the slit at *ab* will be shifted in the direction opposite to the part of the same spectral line corresponding to light entering the slit at *cd*. If the velocity at the periphery of a ring is different than that on the inside, the spectral lines will be inclined.

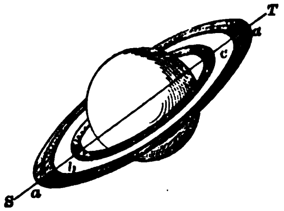


Fig. 546.

It is found that the spectral lines are inclined in the direction which indicates that the inner parts of the ring move faster than the periphery. Therefore the rings cannot be solid disks. From the measured shift of the spectral lines it is found that each point of the rings has the velocity required to maintain a separate body in an orbit having a radius equal to the distance of that point from the axis of revolution. Hence we conclude that the rings of Saturn consist of a cloud of small bodies revolving in the equatorial plane of the planet.

447. Quantitative Spectrum Analysis.—The degree of blackness of an absorption band depends upon the amount of absorbing material through which the light travels. For instance, the fraction of the light incident upon a cell of absorbing solution that emerges from the cell depends upon the concentration of the solution. Thus, by using cells of the same thickness we can compare the concentrations of two absorbing solutions of the same substance from determinations of the fraction of the incident light transmitted by each.

For such a determination a spectroscope is arranged so that part of the light from a luminous source traverses the cell in front of the slit while another part enters a different portion of the slit without traversing the cell. In the eyepiece now appears a continuous spectrum of the luminous source beside the absorption spectrum of the solution. By means of a diaphragm in the focal plane of the eyepiece all of the two spectra can be obscured except [illegible] selected portion of one absorption band, and the corresponding portion of the continuous spectrum. The observer now sees side by side two rectangular patches of light of the same wave-length, one due to light that has traversed the solution, and one due to light from the same source that has not traversed the solution. A device is provided which permits the observer to diminish the brightness of the continuous spectrum by known amounts till the two parts of the field of view are equally bright. A spectroscope provided with a device for comparing the brightness of corresponding parts of two spectra is called a *spectrophotometer*.

By means of a spectrophotometer, the concentration of an absorbing solution can be quickly determined with considerable accuracy. In the case of certain classes of substance the degree of accuracy possible is as great as by chemical analysis. The method is available for the determination of the speed of chemical reactions in a considerable number of cases.

CHAPTER XXVII

DIFFRACTION AND SCATTERING OF LIGHT

448. Half-period Elements.—Consider the illumination at a point *M*, Fig. 547, due to a spherical wave advancing from a point source *S*. The effect at *M* is the resultant of the individual effects produced by all points of the advancing wave front. Draw a line from *S* to *M*. The point of intersection, *P*, of this line with the wave front is called the *pole* of the wave surface with respect to *M*. Denote the distance *PM* by the symbol a and the wave-length by λ. With *M* as center and with radii $a+\lambda/2$, $a+2\lambda/2$, $a+3\lambda/2$, etc., describe a series of spheres. These spheres divide the advancing wave front into a series of zones. Looking at the advancing wave front in the direction *MP*, the appearance of these zones, if they could be rendered visible, would be as represented in Fig. 548. These concentric zones are called *half-period elements.* It can be shown that the areas of these half-period elements are approximately equal. If the areas were exactly equal, and the distances of the various elements from *M* were exactly equal, and the inclination of each element to the line joining it to *M* were equal to the corresponding inclination for every other one, then the illumination produced at *M* by any half-period element

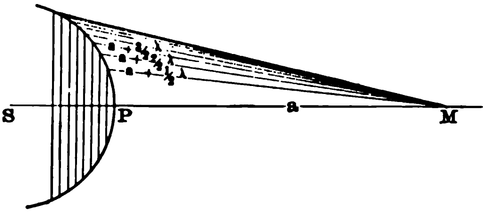


FIG. 547.

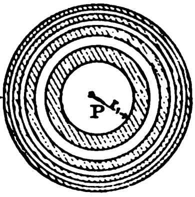


FIG. 548.

would equal that produced by any other. But in an isotropic medium, these conditions are fulfilled only when *M* is a focus toward which a spherical wave is converging. In the case represented in Fig. 547, as the radius of a half-period element increases, there is a gradual increase in the area of the element, an increase in the distance of the element from *M*, and an increase in the inclination of the element to a line drawn from it to *M*. The illumination which any element produces at *M* turns out to be not quite equal to that which the next element produces, the difference being greatest for the elements close to the pole.

The disturbance that reaches *M* from any point of one half-period element is exactly one-half wave-length behind the disturbance from some point in the next inner element. Hence, the disturbance at *M*, at any distance, due to an entire half-period element, is, on an average, one-half wave-length behind the disturbance due to the next inner element. Therefore, the illumination due to any two adjacent half-period elements is considerably less than that due to the inner one alone. In fact, it can be shown that the illumination at *M* due to the entire wave very nearly equals one-fourth that which would be produced by the central half-period element if this alone had been effective. Thus, if all the wave had been screened from *M* except a certain part of the first half-period element, the illumination at *M* would be the same as if no part of the wave were screened. Consequently, the illumination at *M* may be regarded as due to a very small portion of the advancing wave front about the pole. All of the remainder of the wave is ineffective so far as the illumination of *M* is concerned.

If an opaque circular disk of the diameter of the first half-period element be placed at *P*, the illumination at *M* will be almost exactly a quarter of that which would be produced by the second half-period element if that alone were sending light to *M*. If the disk cover two half-period elements, the illumination at *M* will be very nearly a quarter of that which would be produced by the third half-period element if that alone were sending light to *M*. As the illumination due to each half-period element is practically the same, it follows that the illumination at *M* will be but slightly affected by placing a small opaque disk in the path of the light wave.

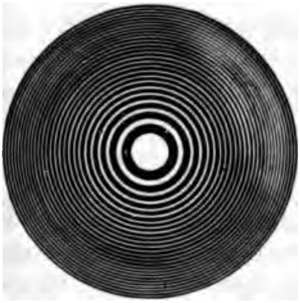


FIG. 549.

If at P there be placed a diaphragm with transparent annular spaces, so arranged that the alternate half-period elements are uncovered, the illumination at M will be several times greater than if the entire wave front were uncovered. That is, the light is brought to a focus, somewhat as it is by a converging lens. Such a diaphragm is called a " zone plate " and has the appearance of Fig. 549.

449. The Rectilinear Propagation of Light.—The approximate radius of any half-period element will now be determined. Let A, Fig. 550, be on the outer edge of the nth half-period element, with respect to M, of a wave front advancing from a point source S.

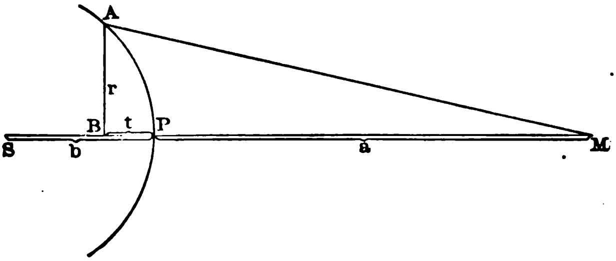


FIG. 550.

From the figure, the radius r of this element is given by the equation

$$r^2=(AM)^2-(BM)^2.$$

But $$AM=a+\frac{n\lambda}{2} \text{ and } BM=t+a,$$

where λ is the wave-length of light.

Hence,

$$r^2 = \frac{2an\lambda}{2} + \frac{n^2\lambda^2}{4} - 2at - t^2.$$

For any case that would arise in experiment, t and λ are so small compared with r that for the degree of precision here sought it will be justifiable to neglect the square of either of them. Consequently,

$$r^2 \doteqdot a(n\lambda - 2t). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (246)$$

Again, from the figure,

$$\begin{aligned} r^2 &= b^2 - (b-t)^2 \\ &= 2bt - t^2 \\ &\doteqdot 2bt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (247) \end{aligned}$$

Eliminating t between (246) and (247),

$$r^2 \doteqdot \frac{nab\lambda}{a+b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (248)$$

If a circular aperture placed five meters from a point source is to permit only the first half-period element of light of $\lambda = 0.00006$ cm. to reach a screen 5 meters from the aperture, the radius of the aperture is given by

$$r^2 = \frac{1(500)(500)(0.00006)}{1000} = 0.015 \text{ cm.}^2$$

$$r = 0\ 12 \text{ cm.}$$

The aperture necessary for the production on the screen of illumination equal to that which would be produced by the entire unobstructed wave is even smaller than this value. That is, the illumination at a point is equal to that which would be produced by a very small area of the wave front normal to the direction of propagation. For this reason light is said to travel in a straight line from the source to the point under consideration.

450. Diffraction around the Edges of an Aperture.—In the model illustrated in Fig. 551 the spherical surface F represents a portion of the front of a wave originating at *S*. This spherical surface is marked off into half-period elements with respect to the point *M*. The board *A* represents a diaphragm pierced by a circular aperture[1] of an area equal to one-half of the central half-period element.

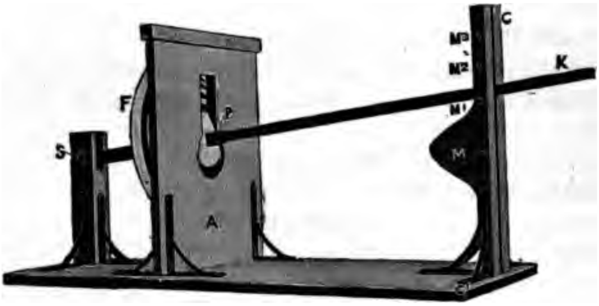


FIG. 551.

Consider the illumination on a vertical screen through *C* and *C′*. Fig. 552 represents a cross-section of the model through the line *SM*, and Fig. 553 represents the wave front advancing through the aperture as seen from the point *M*. (The diagrams in this Article are not drawn to scale.) Since all of the wave front not covered by the diaphragm belongs to the same half-period element, light from no two points will arrive at *M* in opposite phase. Consequently, the illumination at *M* is great.

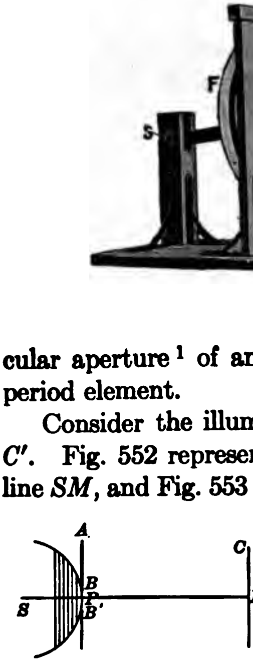


FIG. 552.

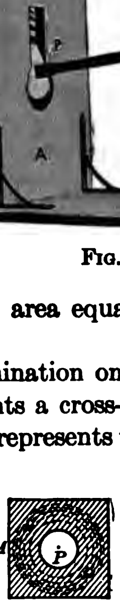


FIG. 553.

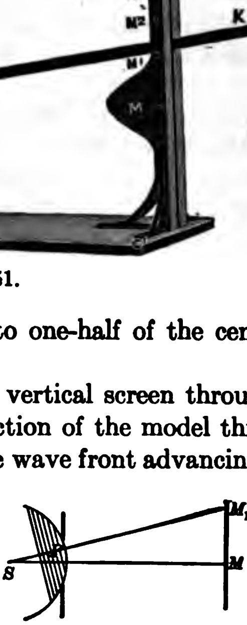


FIG. 554.

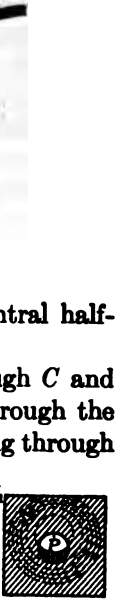


FIG. 555.

On raising the end *K* of the rod *SK*, Fig. 551, the appearance at the diaphragm of the half-period elements with respect to some point M_1 will be as represented in Fig. 555. In this case part of the light that comes from the second half-period element is opposite in phase to part of that from the first half-period element, so that the illumination at M_1 is considerably weaker than at M. If we imagine that the rod SK can move out of the vertical plane and that it is made to move in such a manner that while S remains fixed K describes a vertical circle in a plane perpendicular to SM, we see that the effect which this movement would have in Fig. 555 is simply to turn that figure gradually around in the plane of the paper—without any change in the fractions of the zones that are seen through the aperture. It follows that the bright central spot at M is surrounded by a dark circular band of radius MM_1.

---

[1] In the figure is shown a slot extending upward from the circular aperture. This does not represent an aperture for the passage of light. It is cut in the board so as to permit the upward movement of the rod *SK*.

At some place above M_1 is a point M_2 for which the half-period elements would be as shown in Figs. 556 and 557. In this case the light from the exposed part of the third half-period element interferes with a considerable part of that from the second half-period element. Thus, only a part of the light from the second half-period element is left to interfere with that from the first. The illumination at M_2 is, therefore, greater than at M_1, but is not nearly so great as that at M.

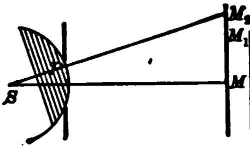


FIG. 556.

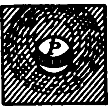


FIG. 557.

Proceeding in this manner, it is found that the central bright spot is surrounded by a series of bands, alternately dark and bright, that extend to some distance beyond the geometric shadow of the edges of the aperture.

The phenomenon of the bending of light, due to interference, around the edges of opaque objects is called *diffraction*. The alternate dark and bright bands that accompany diffraction are termed *diffraction fringes* or *diffraction bands*.

The distribution of illumination on a screen produced by light from a luminous point, traversing apertures of different sizes, is shown in Fig. 558. In this diagram the curve drawn with a dashed line represents the distribution of illumination when the aperture uncovers one-quarter of the first half-period element with respect to M; the curve drawn with a full line represents the

distribution when the aperture uncovers one-half of the first half-period element; and the dotted curve represents the distribution when the aperture uncovers the first eight half-period elements.

Diffraction fringes are produced when light passes across the edge of an opaque object. The fringes are always parallel to the edge. One sees diffraction fringes on looking through a silk umbrella toward a bright street light on a dark night. The two sets of fringes parallel to the threads of the cloth form a distinct cross.

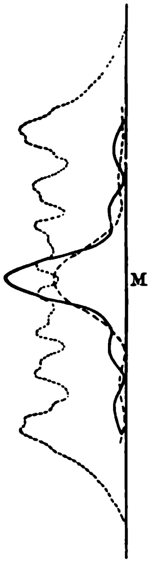


FIG. 558.

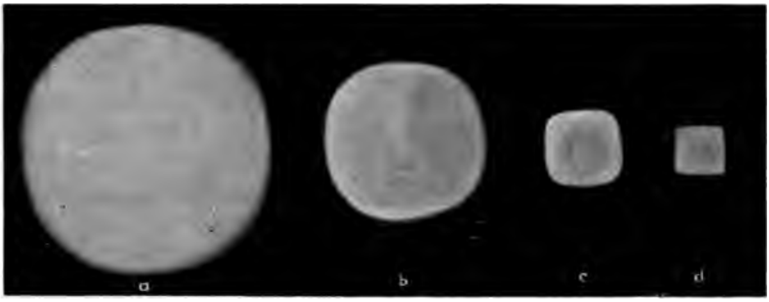
If one looks toward a small light source through a glass plate on which have been dusted small uniformly sized particles, one sees the light source surrounded by a series of concentric rings. Each ring is reddish on the outer edge and bluish on the inner. Sometimes such rings, called coronas, are seen around the moon when the sky is dark and the air is charged with moisture. Less frequently they are seen around the sun. They subtend at the observer an angle which depends upon the size of the particles. The larger the particles the smaller and brighter is the corona. The popular notion that coronas betoken rain is founded on the fact that they occur only when there is considerable moisture in the atmosphere.

There are sometimes seen about the sun or about the moon large rings having diameters, which subtend at the observer an angle of either about 22° or 46°. These so-called haloes are not diffraction effects but are due primarily to refraction of light through ice crystals suspended in the atmosphere.

451. The Formation of Images.—In the preceding Article it was shown that when light from a luminous point traverses a small aperture of any form and falls on a screen there is formed on the screen a small bright dot of light surrounded by a series of faint diffraction fringes. When the aperture is not small, the spot on the screen is large. If, instead of a single point, the luminous object consists of a number of points, each will produce on the screen its own effect. If the image dots of all points of an extended object be small and bright, the image of the object will be distinct;

while if the image dots be large they will overlap, and the image of the object will be indistinct.

Now the image dot of an object point will be small if it be due to light from less than the first half-period element of the incident wave. The distribution of illumination in such a dot is shown by the full line in Fig. 558. In this case most of the light is concentrated in a small dot and this dot is sharply marked off from the surrounding region by destructive interference. But if the image dot of an object point be due to several half-period elements of the incident wave, the distribution of illumination will be something like that represented by the dotted line in Fig. 558.

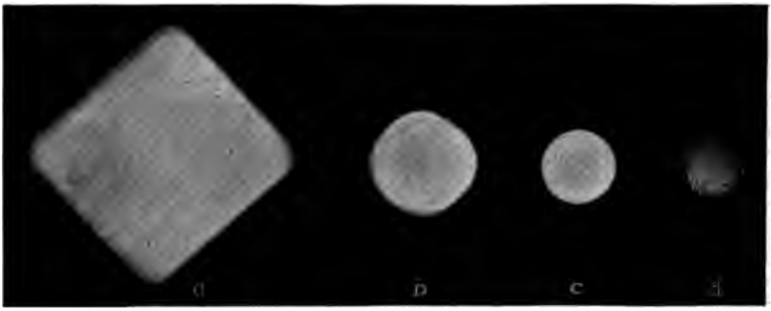


$n=206$ $n=54$ $n=3$ $n=0.3$

Fig. 559.

The dependence of the sharpness of the image on the number of half-period elements of the light wave that are incident on each image dot is shown by the series of pictures constituting Fig. 559. The photographs from which these engravings were made were produced as follows: Several small circular apertures were placed, one at a time, about half way between a luminous square and a photographic plate. The square was 1.3 cm. on a side and the distance from the square to the plate was somewhat over 10 m. The diameters of the round apertures were 3.0 cm., 1.53 cm., 0.36 cm., and 0.11 cm., respectively. The wave-length of the light used was about $\lambda = 0.00004$ cm. The number of half-period elements uncovered by the various apertures was found by means of (248) to be 206, 54, 3, and 0.3, respectively.

Fig. 559*a*, shows that when the aperture contains a large number of half-period elements the bright dot produced on the screen by light from each point of the object is so large that the resultant figure has not the form of the object, but has the form of the aperture. As the size of aperture is diminished, the figure on the screen becomes more and more like the object until when the aperture is too small the outline of the figure becomes nebulous. In the case now being considered the aperture that would give the best image would have a size between that used for *c* and *d*, Fig. 559.



$n = 122$ $n = 5.75$ $n = 0.52$ $n = 0.08$

Fig. 560.

Fig. 560 shows a similar progression. Here the object was a luminous circular disk 1.56 cm. in diameter, and the apertures were squares of different sizes. For these photographs the squares were 2.30 cm., 0.50 cm., 0.15 cm., and 0.06 cm., respectively.

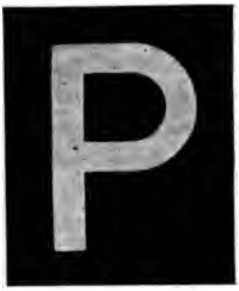


Fig. 561.

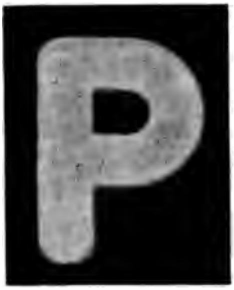


Fig. 562.

The photograph reproduced in Fig. 561 was taken with a circular aperture of such a diameter that one half-period element was uncovered. The distances from the diaphragm to the object, and from the diaphragm to the receiving screen, were such that the diameter of the image dot of a luminous object point was 0.22 mm. The photograph reproduced in Fig. 562 was taken with a circular aperture that uncovered 49 half-period elements, and the diameter of the image dot of each object point was 1.53 mm. The distribution of the illumination in the image dots of Fig. 561 and Fig. 562 have been computed and plotted in Fig. 563.

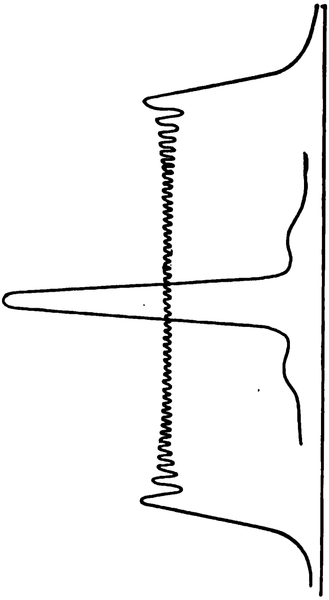


Fig. 563.

Fig. 564 shows the image of an object produced by light that has traversed an aperture of 0.9 half-period element when the distances are such that the diameter of the image of an object point is 0.14 mm.

452. The Functions of a Lens.—In the two preceding Articles it has been shown that at an image the wavelets from the different points of the aperture reinforce one another, and that this bright spot is marked off from the surrounding region by a dark space

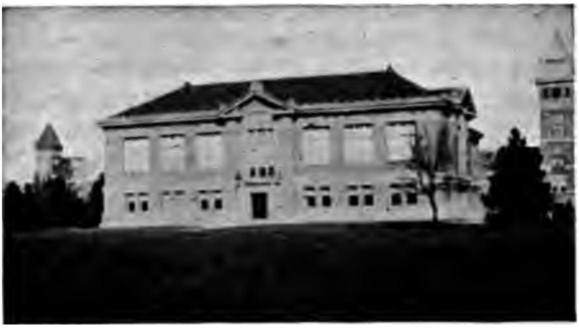


Fig. 564.

produced by destructive interference of wavelets from the different points of the aperture. A satisfactory image of a luminous point should be bright, small, and sharply differentiated from the surrounding space.

In order that an image may be sharp, the diffraction fringes must not be bright. In order that an image may be bright, much light must reach it. The sharpest image is that produced when the aperture uncovers somewhat less than one half-period element. This is on account of the interference of light from different half-period elements. If by any means light from all parts of an aperture could be made to reach a given point in the same phase,

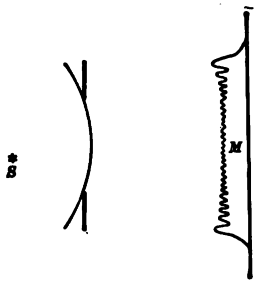


Fig. 565.

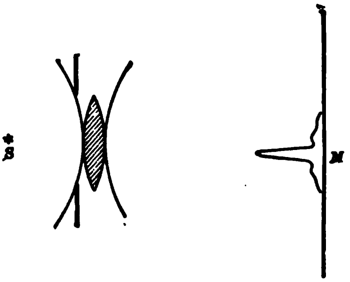


Fig. 566.

then the larger the aperture the brighter would be the image without diminishing the sharpness. This is approximately what is accomplished by the use of a zone plate, a converging lens or a converging mirror.

Consider a screen illumined by light which comes from a point source through a large aperture. If the aperture uncovers a large number of half-period elements the distribution of illumination on the screen will be about as represented in Fig. 565. If there be placed in the aperture a lens of such curvature that the emergent wave converges to M, the spot of light at M becomes small and bright. The lens causes the first half-period element for the point M to be very large. In fact, practically the whole of the converging wave lies inside of the first half-period element for M. But for points which lie very close to M this is not the case. When the lens is used a diffraction pattern is produced just as if the lens were not present, but the size of the pattern is greatly reduced, and the brightness of the central dot is vastly increased. The distribution of illumination on the screen when a lens is used is somewhat as represented in Fig. 566. If no converging lens or mirror be used, the image dot of an object point will be sharp only when the aperture in the diaphragm is so small that it uncovers not more than one half-period element. With an aperture alone, the image dot cannot be both bright and small at the same time.

When a lens is used, the diaphragm with the aperture can be omitted. If a lens be broken into pieces and only one part is used, or if a lens be partly covered by an opaque object, the image will be the same as when the entire lens is used, except that the brightness will be less. This corresponds to the fact that without a lens, if the aperture is smaller than one half-period element, the shape of the bright spot is that of the object and is independent of the shape and size of the aperture.

Optical images are formed only by the reinforcement and destructive interference of light waves. The purpose of the lens is to imprint on the wave front traversing it such a curvature that the light which arrives at the image from the various parts of the emergent wave front shall be in the same phase, whatever the area of the lens may be. When a converging lens or mirror without aberration is used, the image of each object point will be sharp, whatever the size of the aperture.

453. Resolution.—Two point sources may be so close together that for a certain aperture the two image dots will overlap, whereas for a greater aperture the two will be separate. When the two can be distinguished as separate images the point sources are said to be *resolved.*

When the linear distance between the centers of two image dots is greater than the radius of one of them, the two can be distinguished as separate images. It is commonly assumed that the smallest distance between two point sources that can be resolved is that which will produce image dots whose centers are separated by a space equal to the radius of one of them.

Fig. 567$_1$ is from a photograph of the image dot due to one half-period element of the wave from a luminous point. Fig. 567_2 shows the image of two luminous points very close together. Fig. 567_3 shows the image of two luminous points so close together that the distance between the centers of the image dots equals the radius of one of them. Fig. 567_4 shows the image of two widely separated luminous points. In Fig. 567_2 the points are not resolved; in Fig. 567_3 they are barely resolved; in Fig. 567_4 they are widely resolved.

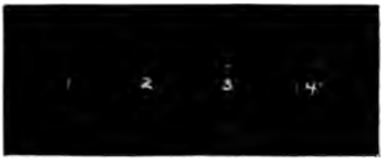


FIG. 567.

It can be shown that shorter distances can be resolved by a short focus lens than by a long focus lens, and by a lens of large aperture than by a lens of smaller aperture. Also, that by using light of short wave-length, shorter distances can be resolved than by using light of longer wave-length. In astronomical telescopes, points apparently coincident are resolved by the use of lenses of large aperture.

In microscopes, resolution is increased by the use of short focus lenses and by illuminating the object with light of short wave-length. If the space between the object and the objective of the microscope be filled with a liquid of index of refraction μ compared with air, light entering the objective will not have the wave-length λ_1 it had in air, but will have a different value λ_2, (230), such that

$$\lambda_2=\frac{\lambda_1}{\mu}.$$

Consequently, by filling the space between object and objective with a liquid of index of refraction relative to air greater than unity, smaller distances can be resolved than when the objective is not immersed. By using a liquid of about the same index of refraction as glass, an added advantage is secured—the object can be illumined by convergent light without loss by reflection at the surface of the objective. For this reason, the space between the object and the objective is frequently filled with a drop of cedar oil of refractive index 1.5. The image may be magnified by the eyepiece, however much, but no finer detail can be distinguished than is resolved by the objective.

In any optical system, each succeeding lens should have a resolving power not less than that of the objective. Greater resolving power is useless.

454. Penumbra.—Consider the illumination due to a uniformly luminous area S in front of a small aperture in a diaphragm D, Fig. 568. If there were no diffraction, the illumination at any point of a screen XX' would depend upon how much of S could be seen from that point. The illumination would be a maximum at C, and would gradually diminish outwards from this point. At the edges A and B of the geometric shadow of the diaphragm, the illumination would be zero. The space from which no part of S can be seen is in complete darkness and may be said to lie in the shadow of D, or in its *umbra*. The space from which a part of S can be seen is not as brightly illumined as that from which all of S can be seen, and is said to lie in the *penumbra* of D. The penumbra is indicated in Fig. 568 by shading. The distribution of illumination on a screen through C parallel to the diaphragm would be about as represented in Fig. 569. The edge of the spot is not sharply defined. Due to diffraction, the luminous spot on the screen XX' will be crossed by bright and dark lines parallel to the edges of the aperture. The actual distribution of the illumination of the spot may be approximately as represented in Fig. 570.

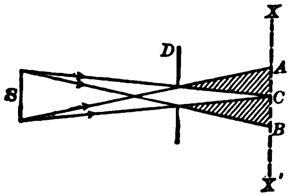


FIG. 568.

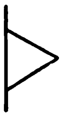


FIG. 569.

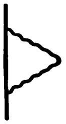


FIG. 570.

If a spot of light be incident on a photographic plate and the plate be developed, the picture on the plate will be larger than the spot of light as seen by the eye. This is due to two facts: The photographic plate is more sensitive than the eye, and the effect of the light spreads laterally outward through the sensitive film. The lateral spreading of the light effect is called *irradiation.* Due to irradiation, the size of a spot on a photographic plate depends upon the size and brightness of the light spot as well as upon the duration of exposure. This is the basis of a method for the determination of stellar magnitude. There is a definite law connecting the size of the developed star image on a given brand of plate and the brightness of the star.

455. Half-tone Engraving.—At the present time most printed pictures are made from plates engraved by the half-tone process. As the process is a very interesting and important application of the principles of diffraction, penumbral gradation and irradiation, it will now be briefly described. The surfaces of the printing plate produced by the half-tone process are broken up into dots of various sizes—large dots where the picture is to be dark, and smaller ones where it is to be lighter. In coarse half-tone work the dots are so far apart that they are very noticeable, but in fine half-tone work they are so close together that they are scarcely observable without a magnifying glass. Figs. 559 to 564 are half-tone engravings from photographs.

To produce the dots a photograph of the object is taken through a "half-tone screen." This screen is a glass plate on which equally spaced opaque parallel lines have been ruled in two perpendicular directions. The distance between adjacent lines equals the width of a line. Thus, the transparent spaces, or apertures, of a half-tone screen of 133 lines to the inch are squares of $^1/_{266}$ of an inch on a side. The half-tone screen is placed in the camera between the plate and the lens and close to the photographic plate.

In Fig. 571 suppose that the object of which an engraving is required consists of a photographic print of a landscape. Parts of the print are dark, parts are high lights and other parts are in half tones. P is the stop of the camera lens, HT is the half-tone screen, and XX' is a photographic plate. Light from any object point S_1, after traversing the camera stop and one of the screen openings, forms at i_1 an image of the camera stop. When the plate is developed, the sizes of the image dots will depend upon how brightly the camera stop was illuminated by light from S_1. Thus, on the negative that has been taken through the half-tone screen, the bright sky will consist of large dots, the parts corresponding to the dark partions of the original will consist of small dots, and the parts in half tone will consist of dots of intermediate size.

The developed screen negative is next placed on top of a sensitized metal plate in an ordinary photographic printing frame. The sensitive film on this metal plate consists of gelatine containing either potassium bichromate or ammonium bichromate. Light renders chromatized gelatine insoluble in water. Consequently, after exposure to light, the parts of the chromatized gelatine that were protected by the opaque portions of the screen negative can be washed away, whereas the parts exposed to the light will remain attached to the plate. After washing (called developing), the plate shows a picture of the original in raised gelatine dots on a polished metallic surface. After the gelatine has been thoroughly dried, the plate is heated over a flame till the gelatine becomes a hard enamel. The back of the plate and the dots of enamel on the face are next coated with an acid-resisting film. The plate is now immersed in an etching solution of iron perchloride till the exposed metal surface between the dots is dissolved away to such a depth that impressions of the plate can be taken in an ordinary printing press.

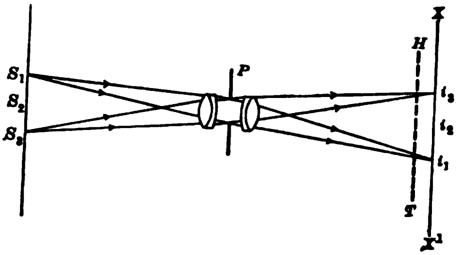


FIG. 571.

456. The Concave Diffraction Grating.—This important device for producing spectra consists of a highly polished concave metallic surface on which are a large number of fine, equally spaced, parallel, non-reflecting lines. Light incident on the intermediate reflecting lines is diffracted in all directions. The distance between adjacent reflecting lines is usually from 0.0001 to 0.00005 of an inch.

In Fig. 572 are represented seven reflecting lines of a concave grating G on which light is incident from a slit S situated at the center of curvature of the grating. The slit is parallel to the grating lines. At first suppose the slit is illumined with monochromatic light. From the center of each reflecting line represented in the figure are drawn several spherical wavelets, one wave-length apart. Surfaces tangent at every point to these wavelets are wave fronts advancing from the grating (Art. 157). In the figure two such wave fronts are represented—one converging at I_1 and another converging at I_2. Other wave fronts will converge at I'_1, I'_2 etc. In this manner there will be formed two series of images of the slit—at I_1, I_2, etc., and at I'_1, I'_2, etc.

If the source emits light of several wave-lengths, there will be at I_1, I_2, etc., and at I'_1. I'_2, etc., an image of the slit for light of each wave-length. These multiple images are spectra of the given source. The spectra at I_1 and I'_1

are called spectra of the "first order;" those at I_2 and at I'_2 are called spectra "second order," etc.

A concave grating spectrometer is shown in Fig. 573. At one end of a rod of length equal to the radius of curvature of the grating is mounted the grating G, and at the other end is mounted a camera C. This rod is supported on trucks which can be rolled along two tracks at right angles to one another.

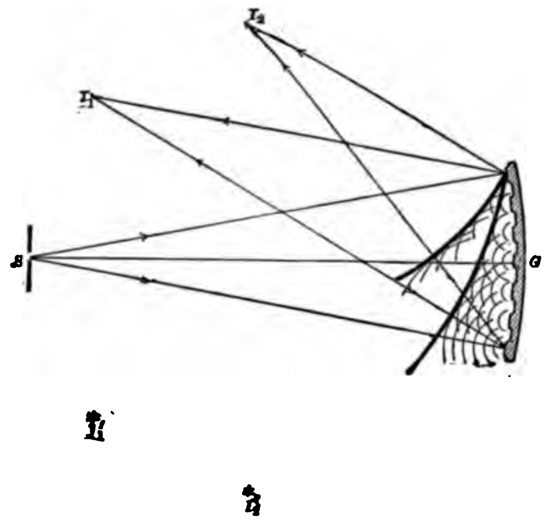


Fig. 572.

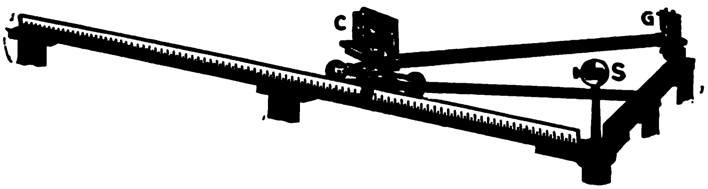


Fig. 573.

At the intersection of the tracks is the slit S. It can be shown [1] that with this arrangement, the wave-length of any spectrum line is directly proportional to the distance of the line from the intersection of the tracks. Wave-length measurements are most easily made by means of gratings.

[1] Ferry—Physics Measurements, Vol. I, p. 220.

457. Scattering of Light by Fine Particles.—If light be incident on a thick cloud of large particles, light of all wave-lengths will be reflected and very little light will be transmitted. The reflected light will be scattered in all directions. A dense cloud of large particles of dust or water vapor between an observer and the sun appears black. When white light is reflected from the cloud to the observer, the same cloud appears either white or gray.

A cloud of particles of smaller size may reflect light of the shorter wave-lengths, and transmit light of the longer wave-lengths. Thus, when illumined by white light, the light reflected will be of one color and the light transmitted of a different color. The colors will depend upon the size of the scattering particles. This effect is called the Tyndall phenomenon. The formation of colors by the scattering of light can be readily shown by sending a beam of light from the sun or an electric arc lamp through a glass tank of water in which is a cloud of fine opaque particles. Sufficiently small particles to reflect blue and transmit red can be produced by pouring into water an alcoholic solution of gum mastic.

The gas molecules and the particles of dust and water vapor high up in the atmosphere are so small that they reflect little but blue. This is the reason that the sky is blue. In the hotter parts of the earth the particles of water vapor high up in the atmosphere are smaller than in the cooler regions. For this reason the sky in hot regions is bluer than in cooler regions. Over an industrial city where there is much smoke, the sky is whiter or grayer than over a rural district.

The red and orange of the horizon at sunset and at sunrise are due to the fact that only the waves of great length are transmitted by the long layer of large dust and water particles close to the earth.

The iris, or colored part of the eye, consists of a layer of colorless medium containing fine opaque particles, backed by a layer containing a yellowish-brown pigment. The color of the eyes is due to the action of these two layers. When the particles in the front layer are sufficiently numerous and sufficiently small, the eyes are blue. As a person grows older these particles increase in size and the eyes become less blue.

Light from automobile headlights is reflected back into the driver's eyes by a haze of dust or water particles ahead of the machine. This glare consists largely of waves of the shorter lengths. To reduce the annoying glare, the lamps are sometimes provided with orange-yellow glasses which absorb the shorter waves.

458. Color.—The term color is
distinct senses. Sometimes it is u
times to denote the cause of the se
to pigments or paints. The word
the cause of the sensation. Hue i
by the particular wave-length or
the light. A given color sensation,
different combinations of hues.
light and green light can be produ
indistinguishable from monochrom
sensation of white may be produc
light waves of all lengths as foun
various mixtures of waves of but t
following are a few of the pairs of c
together give a sensation of white t
tinguishable from the white of noon

Blue and yellow;
Purple and green;
Blue-green and red.

White is also produced by the
lights. Any hue can be produced
proportion of red, green and blue lig
can be produced by the combina

give white. Color mixture by addition can be represented by equations as flows:

$$
\begin{array}{llllll}
R+B=P & \text{and} & R+G+B=W. & \text{Hence,} & P+G=W & \text{or} \quad P\ =W-G \\
R+G=Y & \text{and} & R+G+B=W. & \text{Hence,} & Y+B=W & \text{or} \quad Y\ =W-B \\
B+G=BG & \text{and} & R+G+B=W. & \text{Hence,} & BG+R=W & \text{or} \quad BG=W-R
\end{array}
$$

The colors of two lights which when added give white are called *additive complementary colors.* Yellow and blue, purple and green, blue-green and red are additive complementaries. Theoretically, there is an infinite number of pairs of additive complementaries.

460. Color Mixture by Subtraction.—The colors of most bodies are due entirely to selective absorption. In opaque bodies the absorption occurs within a thin layer at the surface. After traversing this thin layer, the light that has not been absorbed will be diffusely reflected and will emerge. The color of the emergent light, that is, the color of the body, is that of the incident light minus the colors that have been absorbed. No body is monochromatic. The colors of all bodies are due to waves of more than one frequency. For example, all blue and all yellow pigments transmit green in addition to the dominant colors. Some also transmit other colors. Color names are quite indefinite. The same name is used for colors of a given dominant hue even though the accompanying hues are different.

The colors produced by mixing pigments result from a process of selective absorption or subtraction. Let us represent the dominant component of a color by a capital initial letter and the accompanying components by small initial letters. We will now consider a mixture of blue and yellow pigments which transmit $B+g$ and $Y+g$, respectively. Suppose that white light first traverses a Bg particle, afterward a Yg particle, and is then reflected out of the medium. The process can be represented by a diagram as in Fig. 574. Of the incident white light, the blue particle to the left transmits only B and g. Of these two waves the lower yellow particle transmits only the green. Thus, after traversing the two particles, all the light has been absorbed except green. This color can be transmitted by subsequent particles of either sort. In this manner a mixture of blue and yellow pigments gives green. This is a subtractive process. It will be recalled (Art. 458) that a mixture of blue and yellow lights gives white. This is an additive process.

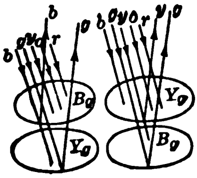


FIG. 574.

There will be some blue and yellow light reflected from the top layer of particles. The white light resulting from the combination of these two colors dilutes the green produced by the subtractive process.

Instead of mixing the pigments before they are applied, suppose that yellow is first applied and then a transparent blue is superposed. In this case green is produced as before. But only blue and green are reflected from the upper layer. Therefore the resulting color is green slightly tinged with blue, instead of green diluted with white. In water color painting the pigments are so transparent that they copiously transmit light reflected from the white paper. When a saturated mixed color is desired, that is, a color undiluted by white, one component color is first applied and, after drying, another color is superposed.

The color of a mixture of dyes is governed by the portions of the absorption spectra of the components which are common to the components. The greater the proportion of the parts of the spectra which are in common, the stronger will be the resultant hue. The smaller the proportion of the parts in common, the duller the resultant hue. If there is no color in common, black is formed.

A mixture of blue, yellow and red pigments gives brown or rusty black. Brown is also produced by a mixture of either orange and green pigments, or red and green pigments of certain selective absorptive powers.

461. Additive and Subtractive Primary Colors.—By mixing a certain monochromatic red light and a certain monochromatic green light, a hue will be produced that to the unaided eye is indistinguishable from a certain monochromatic yellow. Certain other color sensations can be produced either by a monochromatic hue or a mixture of monochromatic hues. But red, green and blue light cannot be obtained by mixing other monochromatic lights. It is found that white or any other color can be produced by mixing lights of these three hues. Consequently, red, green and blue are called the *additive primary colors.*

The colors of natural bodies are due to selective absorption and always consist of a mixture of two or more monochromatic colors. By means of three pigments which together reflect all of the colors of the spectrum, it is possible to make a color mixture which will match any pigment color except white. The colors of these three pigments are a certain purple, a certain yellow and a certain blue-green. It will be noted that purple, yellow and blue-green are the complementaries of the three additive primaries green, blue and red, respectively (Art. 459). The three colors purple, yellow and blue-green are called the *subtractive primaries.* In popular language, these colors are usually indicated by the more general names red, yellow and blue. Henceforward we shall use the latter names to indicate the subtractive primary colors.

The color obtained by mixing any two of the subtractive primaries, red, yellow and blue, is called a *subtractive secondary* color. Each secondary is said to be *complementary* to the subtractive primary which it does not contain. Thus,

Subtractive			
Primaries		Secondaries	Complementaries
$Y+R$	gives	O	B
$Y+B$	gives	G	R
$R+B$	gives	P	Y

462. Three-color Photoengraving.—A picture in natural colors can be made from three printing blocks using transparent inks of the three subtractive primary colors. The color at any part of the picture is due to the colors there superposed on the white paper. The three printing blocks are usually made by the half-tone process (Art. 455.)

In the most common practice, twelve separate photographic plates must be made to produce a set of three-color half-tone printing blocks. First, three "color record negatives" are made—one for each of the primary colors. The color record negative for blue is made with a light-filter which absorbs blue and transmits the other colors. This negative is transparent at places corresponding to the blue parts of the object, and opaque at the other places.

Similar color record negatives are made with light-filters which absorb all light except yellow and red, respectively.

Secondly, a glass positive is made of each of the three-color record negatives.

Thirdly, a screen negative (Art. 455) is made from each of the glass positives.

Fourthly, an etched metal half-tone block is made from each screen negative.

463. Color Mixture by Juxtaposition.—If a surface covered by small spots of pigments of different colors be viewed from a sufficient distance, the surface will appear of uniform color. Consider a surface covered with small dots of blue and yellow pigments. The light reflected from the surface will be

$$(B+g)+(Y+g)=2g+B+Y.$$

But $B+Y$ light $=W$ light. Therefore, the light reflected from the dotted surface will be green diluted with white.

The intensity of the light of a given color reflected by a pigment is decreased by mixing pigments of other colors with the first. The green obtained from small dots of blue and yellow pigments is more intense than the green of a mixture of the same pigments. Any pigment color can be matched by dots of pigments of the additive primary colors. And the color reflected from a surface dotted with the primary colors is of higher chroma, or intensity of hue, than that obtainable from a surface painted with a mixture of the same pigments.

In one style of impressionistic painting great brilliance is obtained by the use of spots of the additive primaries instead of by use of mixtures of the pigments. To obtain the proper fusion of such spots, the observer must view the picture from a distance.

In the laundry, yellow linen is made to appear white by rinsing in water tinged with blue. Some ladies of sallow complexion use face powder tinged with blue.

464. Color Photography.—Most methods of making photographs in colors depend upon the principle of color mixture by juxtaposition. In the Lumière method, the color spots of the picture consist of three sets of starch granules dyed blue, green and red, respectively. The dyed granules are mixed in such proportion that the mixture is of a neutral gray. A close layer, one granule thick, is spread over a sticky glass plate; the spaces between the granules are filled with an impalpable black powder; and the whole is coated with a water-proof varnish. Over the varnish is spread a film of sensitive silver emulsion. This prepared plate is placed in a camera with the glass side toward the lens.

Suppose that the object being photographed is of the one color green. The green light from the object will be transmitted by the green granules, but not by the others. On developing the latent image, black dots of reduced silver will be formed behind the green granules which transmitted light. The emulsion behind the other granules is unaffected, Fig. 575. Now, instead of "fixing" the plate as in the development of an ordinary negative, the silver is dissolved out of the black spots by means of a solution of potassium bichromate and sulphuric acid. The spots formerly black are now transparent. The emulsion behind the red and blue granules is still unaffected.

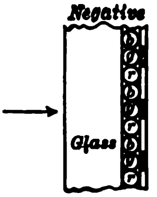


FIG. 575.

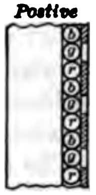


FIG. 576.

The plate is again washed, exposed to white light, developed and washed. This exposure to white light and second development has produced opaque spots behind the red and blue granules, Fig. 576. The picture is a positive made up of green dots.

On looking through the plate one sees a green image of the green object. If the object emitted light of all colors, the picture on the glass would be in the natural colors made up of dots of the three additive primaries blue, green and red. The dots are so small that they are not separately distinguishable by the eye.

465. The Young-Helmholtz Theory of Color Vision.—Our color sensations are as though the retina were provided with three sets of sensory elements or nerves, one of which is highly sensitive to blue, one to green and another to red. Each sensory element may be thought of as sensitive to a less degree throughout a considerable range of wave-lengths. The relation between the wave-length of the light which stimulates these nerves and the intensity of the sensation to which a given stimulation gives rise, may be somewhat as represented in

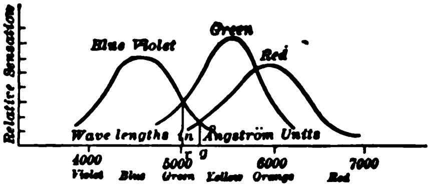


FIG. 577.

Fig. 577. The sensation of white is produced when the three sets of sensory elements are equally stimulated.

466. Color Blindness.—There are a few people whose color sensations are as though their retinæ were provided with but two sets of sensory elements. These persons are said to have dichroic vision or to be blind in one color. Cases have been described in which two, and even all three, of the color sensations of the normal eye were absent. The most common form of dichroic vision is red blindness. A few people are green blind. Blue blindness is extremely rare.

The spectrum as seen by a red-blind person does not extend so far toward the long wave-lengths as it does to a person with normal or trichroic vision. There is a region at *r*, Fig. 577, at which a neutral gray is perceived. The spectrum as seen by a green-blind person is without green and has a neutral gray band at *g*, Fig. 577. To a red-blind person the blues will match rose, and dark greens and browns will match dark red. To a green-blind person the light greens and grays will match rose, and the light greens and browns will match dark red.

Color blindness is organic, inherited and incurable. It is not affected by training in color matching. An artificial color blindness is sometimes caused by tobacco or alcohol. About 4 per cent of men are color-blind and about one-tenth of 1 per cent of women. A larger number have color weakness. The proportion of color blind is greatest among the Quakers and the Jews.

Dalton, the English chemist, who was red-blind, made the first systematic study of the phenomenon. Red-blindness is still sometimes called *daltonism.* It is related that the Quaker meeting of which Dalton was a member was once shocked to see him enter attired in the drab coat and knee breeches of the sect—and brilliant red stockings.

A person who is either red-blind or green-blind cannot distinguish by sight red fruit from green fruit. To a red-blind person the azure of the sky produces a color sensation similar to that produced by a blush on a maiden's cheek.

467. Color Contrast.—Lay a piece of red paper, about an inch in diameter and attached to a thread, on a sheet of white paper.

Fix the glance attentively on the red spot. If after about a minute an assistant jerks away the red paper, the spot of white paper uncovered by the piece of red paper will appear greenish. If instead of red, a piece of yellow paper had been used, the after image would have been bluish. Whatever the color of the small piece of paper, the after image is of approximately the complementary color. The formation of an after image of a color approximately complementary to the color on which the glance had previously rested is called *successive contrast.*

As the glance moves from one color to another the complementary of the first color is carried along and modifies the sensation produced by the second color. If green and red spots are in contact, the sensation of green will be strengthened by the green after image of red, and the sensation of red will be strengthened by the red after image of green. If black be placed next to white, the black will appear blacker and the white will appear whiter. Two complementary colors are strengthened by juxtaposition. The sensation of any color is modified by the complementary of an adjacent color. If yellow be placed next to red, the yellow will appear greenish and the red will appear bluish.

Many pairs of colors placed side by side appear to "run" at the line of contact. This effect can be prevented by separating the colors by a black or a white line.

After images and successive contrast are probably due to retinal fatigue. In terms of the Young-Helmholtz Color Theory, let us suppose that the red sensory elements at a spot of the retina have become fatigued. If now white light be incident on that spot of the retina, only the blue and the green sensory elements will be strongly affected. Hence, the after image will consist largely of the complementary color to that which produced the fatigue. The after image will be weak if the fatigue be slight.

468. The Color of Land and Sea Produced by Scattering of Light.—The color of a distant mountain depends upon the color of the incident light and upon the color absorbed from the reflected light while traveling from the mountain to the observer. On a clear day an inland mountain is illumined by blue skylight and consequently appears bluish. But if the mountain be separated from the observer by either a dusty desert or a large body of water, a

reddish hue will be imparted to the light traveling to the observer, and the mountain will appear reddish-blue, violet, or purple (Art. 457). Violet tones cannot be given to objects in the foreground or when the sky is overcast.

Clear water is very slightly blue-green. When water is still, it appears of the color of the sky, whatever its own color. But when broken into waves its own color modifies the blue from the sky. It is impossible to have a blue sea when the sky is overcast.

The color of ice is that of water except as modified by the presence of air bubbles. When light goes from water or ice to air, much of the light is totally reflected. For this reason a mass of foam, snow or ice containing bubbles of air, appears white.

When an object is in shadow it is less illumined than when not in shadow, but the color is unchanged except as modified by light from the sky or surrounding objects. Under a blue sky shadows are bluish. The weak violet shadows which are noticeable when a snowy landscape is illumined by a yellow-red sunset are due to a different cause. This is an effect of contrast. If the eye has looked at yellow-red and is then turned away, the sensation of violet is always induced. And so, as the eye moves from the yellow-red light on the snow to the gray shadow, a sensation of violet is induced and the shadow appears violet.

469. The Color of a Body Dependent upon the Color of the Incident Light.—Two bodies may match in color when illumined by white light and be quite different in color when illumined by colored light. For example consider the coloring matter of plants called chlorophyll. This substance transmits green freely and yellow and red to a considerable degree. The red and part of the green combine to produce the sensation of white. Thus, the color sensation produced by the transmitted and reflected light is a mixture of yellow, green and white. It is usually called yellow-green. When illumined by red light, chlorophyll shows red. But a yellow-green pigment, containing no red, which matches chlorophyll in white light will appear black in red light. If a green leaf and a board painted green are viewed through a piece of red glass the former will appear red and the latter black.

During the first part of the Great War certain military objects situated amid green foliage were painted green for the purpose of reducing their visibility to distant observers. It was soon found, however, that the camouflage was ineffective. The enemy aviators had been supplied with glasses of the proper color to distinguish the green containing red from ordinary pigment green. Afterward the composition of the pigment green was corrected.

The American army uniform is of a dull orange-yellow color which at a distance is inconspicuous in either a background of foliage or of barren ground. But when observed through yellow-orange glass the uniform appears lighter than surrounding foliage, and when observed through blue-green glass the uniform appears darker than the foliage.

QUESTIONS

1. Upon what does the color sensation produced by an object depend?
2. What will be the appearance of a " blue print " in red light?
3. Why do purple flowers appear red by lamplight?
4. What color of hat or waist would cause the complexion of a pale blonde to appear more rosy?
5. What color of dress would diminish the yellow of a sallow complexion?

CHAPTER XXIX

DOUBLE REFRACTION

470. The Phenomena of Double Refraction.—In the case of glass and other isotropic media, to which our attention has been limited up to the present time, light is transmitted with the same speed in all directions. But in a large proportion of crystals, light, heat, and mechanical vibrations are transmitted with different speeds in different directions. Such substances are called *anisotropic* media. Calcite, sometimes called Iceland spar, is a striking example, while quartz and tourmaline have the same property to a less degree.

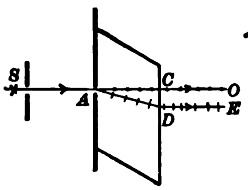


FIG. 578.

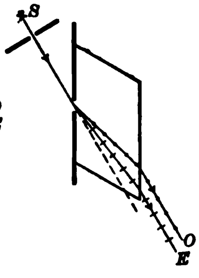


FIG. 579.

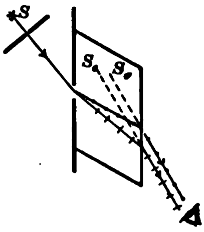


FIG. 580.

On looking through a crystal of calcite placed in front of a brightly illumined aperture one will usually see two spots of light instead of one. If the direction of the incident light, SA, Fig. 578, be normal to the base of the crystal, one portion of the light will be transmitted undeviated as in the case of glass, while another portion will be deviated at A, again at D, and will emerge parallel to the original direction. On rotating the crystal, the spot of light O will remain stationary while the spot E will rotate about it. The light propagated in the direction AO obeys the ordinary laws of refraction (Art. 165), but the light propagated along the path ADE does not obey these laws. If the direction of the incident light is oblique, Fig. 579, one of the refracted rays obeys the ordinary laws of refraction whereas the other does not. The ray that obeys the ordinary laws of refraction is called the *ordinary* ray; the ray that does not obey these laws is called the *extraordinary* ray.

Since the light along the ordinary and along the extraordinary rays has been deviated from its original direction by different amounts, the speed along these two rays must be different. On looking through the crystal toward the illumined aperture in the diaphragm one will observe that the bright spot corresponding to the ordinary ray appears to be nearer the observer than the other, Fig. 580. Therefore the light propagated along the ordinary ray suffers the greater change of speed on entering the crystal and on emerging into the air. Consequently, in a crystal of calcite, the speed along the ordinary ray is less than the speed along the extraordinary ray.

There is one direction in which light can be propagated through a crystal of calcite without suffering double refraction. When light traverses a crystal in this direction, the ordinary ray and the extraordinary become coincident. A direction in which light can be propagated in an anisotropic substance without the occurrence of double refraction is called the *optic axis* of the substance. Some substances have two optic axes.

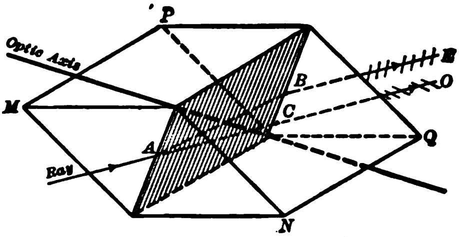


FIG. 581.

The optic axis of a crystal of calcite is the direction of a line that makes equal angles with the three edges of one of the obtuse trihedral angles of the crystal. A plane parallel to the optic axis of a crystal and perpendicular to the face on which light is incident is called a *principal plane* of the crystal. For a crystal of calcite having all the edges of equal length, Fig. 581, the optic axis is parallel to the line joining the obtuse trihedral angles. Any plane parallel to the shaded section is a principal plane of the crystal relative to the faces *MN* and *PQ*.

If the light emerging from the crystal be examined by the aid of a mirror placed at the angle of maximum polarization (Art. 383), or by the aid of any other detector of plane polarized light, it will be found that the light along each ray is plane polarized, and that the planes of polarization of the light in the two rays are at right angles to one another.

Ordinary light becomes polarized on traversing a piece of glass or other isotropic substance that is under stress. That is, glass, or other isotropic substance is rendered doubly refracting by the application of mechanical stress. By examining the transmitted light with the aid of a detector of polarized light the direction of the stress can be determined and also the approximate magnitude of the stress. When a mass of hot glass cools too suddenly, stresses are set up in it which diminish its ability to withstand shocks. The presence of these stresses can be easily detected by examining the specimen with the aid of some detector of polarized light. To prevent these internal stresses, glass must be annealed by cooling so slowly that the internal strains have time to be relieved before the molecules acquire fixed positions.

471. Fresnel's Theory of Double Refraction.—A theory of double refraction must coordinate the following facts:

(*a*) Light traverses certain substances with speeds which are different in different directions.

(*b*) In some substances there is one and in some substances there are two directions in which light can be transmitted without double refraction.

(*c*) In all other directions an incident wave is divided into two waves which traverse the substance with different speeds.

(*d*) The light in each of these waves is plane polarized, and the planes of polarization of the two waves are at right angles to one another.

A crude and imperfect analogy suggests itself. Consider a long steel rod of rectangular cross-section with one end fastened in a vise and the other end free. If the free end, Fig. 582, be displaced in the direction *AA'* and then released, a restoring force will be developed in the direction *A'A* which will cause the free end to vibrate along this line. This vibration will be propagated along the length of the rod with a certain speed. If the free end had been displaced in the direction BB' a greater restoring force would have been developed in the direction $B'B$ and the vibration would have been transmitted down the rod with greater speed. If, however, the free end is displaced in the direction CC' the restoring force is the resultant of forces in the directions $A'A$ and $B'B$. This produces a vibration of the free end of the rod which is the resultant of a vibration in the line AA' and another in the direction BB'. Since these vibrations are of different periods, two waves of different speeds will be sent simultaneously down the rod. Consequently, the disturbance produced by the original displacement in the direction CC' has been resolved into two waves polarized at right angles to one another and traveling with unequal speeds.

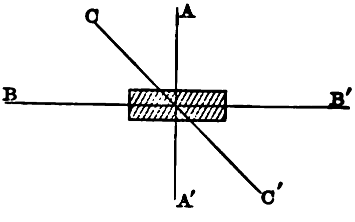


Fig. 582.

We may think of ordinary or unpolarized light as consisting of vibrations which occur successively in all directions normal to the line of propagation. Plane polarized light consists of vibrations in a single direction normal to the line of propagation. We may then imagine that on entering a doubly refracting substance, the vibrations of a wave of ordinary light are quickly altered in direction till all the vibrations are limited to two directions at right angles to one another. That is, the incident wave has been polarized in two planes at right angles to one another. This change has been effected without absorption of energy and the energy in the incident wave has been equally divided between the two transmitted waves.

The phenomena of double refraction suggest that light vibrations in a doubly refracting crystal, like mechanical vibrations in a flat spring, have different speeds for different directions of vibration. It is found that in the case of substances of which calcite is an example, the speed parallel to the optic axis is least, and the speed perpendicular to the optic axis is greatest. In terms of the previously considered analogy, the light vibrations in a specimen of calcite *C*, Fig. 583, in which the optic axis *AX* is in the plane of the paper, correspond to the mechanical vibrations of a flat spring *SP* which is wider in the plane of the paper than in the direction normal to the plane of the paper. If the spring vibrates perpendicular to the paper, waves travel slowly in the direction *SP*, and if the spring vibrates in the plane of the paper waves travel faster in the direction *SP*. So it may be that if the vibrations of the light waves in the crystal are perpendicular to *AX*, the waves which travel in the direction perpendicular to *AX* go slowly; whereas if the vibrations of the light waves in the crystal are parallel to *AX*, the waves which travel in the direction perpendicular to *AX* go faster.

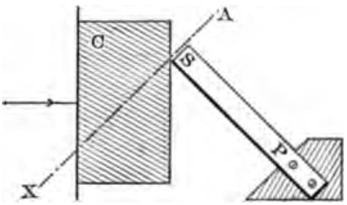


FIG. 583.

When the free end of the spring is displaced in any direction inclined to the long axis of its cross-section, and then released, two plane polarized waves will be produced. These move along the length of the spring with different speeds, and their planes of polarization are perpendicular to one another. Since light vibrations have different speeds in different directions of a doubly refracting substance, we should expect that if a wave consisting of transverse vibrations in all directions is incident on such a substance, the wave will, in general, be resolved into two components polarized at right angles to one another. In the section represented in Fig. 583, one set of vibrations will be in the plane of the paper and the other in a plane perpendicular to the plane of the paper. If, however, the incident wave advances in the direction of the optic axis, all the vibrations will be perpendicular to the axis and no polarization will be produced.

Fig. 584 represents a section of a doubly refracting substance cut parallel to a principal plane. Each point of the surface *BD* on which a wave impinges will be a center from which two waves will be propagated in all directions. Consider that part of the light which travels in the plane of the paper. We may think of the vibrations of one of the waves traveling in this plane as being in the plane of the paper, and the vibrations of the other wave traveling in this plane as being perpendicular to the plane of the paper. In Fig. 584 we shall consider the waves in which the vibrations are in the principal plane, and in Fig. 585 we shall consider the waves in which the vibrations are perpendicular to the principal plane.

In Fig. 584 the wave originating at some point *C* will, in a certain interval of time, have traveled a distance *CE* parallel to the optic axis and a *greater* distance *CF* perpendicular to the optic axis. The trace of this elementary wave front on the principal plane of the crystal is not circular but is elliptical. Traces of the waves originating at points *B* and *D* are also shown in the diagram. The envelope of these elementary wave fronts, *GH*, is the wave front of the refracted wave. Light from *C* meets the new wave front at the point of tangency of the elementary wave and the envelope. In an anisotropic crystal the ray is generally not normal to the wave front. When the wave emerges from the crystal, each point of the surface becomes a center of disturbance from which spherical waves are sent into the air. Consequently the emergent ray is normal to the wave front in the air.

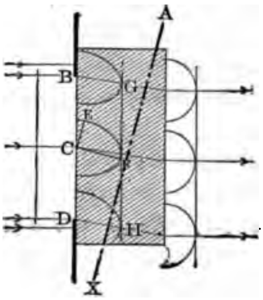


FIG. 584.

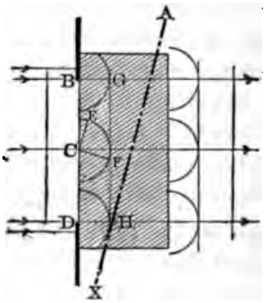


FIG. 585.

By the aid of Fig. 585 we shall consider the wave in the crystal due to vibrations normal to the principal plane. In this case, the disturbance originating at some point C will in a certain interval of time have traveled a certain distance CE parallel to the optic axis and an *equal* distance CF perpendicular to the optic axis. Consequently the trace on the principal plane of the elementary wave from C is circular. The envelope of the elementary wave fronts from all the points from B to D is the refracted wave front GH due to the component under consideration. A ray from C will be normal to this wave front and will obey the ordinary laws of refraction.

472. The Planes of Polarization of Light in an Anisotropic Medium.—When light from any source is copiously reflected by a mirror set in one position, but is not reflected when the mirror is rotated 90° about an axis coincident with the incident ray, the incident light is said to be plane polarized. The plane of polarization is defined as that particular plane of incidence in which light is most copiously reflected.

When plane polarized light is incident upon a crystal of calcite, it is found that if the plane of polarization of the incident light is parallel to the principal plane of the crystal, only the ordinary ray is transmitted, whereas, if the plane of polarization of the incident light is perpendicular to the principal plane of the crystal, only the extraordinary ray is transmitted. Consequently, the ordinary ray is polarized in the principal plane of calcite, while the extraordinary ray is polarized perpendicularly to the principal plane.

According to the generally accepted theory of light, the vibrations of plane polarized light are perpendicular to the plane of polarization.

473. Polarizing Prisms.—The most effective means of producing polarized light is by the use of doubly refracting crystals. When transmitted by a doubly refracting substance, ordinary light is separated into two parts each consisting of plane polarized light. On emerging from the crystals these two parts usually recombine and form ordinary light. But if one of these parts can be eliminated by being absorbed or by being reflected to one side, the emergent light will be plane polarized.

The polarization produced when light traverses tourmaline is due to the absorption of one of the components into which the incident light is resolved. A tourmaline polariscope consists of two plates of tourmaline cut parallel to the axis of the crystal, Fig. 151, p. 197. But as tourmaline crystals are small and usually strongly colored, this type of polariscope is unavailable for most purposes.

A commonly employed device for producing plane polarized light consists of a piece of calcite in which the second face makes such an angle with the ordinary and extraordinary rays that one of these components is transmitted whereas the other is totally reflected to one side.

Foucault's prism consists, Fig. 586, of an equilateral rhomb of calcite cut by a plane *BC*. The two sides of the cut are polished and separated by a thin film of air. On entering the calcite light is resolved into an ordinary and an extraordinary ray which meet the face *BC* at different angles. The light in the ordinary ray is incident on the air film at an angle greater than the critical angle (Art. 397) and is totally reflected to the side of the prism where it is absorbed by black paint. The light in the extraordinary ray is incident on the air film at an angle less than the critical angle and is not totally reflected. The emergent light *E* is plane polarized. The second half *BCD* of the prism prevents dispersion and deviation.

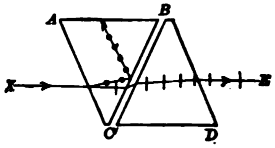


FIG. 586.

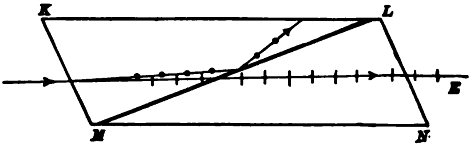


FIG. 587.

An objection to the prism is that a considerable part of the light in the extraordinary ray is lost. This is because the refractive index of calcite differs considerably from that of air. To prevent the excessive loss of light in the emergent ray due to reflection at the air film, Nicol substituted for the air film a thin layer of Canada balsam. As the index of refraction of Canada balsam is nearly that of calcite, there will be little reflection at the surface of the Canada balsam unless the light is incident on it

at an angle not less than the critical angle. The critical angle from calcite to Canada balsam being greater than that from calcite to air, light must be incident on the Canada balsam film, Fig. 587, at a larger angle than in the case of the air film, Fig. 586. This requires that the Nicol prism shall be about three times as long as a Foucault prism. But although large clear crystals of calcite are expensive, there is so much more light transmitted by a Nicol prism than by a Foucault prism that the Nicol prism is much more commonly used.

According to the generally accepted relation between the direction of vibration and the plane of polarization, the direction of vibration of the light emerging from either a Foucault or a Nicol prism is parallel to the shorter diagonal of the end face. A plane parallel to the shorter diagonals of the two end faces of a polarizing prism is called a *principal section* of the prism.

Either a Foucault or a Nicol prism can be used as an analyzer or as a polarizer. When two polarizing prisms are placed end to end, with their principal sections parallel, the light transmitted by the first is transmitted by the second with little diminution. If one of the prisms be rotated about an axis coinciding with the incident ray, the intensity of the light emerging from the second prism gradually diminishes, until, when the prisms are "crossed"—that is, when their principal sections are at right angles to one another—the intensity of the emergent light is zero. If the rotation be continued, the intensity of the emergent light increases until it attains a maximum value when the principal sections of the two prisms are again parallel.

Polarizing prisms are also made which have the two end faces perpendicular to the sides. With these prisms there is very little loss of light by reflection at the end faces. If such a prism be rotated about the optic axis, there will be no lateral shifting of the transmitted beam.

474. Rotation of the Plane of Polarization.—If a plate of quartz cut perpendicular to the optic axis be interposed in the path of light that traverses a polarizer and analyzer set for extinction, the field of view of the analyzer becomes bright. If the light be monochromatic, the light emerging from the quartz can be quenched by rotating the analyzer through a certain angle. Thus, the light transmitted by the quartz plate is plane polarized in a plane inclined to the plane of polarization of the incident light. This fact is expressed by the statement that in traversing the quartz plate, the plane of polarization of light is rotated through a certain angle. The ability to rotate the plane of polarization of light is possessed by many substances, solid, liquid, and gaseous. Some produce a rotation in the clockwise direction, while others produce a rotation in the counterclockwise direction.

Biot found, (*a*) that the amount of rotation produced by any substance is proportional to the thickness; (*b*) that when light traverses more than one substance, the rotation equals the algebraic sum of the rotations due to the separate substances; (*c*) the rotation depends upon the wave-length of the light transmitted; (*d*) in the case of solutions of active substances in inactive solvents, the rotation is proportional to the concentration.

A plate of quartz 1 mm. thick, cut perpendicular to the optic axis, rotates the plane of polarization of red light about 18°, and of yellow light about 22°. A column of 50 per cent aqueous cane sugar solution, 10 cm. long, produces a rotation of the plane of polarization of yellow light of about 21.7°.

475. Elementary Explanation of the Rotation of the Plane of Polarization.—It can be shown that any simple harmonic motion can be resolved into two uniform circular motions of equal period in opposite directions. This fact can be illustrated by the following device: Let a bead M be so mounted on a rod OM that it can rotate clockwise in its own plane at a uniform rate about an axis through O perpendicular to the plane of the paper; and let this axis through O simultaneously rotate with the same angular speed in the counterclockwise direction about another parallel axis through C. In Fig. 588, O_1, O_2, O_3 and O_4 represent positions of the end O of the rod, and M_1, M_2, M_3 and M_4 represent the corresponding positions of the other end M.

In the first position M is at C and is moving horizontally to the left with respect to O, while at the same time O is moving to the left with respect to C. The resultant motion of M is therefore directed toward the left, and its speed is twice as great as that of O.

In the second position M is at X and is moving upward with respect to O, while O is moving with equal speed downward with respect to C. At this position, then, the motion of M with respect to C is zero.

Similarly we see that in the third position, M is again at C, but this time is moving to the right. In the fourth position, M is at rest at X'.

If we consider any other position, we find that M still lies on the line XX'. For example, if the selected position be half way between the first and the second, M is somewhere between O_2 and M_2; its velocity with respect to O is directed upward toward the left and the velocity of O with respect to C is directed downward toward the left. The resultant velocity of M is horizontal and toward the left. Thus we see that as a result of the two component circular motions the bead travels back and forth along a straight line.

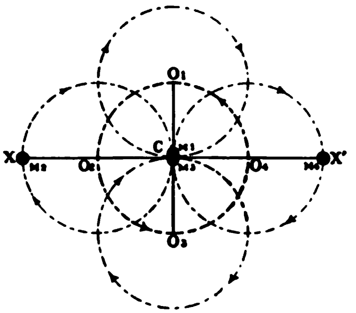


Fig. 588.

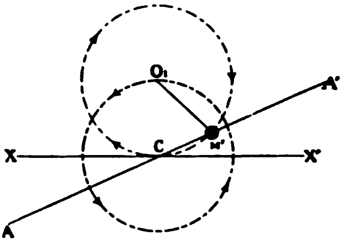


Fig. 589.

It will now be shown that if the speed of one of the circular components be retarded for a time, and then be allowed to resume its former value, the path of the resultant simple harmonic motion will be rotated from the axis XX' to some new position AA', Fig. 589. To fix the ideas, suppose that while O has traveled 360° with respect to C, M has traveled only 330° with respect to O. At this instant the bead M is at the position M', Fig. 589. If from this instant the retardation ceases and both components become of the same speed, the resultant motion will be back and forth along the axis ACA'.

The explanation of the rotation of the plane of polarization is based on the principles above illustrated and has been fully verified by Fresnel both theoretically and experimentally. According to the work of Fresnel it appears that plane polarized light may be regarded as being composed of two portions, circularly polarized in opposite directions. In traversing any substance, plane polarized light is resolved into its two circularly polarized components, and these components may traverse the substance with different speeds. If the two components traverse the given substance in any direction with equal speed, they combine on emergence into plane polarized light polarized in the same plane as before entering the given substance. If, however, the two components traverse the given substance in the direction of the optic axis with unequal speeds, the emergent light is plane polarized in a plane inclined to the plane of polarization of the entrant light.

If plane polarized light be incident normally upon a plate of doubly refracting substance cut parallel to the optic axis, the emergent light may be plane polarized, circularly polarized, or elliptically polarized, depending upon the relative retardation of the two components produced by the substance.

476. Laurent's Half-shade Analyzer.—The obvious method of determining the amount of rotation of the plane of polarization produced by any substance would be to set two Nicol prisms for extinction; place the substance under investigation between the prisms; and rotate one prism until the field of view again becomes dark. The troubles with this method are that the eye is not very sensitive to small changes of brightness, and the mind cannot accurately compare the brightness of two things unless seen simultaneously and in juxtaposition. To overcome these difficulties several methods have been devised in which the plane polarized light entering the analyzing Nicol is divided into two plane polarized portions with the planes of polarization inclined at a small angle to one another. Thus, suppose that in advancing toward the observer, monochromatic plane polarized light in which the vibration is parallel to OB, Fig. 590, is divided into two parts—one which illumines the right half of the field of view and retains its original plane of

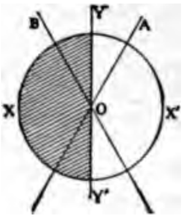


FIG. 590.

vibration, and another portion of equal brightness which illumines the left half of the field of view and has its plane of vibration parallel to OA. Without an analzying Nicol, both halves of the field of view are of the same brightness. But with an analyzing Nicol the two halves are unequally bright except when the principal plane of the analyzer is parallel to YY'.

The device used by Laurent to produce this separation of plane polarized light into two portions consists of a plate of quartz YXY' and a plate of glass $YX'Y'$, Fig. 590, joined together along one edge. The plate of quartz is cut with the optic axis parallel to the joint YY', and is of such a thickness that during the passage of light through it, light in the extraordinary ray is retarded more than light in the ordinary ray by an amount equal to one-half wave-length of the monochromatic light used. The glass plate is of such a thickness that the light which traverses it is reduced in brightness, through absorption and reflection, by the same amount as the light that traverses the quartz plate.

Suppose that in the monochromatic plane polarized light incident on the compound quartz-glass plate the vibration is parallel to some line OB. The part of the light incident on the glass plate emerges with the vibration in the same plane OB, but the portion incident on the quartz emerges as plane polarized light with the vibration in some other plane OA. It can be shown that the planes of vibration of the emergent light, OB and OA, are equally inclined to the joint YY'. Consequently, when a Nicol prism is placed in front of the quartz-glass plate with the principal plane parallel to the joint, the field of view is uniformly bright. With the Nicol turned out of this position, even very slightly, the two halves of the field of view are of unequal brightness. This quartz-glass plate is called Laurent's *half-shade analyzer*.

As usually employed, the half-shade analyzer is placed between two Nicol prisms with the joint between the quartz and glass plates slightly inclined to the principal plane of the polarizing Nicol. The polarizing prism is illumined with monochromatic light and the analyzing prism is turned till the field of view is uniform. The specimen under investigation is then placed between the half shade analyzer and the analyzing prism. If a rotation of the plane of polarization has been produced, the two halves of the field of view are no longer equally bright. The angle through which the analyzing Nicol must be turned to bring the two halves to equal brightness is the amount of rotation produced by the specimen.

477. The Laurent Saccharimeter.—In customs houses and sugar refineries it is necessary to have an accurate method for quickly determining the percentage of pure sugar in a given specimen of syrup or solid sugar. The most convenient means, and the one usually employed, is afforded by the fact that sugar rotates the plane of polarization of light passing through it. A tube with glass ends is first filled with a solution of pure sugar of known concentration, and the amount of rotation of the plane of polarization produced by it is observed. The same tube is then filled with a solution of the given specimen and the amount of rotation produced by it is observed. Since for a layer of constant thickness and temperature, the rotation depends directly upon the concentration, the per cent of sugar in the given specimen can be readily computed. An instrument for determining the sugar content of a solution is called a *saccharimeter*.

Laurent's saccharimeter consists of a lens *O*, Fig. 591, for parallelizing the light emitted by some source not shown in the engraving, a polarizing prism *P*, a half-shade analyzer *D*, a specimen tube *S*, an analyzing Nicol *A*, an eyepiece *E*, and a divided circle *V* for reading the angle through which the analyzing Nicol is turned. The half-shade analyzer is usually made for yellow light. To produce light of the proper color, a gas flame supplied with common salt may be used, or an absorptive plate of potassium bichromate may be interposed between the polarizer and any light source.

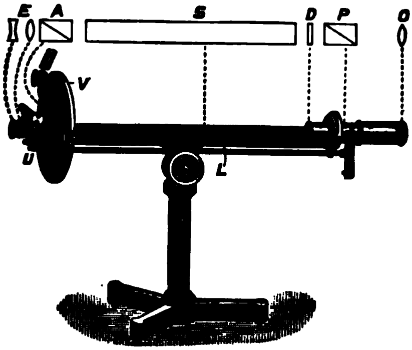


Fig. 591.

There are several sorts of sugar, some of which produce right-handed rotation, others of which produce left-handed rotation. Oftentimes a specimen is a mixture of right-handed and left-handed sugar. The concentration of such specimens can also be determined, but the methods employed in such cases belong particularly to the laboratory and will not here be described.

PROBLEMS

The Solving of Problems.—Our pleasure in pursuing a subject is in direct proportion to the degree of our mastery of it. Mastery of a subject involves the ability to use it. Solving numerical problems is one of the most effective means for developing the ability to make use of the principles of Physics. The present list of problems is designed to furnish experience in the application of the principles considered in the text and to give facility in deducing required relations from assigned data. Securing a correct answer is less important than making a well-planned attack.

A few of the problems at the beginning of each group can be solved by inserting the given data into one of the equations proved in the text and then performing the arithmetical operations indicated. In most cases, however, the required equation can be obtained only by combining two or three other equations. But in any case, it is better to get early into the habit of deducing the equation required to solve the problem than to seek one already made. Never use a proportion without first proving it by means of equations.

The figures necessary to express the accuracy of a number, and not to locate the position of the zero point, are called *significant figures*. It is a principle of calculation that the products and quotients of quantities obtained from measurement need not be expressed with a greater number of significant figures than the original data. Thus,

6481 grams × 78 cm. per sec. per sec. = 50,500 dynes;

21.312 dynes ÷ 980 = 0.0217 gram weight.

To avoid repetition, some data of frequent use are given below. In solving problems, no data are to be used which are not given either below or in the statement of the problems.

1 cm. = 0.394 in.
1 ft. = 30.5 cm.
1 kg. = 2.20 lb.
1 lb. = 0.454 kg.
1 American ton = 2000 lb.
1 metric ton = 1000 kg.
1 horse-power = 550 ft. lb. per sec.
1 force de cheval = 75 kg. m. per sec.

Acceleration due to gravity = 980 cm. per sec. per sec.
= 32.1 ft. per sec. per sec.

When a gas is at 0° C., and under a pressure of 76 cm. of mercury, it is said to be under "standard conditions."

Velocity of sound in air at 0° C. = 331.2 meters (1187 ft.) per sec.
Heat equivalent of vaporization of water at 100° C. = 559 calories per gram.
Heat equivalent of fusion of ice = 80 calories per gram.
Heat equivalent of fusion in lead = 5.8 calories per gram.
Mechanical equivalent of heat, $J = 4.2\ (10^7)$ ergs per calorie = 970 B.t.u. per lb.
All temperatures are Centigrade unless otherwise stated.

DENSITIES

In g. per cc. unless otherwise stated.

Air (standard conditions)	0.0013	
Aluminium	2.6	
Brass	8.5	
Cork	0.25	
Diamond	3.5	
Glass	2.6	
Gold	19.3	
Ice	0.92	
Iron	7.5	
Mercury	13.6	
Silver	10.5	
Water (pure)	1.0	
"	62.5	lb. per cu. ft.
Water (sea)	1.03	
"	64.0	lb. per cu. ft.

SPECIFIC HEATS

Air (p. constant)	0.24
Air (v. constant)	0.17
Aluminium	0.22
Brass	0.09
Copper	0.09
Glass	0.18
Ice	0.50
Iron	0.11
Lead	0.03
Mercury	0.03
Silver	0.05
Steam (p. constant)	0.48
Tin	0.05
Turpentine	0.47
Zinc	0.09

COEFFICIENT OF EXPANSION PER ° C.

Brass (linear)	0.000019
Copper (linear)	0.000017
Glass (linear)	0.000008
Iron and steel (linear)	0.000012
Platinum (linear)	0.0000086
Silver (linear)	0.000019
Zinc (linear)	0.000029
Mercury (cubical)	0.000182

THERMAL CONDUCTIVITY

Calories per cm.^{3} per ° C. per sec.

Air	5.22 (10^{-5})
Iron	0.16
Lead	0.08
Mercury	0.02
Silver	1.10
Water	0.0014
Wood (dry)	0.5 (10^{-3})

RESISTIVITY

	Ohms per Centimeter Cube at 0° C.	Ohms per Circular Mil Foot at 0° C.	Temperature Variation per ° C.
Copper	1.6 (10^{-6})	9.8	0.0039
German silver	21 (10^{-6})	126.	0.0004
Iron	9.7 (10^{-6})	58.	0.0053
Platinum	9 (10^{-6})	54.	0.0036
Silver	1.6 (10^{-6})	9.7	0.0037

Mechanical Advantage; Work and Efficiency

1. Show by a diagram how to rig a pair of double pulleys so that their mechanical advantage shall be 5.

2. By means of a diagram represent a system of two fixed and two movable pulleys with continuous cord. What pull on the free end of the cord will support a weight of 1000 lb. suspended from the movable pulley block, the blocks being so rigged that the operator exerts a downward pull? (Omit the effect of friction.)

3. With the aid of a diagram, show a contrivance, employing a system of pulleys attached to an overhead support by which a person who can exert a force of only 100 lb. wt. can raise a load of a little under 400 lb. wt. How great is the force exerted on the overhead support?

4. The 600-lb. hammer of a pile driver is raised 20 ft. and is then allowed to fall on the head of a pile, which is thereby driven 2 in. into the mud. Find the average force exerted by the hammer on the head of the pile.

5. A machinist exerts upon a file a force of 10 lb. wt. downward and 15 lb. wt. forward. How much work does he do in 40 horizontal strokes, each 6 in. long?

6. Show by a diagram how two single pulleys may be so arranged as to afford a mechanical advantage of 2. If an effort of 3.25 lb. wt. must be used with this device in order to raise a weight of 4 lb., what is the efficiency of the machine?

7. A man is capable of exerting a force of 150 lb. wt. How long an inclined plane must be used in order to push a truck weighing 600 lb. upon a platform 3 ft. above the ground? Neglect friction.

8. A plow making 12 furrows to the rod requires an average pull of 550 lb. wt. Find the work done in plowing an acre. (An acre = 160 sq. rods.)

9. How much work is done against gravity by a man weighing 180 lb., climbing a mountain 4000 ft. high?

10. How much work is expended in raising the brick for building a uniform column 66 ft. 8 in. high and 21 ft. square? Weight of brick per cubic foot is 112 lb.

11. Find the work required to lift the stone from the ground to build a cylindrical reservoir 20 ft. outside diameter, 90 ft. high and 5 ft. thick. Weight of stone is 125 lb. per cu. ft.

12. Find the work done to wind up a 250-ft. chain hanging vertically which weighs 20 lb. per ft.

13. A ladder 50 ft. long rests against a wall, making an angle of 30° with it. Calculate the work done by a man weighing 160 lb. when he carries a 120-lb. load up the ladder.

14. A man draws a box along the sidewalk for a distance of 100 ft. by means of a rope which makes an angle of 30° with the horizontal. The pull in the rope is 25 lb. wt. Calculate the total work done.

15. A laborer carries 1000 lb. of brick to a height of 20 ft. in 20 trips. He weighs 180 lb. and his hod weighs 20 lb. Calculate (*a*) the useful work, (*b*) the useless work.

16. In the preceding problem assume the laborer uses a single pulley and rope, hoisting the bricks in 10 loads in a bucket weighing 30 lb. Neglect any losses due to friction. Calculate (*a*) the useful work, (*b*) the useless work. Compare the efficiencies of the two methods.

17. If it is found possible, by a wheel and axle, to raise a weight of 500 lb. by applying a force of 70 lb. wt., when the diameter of the wheel is 4 ft. and that of the axle is 6 in.; calculate (*a*) the theoretical mechanical advantage, (*b*) the actual mechanical advantage, (*c*) the efficiency.

18. On turning the handle of the windlass of a certain derrick 1 ft. the load is raised 0.2 in. How heavy a load can be raised by applying to the handle a force of 60 lb. wt.?

19. A bucket of water weighing 30 lb. is to be raised from a well 20 ft. deep by means of a windlass having an efficiency of 80 per cent. If the crank arm is 15 in. long and the drum on which the rope is wound is 6 in. in diameter, how great an effort must be applied at the crank, and through what distance must it act?

Static Moments

20. The crank arm of a windlass is 50 cm. long, and the shaft around which the rope is wound is 18 cm. in diameter. What force must be applied at right angles to the end of the crank arm to raise a body weighing 110 kg. attached to the rope?

21. In the preceding problem, at what angle to the crank arm must a force of 20 kg. wt. be applied in order to hold the 110 kg. wt. in equilibrium?

22. A uniform plank 10 ft. long having a body weighing 25 lb. fastened to one end is balanced at a point 3 ft. from the loaded end. Find the weight of the plank.

23. A beam of uniform cross-section is carried by three men, one at one end and two by means of a light bar placed crosswise under the beam.

How far from the middle of the beam must the bar be placed that each man may bear one-third the weight?

24. Two boys make a see-saw by balancing on a fence a uniform board 5 m. long. On one end sits a boy weighing 35 kg. Where must the other boy weighing 40 kg. place himself in order to balance the first boy?

25. A uniform iron rail weighing 100 lb. is supported by two posts 10 ft. apart, the posts being 12 ft. and 8 ft. respectively, from the ends of the rail. Find the weight each post must bear.

26. A stiff uniform pole 12 ft. long sticks out horizontally from a vertical wall. It would break if 28 lb. wt. were applied vertically at the end. How far out along the pole may a boy safely venture who weighs 112 lb.?

27. A uniform beam, 20 ft. long and weighing 100 lb., rests in a horizontal position on a fulcrum 4 ft. from one end which at this end presses against the underside of a second beam. (*a*) How great is the upward force exerted on the second beam? (*b*) How great is the downward force exerted on the fulcrum?

28. A uniform bar 100 cm. long weighs 20 kg. What force must be applied on one end of the bar so that the beam will just balance about a point 20 cm. from that end?

29. A uniform beam 20 ft. long weighs 200 lb. It is supported at a pivot 12 ft. from one end *A*, at which hangs a body of 100 lb. wt. Where must a body of 300 lb. wt. be applied to keep the beam horizontal? What is the force on the pivot?

30. A painter stands on a scaffold hung by its ends from vertical ropes *A* and *B*, 16 ft. apart. The ladder weighs 50 lb., the tension in *A* is 140 lb., and that in *B* is 60 lb. What is the weight of the painter? How far from *A* is he standing?

31. If the handle of a claw hammer is 12 in. long and the distance from point of contact to a nail head is 2 in., how much resistance is offered by the nail when a force of 25 lb. wt. is required to draw it out?

32. Two men, A and B, 12 ft. apart, carry a 150-lb. body between them on a pole. Where must the body be placed in order that A may not carry more than 60 lb. wt.? Give a diagram.

33. A telephone pole 30 ft. long weighs 400 lb. If 15 ft. of its length project beyond the edge of an horizontal surface on which the pole rests, a weight of 80 lb. at the outer end will just cause it to tip. Find the enter of gravity of the pole.

34. A man weighing 150 lb. stands on one end of a railroad rail, 30 ft. long, which balances over a fulcrum at a point 2 ft. from its middle. What is the weight per yard of the rail?

35. A uniform beam 10 ft. long and weighing 50 lb. rests on a support 4 ft. from one end, and is to be kept horizontal by a vertical force at some one other point. (*a*) Where must this vertical force be applied in order that it may be as small as possible? Find its magnitude and direction. (*b*) Where must a vertical force be applied to make the pressure on the support as small as possible, and how great is the force and how great is the load on the support?

36. A rod 18 ft. long and weighing 80 lb. is supported at the end *A* and at a point 4 ft. from the end *B*. Where must a body weighing 40 lb. be hung to produce equal loads on the two supports?

37. A plank *AB* 20 ft. long and weighing 100 lb. rests on top of a box 4 ft. wide with the end *A* projecting 7 ft. beyond the box. Find (*a*), how near the end *A*, a 60-lb. boy can approach without upsetting the plank, (*b*), how near the end *B*.

38. A uniform rod 12 ft. long and weighing 24 lb. rests horizontally on two props distant 2 ft. and 4 ft. from the two ends. Find the force supported by each prop.

39. A uniform beam 12 ft. long and weighing 100 lb. rests horizontally on similar supports at its ends. Find the force supported by each prop when a load of 70 lb. wt. is placed one-third of the distance from one end.

40. Two men carry a body weighing 90 kg. suspended from a light pole 3 m. long. If the body be placed at a distance of 1.2 m. from one end, what weight does each man bear?

41. A man carries on his shoulder a uniform straight pole weighing 2 kg., on one end of which hangs a body weighing 10 kg. He keeps the pole horizontal by holding down the other end with his hand. If the distance from shoulder to hand is 70 cm. and the distance from shoulder to suspended body is 2 m., find the vertical force exerted by the hand, and the weight on the shoulder.

42. A window sash 3 ft. wide and weighing 25 lb. is supported by two sash cords, to each of which is attached a piece of iron weighing 10 lb. If one of the cords is broken, find at what distance from the middle of the sash the hand must be placed to raise it with the least effort.

43. A uniform rod, 14 in. long and weighing 10 lb. is joined so as to be in the same straight line with another uniform rod 16 in. long and weighing 8 lb. Find the point on which they will balance.

44. A uniform beam 50 ft. long and weighing 100 lb. rests horizontally with its ends on two supports. The beam carries loads of 30, 50, and 80 lb. wt. at distances of 10, 20, and 35 ft. respectively from one support. Find the reaction at each support.

Composition and Resolution of Forces

45. Three cords of equal length are attached to a small iron ring. Two boys, at a distance from one another equal to the length of each cord, pull at the ends of two of the cords with forces of 50 and 60 lb. wt. respectively. What force must a third boy exert on the other cord so that the ring will be at rest? Find also the angle the third cord makes with the cord in which there is the tension of 60 lb. wt.

46. A boat is towed along the middle of a canal 50 ft. wide by mules on both banks. Each rope is 72 ft. long and is under a tension of 800 lb. wt. Find the total effective pull on the boat.

47. Three smooth posts fixed in the ground so as to form an equilateral triangle are wrapped about by a tightly stretched rubber band which is under a tension of 2 lb. wt. Find the force acting on each post.

48. Two tug-boats are pulling on a vessel, one with a force of 1200 lb. wt., the other with a force of 900 lb. wt. The cables from the two tugs are at right angles to each other. How much is the resultant pull on the vessel?

49. A piece of wire 26 in. long, and strong enough to support directly a load of 100 lb. wt., is attached to two points 24 in. apart in the same horizontal line. Find the maximum load that can be suspended at the middle of the wire.

50. A string 7 ft. long has its ends attached to two points in the ceiling 5 ft. apart. When a stone is attached to the string 3 ft. from one end there is in the short portion of the string a tension of 8 lb. wt. and in the longer portion a tension of 6 lb. wt. Find the weight of the stone.

51. A fish caught by a rod and line pulls with a force of 4 lb. wt. The inclination between the rod and the line is 50°. What force on the end of the rod normal to its length must the rod be able to bear?

52. A board will just support a load of 200 lb. wt. placed at its middle point when the board is inclined to the horizon at an angle of 25°. What weight would it support when placed horizontally?

53. Two men standing on opposite sides of a pit are drawing up a bucket of earth by means of two ropes. When the bucket is in equilibrium, one man is exerting a force of 70 lb. wt. on a rope inclined 15° to the vertical while the other man is exerting a force of 50 lb. wt. on a rope inclined 20° to the vertical. Find the weight of the bucket.

54. A horse is attached to a wagon so that the traces make an angle of 20° with the ground. If the road is level and offers a resistance to the wagon's motion of 30 lb. wt., find how much a horse must pull in order to keep the wagon moving uniformly.

55. A horse exerts a force of 50 lb. wt. in towing a boat along a canal. If the boat is 10 ft. from the towpath and moves parallel to it, find the effective force of the horse when the towline is 20 ft. long.

56. An automobile weighing two tons stands on a hill which rises 10 ft. in every 50 measured along the incline. What is the direction and magnitude of the least force that will hold it there?

57. A boy drags a sled by exerting a force of 6 lb. wt. on a rope reaching from his arm to the sled, a distance of 5 ft., his arm being 3 ft. higher than the sled. What part of the force that he is exerting is effective in dragging the sled forward?

58. A man who can exert a force of but 50 lb. wt. is required to load a 200 lb. barrel into a wagon whose bed is 3.5 ft. high. Find how long a plank he must use in order that he can roll the barrel into the wagon. (Solve by principle of work and also by resolution of forces.)

59. A balloon capable of supporting a body weighing 200 kg. is held by a rope which makes an angle of 60° with the horizontal. Find the tension of the rope and the horizontal pressure of the wind on the balloon.

60. A picture weighing 25 lb. is suspended from a nail in the wall by means of a wire whose ends are fastened to the sides of the frame. Find the tension in the wire; first, when the two halves of the wire make an angle of 60°; second, when this angle is 30°.

61. A man weighing 160 lb. sits in a hammock suspended by ropes which are inclined at 30° and 45° to vertical posts. Find the tension in each rope.

62. A body weighing 125 kg. is suspended by a rope. A second rope attached to the body is drawn in a horizontal direction until the suspended rope has been deflected 30° from the vertical. Find the tensions in the two ropes.

63. Find the force required to sustain a body weighing 75 kg. on a plane inclined at 45° to the horizontal: first, when the direction of the force is horizontal; second, when the direction of the force is parallel to the plane. Find also the force perpendicular to the plane in each case.

64. A chandelier weighing 75 kg. is suspended at the intersection of two rafters inclined to one another at an angle of 120°. Find the thrust along each rafter due to the chandelier.

STATIC EQUILIBRIUM

65. A uniform beam, 12 ft. long and weighing 50 lb., rests with one end at the bottom of a vertical wall, while a point in the beam 10 ft. from the bottom is connected by a horizontal string with a point in the wall 8 ft.

above the ground. Find the tension of the string, and the pressure against the wall.

66. Two uniform beams, each 24 ft. long and weighing 110 lb., joined at one end, rest with their other lower ends fixed to the top of two vertical walls of the same height and 36 ft. apart. Find the horizontal thrust tending to overturn each wall.

67. A uniform rod 3 ft. long and weighing 25 lb. is supported horizontally with one end hinged to a vertical wall and the other end attached by a string to a point 4 ft. above the hinge. A body weighing 50 lb. is suspended from the free end of the rod. Find the tension in the string and the horizontal thrust on the hinge.

68. A uniform horizontal bar *AB*, 3 m. long and weighing 50 kg., has the end *B* hinged to the vertical side of a building, while the end *A* is supported by a light rope tied to it and to the building at a point 4 m. above *B*. Find the tension in the rope, and also the horizontal and vertical components of the reaction at the hinge.

69. A horizontal beam 12 ft. long is supported from a vertical wall by a bracket which holds one end, and by a rope attached at its middle point and fastened to a point on the wall 6 ft. above the bracket. A load of 100 lb. wt. is fastened at the outer end of the beam. The weight of the beam is 60 lb. Calculate (*a*) the tension in the rope; (*b*) the horizontal, and (*c*) the vertical reaction at the bracket.

70. Two vertical posts are 20 ft. apart and between them a rope is stretched horizontally from hooks. When a force of 600 lb. wt. is applied at the middle of the rope the sag is 1.944 ft. Neglecting the weight of the rope calculate

(*a*) The tension in the rope;
(*b*) The magnitude and direction of the force on each hook;
(*c*) The horizontal and vertical forces on each hook.

71. A uniform horizontal beam 5 m. long has one end supported on a ledge in a vertical wall, while the other end is supported by a light rope which makes an angle of 30° with the beam. The beam weighs 5 kg. and supports a load of 20 kg. wt. placed 3 m. from the wall. Find the tension in the rope, and also the vertical and horizontal reactions of the wall.

72. A uniform beam 36 ft. long, weighing 150 lb., rests with one end against a smooth wall, and the lower end, which rests on the ground, is prevented from slipping by a peg in the ground. If the inclination of the beam to the horizon be 30°, find the thrust against the wall and the vertical and horizontal components of the reaction at the peg.

73. A uniform beam 20 ft. long and weighing 100 lb. rests with one end on a smooth floor, and the other end against a smooth vertical wall. If a string 5.4 ft. long connects the lower end with the foot of the wall, find the tension in the string, and the reactions exerted against the floor and the wall.

74. Two equal uniform boards, each 10 ft. long, and weighing 20 lb., are hinged together at one end, while the other ends rest on a smooth floor. If a string 5.4 ft. long connects the lower ends, find the tension in the string and the reaction at the hinges.

75. A derrick has a uniform boom 16 ft. long and weighing 150 lb., with one end hinged to a vertical mast 16 ft. from the top. A rope extends from the other end of the boom to the top of the mast. When the boom makes an angle of 60° with the mast and supports at its end a load of 900 lb. wt., find the tension in the rope, and the vertical and horizontal thrusts on the pin of the hinge connecting the boom to the mast.

FRICTION BETWEEN SOLIDS

76. Water presses against the vertical gate of a large valve with a total force of 31,500 lb. wt. The gate weighs 1800 lb., the coefficient of static friction between the gate and its ways is 0.20 and the coefficient of kinetic friction is 0.15. Calculate the vertical force, (*a*) necessary to start the gate, (*b*) necessary to keep it moving upward with constant speed.

77. A horse draws a load weighing 2000 lb. up a grade[1] of 1 in 20. The resistance on the level is 100 lb. wt. per ton. Find the pull on the traces when they are parallel with the incline.

78. How much work is done in drawing a mass of 100 kg. at uniform speed up a plane 6 meters long, inclined 30° to the horizontal? Coefficient of friction is 0.2.

79. A body weighing 50 lb. is held by friction on a plane inclined 30° to the horizontal. Find the frictional resistance and the pressure on the plane.

80. A uniform beam weighing 60 kg., inclined at an angle of 60° to the horizontal, rests between a rough pavement and a smooth vertical wall. Find the reactions against the pavement and the wall, and, also, find the coefficient of friction between the beam and the pavement when the beam is just on the point of slipping.

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[1] The grade of a hill is usually specified by the ratio of the vertical rise to the horizontal distance. Thus a grade is said to be 1 in 20, or to be 5 per cent, when the tangent of the inclination equals 1/20 or 0.05.

81. On top of a stepladder the sides of which make an angle of 60° with each other stands a man weighing 70 kg. Supposing each side of the stepladder is uniform and weighs 10 kg., find the reaction of the ground at the foot of each side of the ladder. If the feet of the ladder are just on the point of slipping, find the coefficient of friction between the ladder and the ground.

82. The weight on a locomotive drive wheel is 20,000 lb. wt. If the coefficient of kinetic friction between the drive wheel and the brake shoe is 0.3 and the maximum coefficient of static friction between the wheel and the rail is 0.2, find the normal brake shoe force that will produce the maximum retarding force on the locomotive.

83. A train is moving with uniform speed up a 2 per cent grade.[1] Find the tension of the couplings of the car next to the locomotive, assuming that the weight of the train, exclusive of the locomotive, to be 80 tons and the frictional resistance 8 lb. wt. per ton.

84. A boat weighing 1500 lb. is to be drawn up a 20 per cent grade. The coefficient of friction between the boat and the beach is 0.3. A pair of four-pulley blocks are available, as well as suitable rope and a tree for attaching one of the blocks. Make a sketch of the tackle and find the force required to draw the boat.

85. A safe weighing 3500 lb. is to be taken through a door 5 ft. above the ground. There are available some 15-ft. planks which can be used as skids, a rope, various pulley blocks, and sufficient men to exert a pull of 360 lb. wt. The coefficient of friction between the safe and the skids is 0.4. Draw a diagram of the tackle you would use and compute the tension in the rope.

UNIFORMLY ACCELERATED LINEAR MOTION

86. A train is moving on a level track with a speed of 40 mi. per hr. If the brakes are applied so as to produce a retardation in the speed of 10 ft. per sec. per sec. find the time required to bring the train to rest.

87. If an engineer can retard his train at the rate of 4 ft. per sec. per sec., how far from a station must he put on his brakes if he is traveling at the rate of 60 mi. per hr.?

88. A ball is thrown up and 5 sec. later is caught. (*a*) How high did it rise? (*b*) With what velocity did it return to the hand?

89. A rifle bullet leaves the muzzle of the rifle with a velocity of 750 m. per sec. The barrel is 80 cm. long. Assuming the acceleration to be constant, find its value in cm. per sec. per sec. Also find the time taken to traverse the barrel.

90. A lad wishing to time the shutter of his camera, hung a tape line from a second story window, and had a companion drop a bullet from the zero mark on the tape. He photographed the falling bullet, and the negative showed that while the shutter was open, the bullet had moved from opposite the mark 16 ft., to opposite the mark 16 ft. 4 in. Find the time of exposure of the plate.

91. How long must a constant force of 1 kg. wt. act on a kilogram mass to give it a speed of 50 cm. per sec., from rest?

92. A force equal to the weight of 2 kg. acts on a mass of 50 kg. for 30 seconds. Calculate (*a*) the acquired speed, and (*b*) the distance passed over in this time.

93. A car of 50 tons, moving 10 ft. per sec., is stopped by a bumper in 0.5 sec. Find the average force of the impact.

94. A force equal to the weight of one ounce acts upon a pound for 10 sec. Find the speed generated and the distance through which the mass will be moved in 10 sec. if the body starts from rest?

95. What is the force equivalent to the weight of 3 kg. at a place where a body starting from rest falls freely through 44.1 m. in 3 sec.?

96. A force of 4 lb. wt. causes a certain mass to move from rest through 18 ft. in 3 sec. Find the mass.

97. A body is projected along a horizontal plane with a speed of 100 ft. per sec. If the coefficient of friction is 0.1, find how far the body will move.

98. A body moves 10 ft. along a horizontal plane before coming to rest. If the coefficient of friction be 0.1, find the initial speed of the body.

99. A mass of 2000 lb. is moving with a velocity of 20 ft. per sec. A force of 100 lb. wt. opposes the motion. What is the time required to bring the body to rest?

100. Find the mean force acting on a nail which advances 0.25 in. when struck by a 1-lb. hammer moving with a speed of 40 ft. per sec.

101. If the coefficient of friction between the driving-wheels of a locomotive and the rails is 0.2, what must be the weight, in tons, of the locomotive in order to exert a pull of 5 tons' weight?

102. Determine the tractive force required to haul a car weighing 100 tons with constant velocity up a 2.5% grade when the coefficient of friction is 0.005.

103. A body weighing 60 lb. is just set in motion on a rough horizontal plane by a force of 9 lb. wt. parallel to the plane. If the force be withdrawn and the plane tilted, at what inclination of the plane to the horizon will the body begin to slide?

104. The slide valve of a certain steam engine is pushed against the valve seat with a force of 1000 lb. wt., and the coefficient of kinetic friction between the slipping surfaces is 0.04. If the mass of the sliding system is 50 lb., find the force required to give it an acceleration of 100 ft. per sec. per sec.

105. What draw-bar pull must a locomotive exert on a freight train of 5000 tons mass in order to get up a speed of 2 mi. per hr. in 100 sec.?

106. What constant horizontal force is required to stop a train of 100 tons mass running at 50 mi. per hr., (*a*) in 1 min.; (*b*) in 200 yd.?

107. A car of 80 tons starting from rest on a level road, has a speed of 30 mi. per hr. at the end of the first mile. Determine the average tractive force of the engine, (*a*) if there were no frictional resistance; (*b*) if there is frictional resistance of 8 lb. wt. per ton. Also find what tractive force is required to haul the same car over a level road at constant speed.

108. An automobile weighing with load 1500 lb. and running at 30 mi. per hr. bumps into a stone wall. The fender is crushed in 5 in. Find the average force of the blow.

109. A body slides down a smooth plane 326 cm. long, inclined at an angle of 30° with the horizontal. Calculate (*a*) the time of descent, and (*b*) the speed with which it reaches the bottom.

110. A spring balance fastened to the roof of a moving elevator car indicates 75 lb. as the weight of a 100 lb. mass. Find the acceleration of the motion of the car.

111. An elevator having a mass of 200 lb. changes its speed by 10 ft. per sec. in 3 sec. What force in pounds weight is required to thus accelerate its motion?

112. A man who is just strong enough to lift 150 lb. can lift 200 lb. from the floor of a descending elevator. What is the acceleration of the elevator?

113. An elevator of 800 lb. mass is pulled upward with a force of 1000 lb. wt. Find (*a*) the acceleration; (*b*) the distance the elevator will rise in 3 sec.

114. An elevator starts to descend with an acceleration of 3 m. per sec. in a sec. Find the thrust on the floor produced by a mass weighing 75 kg. Find the thrust when the elevator starts to ascend with the same acceleration.

115. A man in the car of an elevator holds a 10-lb. package in his hand. The elevator starts to rise with a uniform acceleration such that the car is raised 10 ft. in 2 sec. The cable then breaks, thus allowing the car to fall freely to the bottom of the shaft. Find the pressure of the package on the man's hand when the elevator was rising and also when it was falling.

116. A body is suspended by a string from the ceiling of a railway car. When the train starts the string is deflected 10° from the vertical. Find the acceleration of the train on starting.

117. An engine winds a cage weighing 3000 kg. up a shaft at a uniform speed of 10 m. per sec. Find the tension of the rope. Find also the tension if the cage rises with a uniform acceleration of 10 m. per sec. in a sec.

118. A mass of 162 g. hanging by a perfectly flexible cord over the edge of a smooth horizontal table, drags a mass of 973 g. along the table. Calculate (*a*) the acceleration of the system; (*b*) the tension in the cord.

119. A cord is hung over a pulley with masses of 10 lb. and 11 lb. on either end. Neglect the weight of the cord, and the mass and friction of the pulley, and calculate, (*a*) the acceleration of the system, (*b*) the tension of the cord.

120. To the ends of a rope passing over a pulley are attached two bodies of unequal masses, one of 300 g., the other of something more than 300 g. The acceleration imparted to the masses is observed to be 45 cm. per sec. per sec. Neglecting friction and the mass of rope and pulley, find the mass of the larger body and the tension in the cord.

121. A locomotive weighs 35 tons. The coefficient of friction between the wheels and rails is 0.18. Find the greatest pull the engine can exert in pulling itself and train. What is the total weight of itself and train which it can draw up a 1 per cent grade if the resistance to motion on the level is 10 lb. wt. per ton?

122. With what force must the wheels of a 10-ton locomotive push against the rails parallel to the ground in order to get up a speed of 15 mi. per hour in a distance of 500 ft.? What is the least coefficient of friction between the wheels and rails?

123. The mass of a certain electric street car is 8 tons. The coefficient of kinetic friction of the bearings is 0.01. While going with a speed of 15 mi. per hr., the current is turned off at the moment the car reaches the beginning of an up-grade of 5 per cent. To what height will the car ascend?

UNIFORMLY ACCELERATED CIRCULAR MOTION

124. A piece of wire of 10 g. mass is attached parallel to the axis of a cylinder of 15 cm. radius. Find the force necessary to hold the wire in place when the cylinder rotates 1400 times per minute.

125. On one end of a shaft is a flywheel of 1000 lb. mass and radius of gyration 6 ft., and on the other end is a pulley of 2 ft. diameter. Find the angular acceleration of the system when the two sides of a belt on the pulley are under a difference of tension of 100 lb. wt.

126. Find the horizontal thrust on the rails when a 20-ton engine runs at 50 mi. per hr. on a curve of 1 mi. radius.

127. A cord can just support a weight of 2 kg. What is the greatest length of it that can be used to whirl a mass of 500 g. in a horizontal circle at a rate of 2 revolutions per sec.?

128. A stone of mass 2 lb. is whirled at the end of a string 3 ft. long in a horizontal circle with a velocity of 30 ft. per sec. What is the tension in the string in pounds weight?

129. A large horizontal rotating platform is started from rest with gradually increasing speed. A boy is sitting on the platform, 3 ft. from the center. The coefficient of friction between the platform and the boy is 0.3. Find the angular speed at which the boy just begins to slip.

130. A flywheel is unbalanced by an amount equivalent to a mass of 20 lb. added at a point 3 ft. from the axis. If the flywheel revolves at the rate of 30 rev. per min., find the magnitude of the force due to the lack of balance tending to lift the flywheel from its bearings.

131. When a 20-ft. swing is vertical, the push of the occupant against the seat is 1.5 of his weight. Find the linear speed at that instant.

132. A pail of water is rotated in a vertical plane in a circle of 1 m. radius. Find the greatest period of revolution the pail can have without the water spilling.

133. A train of mass 500 tons is moving around a curve of radius 500 ft. with a speed of 30 mi. per hr. Find the resultant thrust on the track, and also find the angle of elevation of the track that will prevent any tendency of the train to leave the track or wrench the rails.

134. What is the proper superelevation of the outer rail of a track 4 ft. 8 in. wide at a curve of 2000 ft. radius for trains having a speed of 45 mi. per hr.?

135. A bicyclist goes around a half-mile circular track with a speed of 0.35 mi. per min. What is his inclination to the vertical?

136. A 1-ton automobile is moving with the speed of 60 mi. per hr. around a curve of 500 ft. radius. Find, (*a*) the centripetal force acting on the car; (*b*) the angle which the track should incline to the horizontal in order that there may be no tendency to skid.

137. The flywheel of a punch has a mass of 500 lb., and a radius of gyration of 18 in. Find what must be the initial angular velocity in order that when the punch is exerting a force of 40,000 lb. wt. through a distance of 0.5 in., the velocity shall not be reduced more than 75 per cent. of its initial value.

138. A body of 30 g. mass is constrained to move in a horizontal circle of 60 cm. radius by a central force of 3200 dynes. One sec. after passing the north point of the circle, the constraint suddenly ceases. Find the amount and direction of the immediately subsequent velocity.

ENERGY AND POWER

139. What dead weight will a pile support which sinks 1 in. when struck by a 50-lb. pile-driver hammer falling 25 ft.?

140. A runaway team pulling 100 lb. wt. develops 4 horse-power. Find the speed.

141. A horse hauling a cart at the rate of 4 mi. per hr. exerts a pull of 100 lb. wt. Find the horse-power furnished for hauling the cart.

142. A moving stairway 45° to the horizontal carries a load of 15,000 lb. wt. through a vertical distance of 40 ft. at a speed up the incline of 2 ft. per sec. Assuming that one-half the energy supplied is required to overcome friction, find the horse-power of the engine required to operate the stairway.

143. Find the horse-power of a tractor traveling at 3 mi. per hr. and exerting on a plow a force of 180 lb. wt.

144. A given tractor develops 8 horse-power-hours of useful work on 1 gal. of gasolene. A gang of plows making 3 furrows of a total width of one-fourth of a rod requires a draw-bar pull to 1650 lb. wt. Find the amount of gasolene required to plow a field of 80 acres. (An acre equals 160 sq. rods.)

145. A pile is to be driven into ground which resists penetration with a force equivalent to the weight of 15,000 lb. If the pile is struck by the 300-lb. hammer of a pile driver moving with a velocity of 30 ft. per sec., how far will it be driven at each blow?

146. What pull must a horse exert upon a wagon traveling 5 mi. per hr., in order to develop 1 horse-power?

147. A horse draws a wagon along a level road at a speed of 4 mi. per hr. The traces slant upward at an angle of 20° with the horizontal. The pull in the traces is 100 lb. wt. Find (*a*) the work done in 1 mi. (*b*) The horse-power.

148. A belt traveling with a speed of 50 ft. per sec. transmits 200 horse-power. Find the difference in the tensions in the loose and tight side of the belt.

149. The engines of a steamship develop 10,000 horse-power when the speed of the ship is 15 mi. per hr. Assuming that 25 per cent of the power is utilized in propulsion, find the forward thrust of the screw.

150. How many pounds of water can be pumped per minute from a mine 500 ft. deep by an engine expending 20 horse-power?

151. A boy turns a grindstone at the rate of 20 rev. per min. by exerting an average force of 10 lb. wt. perpendicular to a crank 1 ft. long. Find the horse-power exerted.

152. A boy turns the crank of a grindstone, rotating uniformly 15 times per minute. The crank is 40 cm. long, and his average force perpendicular to the crank is 3 kg. wt. How much work does he do per second? Express his rate in horse-power and watts.

153. How much work is done in pumping 1000 gal. of water out of a well 30 ft. deep in 5 min. if 1 gal. of water weighs 8.3 lb.? What horse-power would be required?

154. How many pounds of coal can be elevated 50 ft. in 1 hr. by a 10 horse-power engine, if the friction of the hoisting mechanism absorbs an amount of energy equal to that required to raise the coal?

155. A 10 horse-power water turbine transforms 55 per cent of the energy of a waterfall into useful work. If the height of the fall be 100 ft., find the mass of water that falls in each sec.

156. A steam pump fills a tank with water in 4 hrs. The capacity of the tank is 5000 gal., and the elevation of its center of gravity is 40 ft. If a gallon of water weighs 8.3 lb., what is the total work done and what is the horse-power of the pump?

157. An eight-oared crew makes 35 strokes per minute. The pull in each stroke is for a distance of 5 ft. and the average pull per man during each stroke is 70 lb. wt. Calculate the horse-power of the crew.

158. The cylinder of a steam engine has a diameter of 15 in.; the stroke is 3 ft.; the number of strokes is 77 per min.; the mean pressure of the steam is 40 lb. per sq. in. What is the horse-power of the engine?

159. A body of mass 300 lb. is raised in an elevator 70 ft. How much is its potential energy increased? What must be the horse-power of the motor which can raise the mass 70 ft. in 5 sec.?

160. Find the available power of a stream that flows at the rate of 1000 cu. ft. per sec. over a dam 15 ft. high.

161. The falls of Niagara are 160 ft. high. It is estimated that 700,000 tons of water pass over per minute. Calculate the equivalent horse-power, if all this energy could be utilized.

162. Steam is admitted to the cylinder of an engine in such a manner that the average pressure is 120 lb. per sq. in. The area of the piston is 54 sq. in. and the length of stroke is 12 in. How much work can be done during a single complete stroke, assuming that steam is admitted to both sides of the piston in succession? What is the horse-power of the engine when it is making 300 strokes per minute?

163. A 15-g. bullet moving with a velocity of 600 m. per sec. penetrates 32 cm. of wood. What is the average resistance to penetration?

164. A ball weighing 5 oz. and moving with a speed of 1000 ft. per sec. pierces a shield and moves on with a speed of 400 ft. per sec. Find the energy lost by piercing the shield.

165. Calculate the horse-power of a man who strikes 25 blows per min. on an anvil with a 14-lb. hammer, if the velocity of the hammer on striking the anvil is 32 ft. per sec.

166. A bicyclist along a level road maintains a speed of 10 mi. per hr., and works at the rate of one-tenth horse-power. What is the resistance to motion which he is obliged to overcome?

167. How much work must be done on a train of total mass 40 tons to give it a speed from rest of 30 mi. per hr.? What force must be applied to bring the train to rest in a distance of 500 ft.?

168. A sled and rider weighing 100 lb. reach the foot of a hill 64 ft. high with a speed of 50 ft. per sec. At the foot of the hill is a level icy road. The coefficient of kinetic friction between the sled and the level road is 0.03. Find (*a*) the amount of work done against the friction of the hill; (*b*) the distance the sled will move along the level road.

169. A mass moving horizontally with a speed of 100 ft. per sec. meets a smooth plane inclined 30° to the horizontal. How far up the plane will it move?

170. Calculate the horse-power of a locomotive which is moving at the rate of 20 mi. per hr. up an incline which rises 1 ft. in 100, the mass of the locomotive and load being 60 tons, and the frictional resistance 12 lb. wt. per ton.

171. Find the horse-power required of a locomotive to haul a train of 100 tons at 30 mi. per hr., the resistance amounting to 8 lb. wt. per ton: (*a*) on a level road; (*b*) up a 1 per cent grade; (*c*) up a 2 per cent grade.

172. A thin rope is wound around the flywheel of an engine of which the radius is 10 ft. When the speed of the flywheel is 300 rev. per min. the difference in tension at the two ends of the rope is 40 lb. wt. Calculate the horse-power of the engine.

173. Find the horse-power required to raise 1000 lb. of water per minute through a height of 20 ft., the water leaving the top of the pipe with a horizontal velocity of 16 ft. per sec.

174. Find the horse-power necessary to turn a 2-in. shaft 100 rev. per min., if it exerts on the bearings a force of 1500 lb. wt. and the coefficient of friction is 0.01.

175. Find the horse-power required to drive an automobile of 1500 lb. up a 5 per cent grade at the rate of 25 mi. per hr., the coefficient of friction being 0.1.

176. A cut is made on a 4-in. iron shaft making 10 rev. per min.; the pressure on the tool is 435 lb. wt. Find the horse-power expended at the tool.

177. A motor car with disconnected engine coasts down a 5 per cent grade with a constant speed. Calculate the resistance to motion in lb. wt. per ton of the car's weight.

178. What power must be developed by the engine of the car considered in the above question, assuming it to weigh 1 ton, in order to propel it with a speed of 20 mi. per hr. (*a*) along the level, (*b*) up the same slope?

179. A shaft transmits 100 horse-power and runs at a speed of 250 rev. per min. Calculate the torque exerted on the shaft.

180. Express the value 4.2 (10^7) ergs per sec. in terms of foot-pounds per min.

181. Two pulleys, each 3 ft. in diameter, are connected by a leather belt pulled so tight that the two parts of the belt not in contact with the pulleys may be regarded as straight and parallel. The linear speed of the belt is 2400 ft. per min. The tension of the tight side of the belt is 500 lb. wt. and the tension in the slack side of the belt is 200 lb. wt. (*a*) How many foot-pounds of work are being delivered every minute by the driven pulley to the machinery to which it is attached? (*b*) How many horse-power are being transmitted?

Fluid Pressure

182. If the area of a man's body is 1.50 sq. m., what is the total atmospheric force to which he is subject when the barometer stands at 753 mm.?

183. Find the force necessary to hold a cover against an opening of area 72 sq. in. in the bottom of a ship when the opening is 15 ft. below the surface of the sea. (1 cu. ft. of sea water weighs 64 lb.)

184. Find the force tending to crush in a door of 2 sq. ft. area in a submarine, when the boat is 100 ft. below the surface. (1 cu. ft. of sea water weighs 64 lb.)

185. A pail 20 cm. in diameter at the bottom, 30 cm. in diameter at the top, 22 cm. high, inside measurements, and weighing 1.5 kg. stands, full of water, upon a table. The capacity of the pail is 10.94 liters. Calculate (*a*) the total force, due to the water, on the bottom of the pail. (*b*) The total force of the pail and contents against the table.

186. The pressure in a city water main is 40 lb. per sq. in. The diameter of the plunger of an hydraulic elevator is 12 in. Loss by friction is one-third. How heavy a load can the elevator lift?

187. At what depth beneath the surface of a lake will the pressure be 200 lb. per sq. in, when the barometer stands at 29 in.?

188. What will be the difference in the readings of two barometers situated at the foot and at the top of a hill 30 m. high?

189. A rectangular scow 30 m. long and 5 m. wide weighs 25,000 kg.; find the depth of fresh water required to float it.

190. The gates of a canal lock have a width of 20 ft. The water-level on one side of the gates is 10 ft. higher than the level on the opposite side of the gate. Find the difference in the total force on the two sides of the gate.

191. A V-shaped open manometer contains kerosene of sp. gr. 0.7. Find the displacement of the liquid column produced by a difference of pressure of 1 mm. of mercury, (*a*) when the sides of the manometer are inclined 45° to the horizontal; (*b*) when inclined 30°.

192. What is the pressure in excess of the atmospheric pressure at a depth of 50 ft. below the surface of the sea?

193. Determine the available water pressure (in lb. wt. per sq. in.) in a laboratory which is supplied from a tank at a height of 40 ft.

194. The lever of a hydraulic press gives a mechanical advantage of 6; the sectional area of the smaller plunger is 0.5 sq. in. and that of the larger plunger 15 sq. in. A 56-lb. mass is hung from the handle. What weight will the larger plunger sustain?

195. If the diameters of the piston of a hydraulic press are in the ratio of 10 to 1, and a force of 14 lb. wt., applied to the handle produces a pressure of 2240 lb. wt. on the larger, what is the ratio of the arms of the lever used for working the piston?

196. The two pistons of a hydraulic press have diameters of a foot and an inch respectively. What is the pressure exerted by the larger piston when a weight of 56 lb. is placed on the smaller one? If the stroke of the smaller piston is 1.5 in., through what distance will the larger piston have moved after ten strokes?

197. What change in the atmospheric pressure is indicated by a fall of 1 in. in the height of a barometric column?

198. To what depth must a diving bell 150 cm. high be immersed in order that the water may rise 100 cm. within it? Barometric reading is 74 cm.

ARCHIMEDES' PRINCIPLE

199. A hollow sphere made of a material of density 7 g. per cc. will float completely submerged in water. How large is the cavity in proportion to the entire sphere?

200. A cubic foot of iron weighs 425 lb. when suspended in oil. What is the specific gravity of the oil? What would the iron weigh if suspended in water?

201. A diamond ring weighs 4 g. in air and 3.72 g. in water. Find the weight of the diamond if the specific gravity of the gold is 17.5 and that of diamond is 3.5.

202. A block of wood of volume 100 cc. and sp. gr. 0.75 floats in a certain liquid with two-thirds of the volume of the block beneath the surface of the liquid. What is the specific gravity of the liquid?

203. What is the greatest weight that a cubic yard of ice floating in sea water will support?

204. A block of wood having a cross-section of 5 cm. ×4 cm. and a height of 3 cm. floats in water immersed to a depth of 2.5 cm. What mass laid on top would be sufficient to cause complete immersion of the block?

205. A balloon displaces 10,000 cu. ft. of air. Assume that air weighs 0.08 lb. per cu. ft., that the gas weighs 0.05 lb. per cu. ft. and that the material of the bag weighs 85 lb. What lifting force will the balloon exert?

206. A balloon contains 1000 cu. m. of a gas whose density if 0.000092 g. per cc. Calculate the total weight which the balloon will lift.

207. What volume of lead attached to the underside of a cork float, of sp. gr. 0.52 and volume 100 cc., will pull the cork down until three-fourths of its volume are under water?

208. If a body has a mass of 150 g. and a volume of 20 cc. what weight will it lose if immersed in water? What is its specific gravity?

209. A mass of iron is placed in a vessel containing mercury. Determine the portion of the iron submerged.

210. A cubic foot of air weighs, at the sea level, 1.28 oz. Disregarding changes due to temperature, what would be the buoyant force of the air on an object having a volume of 10 cu. ft. at an altitude where the mercury in a barometer stands half as high as when on the ground?

211. An iron body weighing 275 g. floats in mercury with 0.556 of its volume immersed. Required the volume and density of the body.

212. A bar of aluminum weighs 54.8 g. in vacuum. What will be the loss of weight when it is weighed in water?

213. A hollow stopper made of glass of density 2.6 g. per cc. weighs 23.4 g. in air and 3.90 g. in water. What is the volume of the internal cavity?

214. What is the total volume of an iceberg which floats with 700 cu. yds. exposed?

215. A solid which weighs 120 g. in air is found to weigh 90 g. in water and 78 g. in a strong solution of zinc sulphate. What is the specific gravity of the solution?

216. Using brass standard masses, a body which weighs 1030 g. in air weighs 905 g. in water at 4° C. What would it weigh in vacuum?

217. A steamer in going from salt into fresh water is observed to sink 2 in., but after burning 50 tons of coal to rise 1 in. Assuming the density of salt water to be 64 lb. per cu. ft. and of fresh water 62.5 lb. per cu. ft. find the steamer's displacement before she entered the fresh water.

218. A mass of 28.1 g. of specific gravity 5.59 and another mass of 35.8 g.,weigh the same when immersed in water. What is the density of the second body?

219. What portion of an iceberg is above water?

220. What is the net lifting ability of a balloon weighing 250 kg. that contains 400 cu. m. of hydrogen? The weight of a cubic meter of air is 1200 g.; of hydrogen is 90 g.

221. The volume of a balloon filled with coal gas is 1,000,000 liters, and its mass, including the car, but not the gas, is 500 kg. If the density of the air is 1.28 and that of the coal gas is 0.52, both being in grams per liter, find what additional weight the balloon can sustain in the air.

222. An ornament made of gold and silver weighs 76.8 g. and has a specific gravity 18.0. Assuming the volume of the alloy is equal to the combined volume of the component parts, calculate the masses of gold and of silver in the body.

223. A boat weighs 1200 lb. and floats with one-third of its volume immersed. A lead keel is to be attached outside so that it will float with two-thirds of its volume immersed. Calculate the mass of the lead required. Would the same mass of lead be equally effective if placed inside the boat?

224. Calculate the amount of work done in forcing a block of wood of volume 8 (10^4) cc. and specific gravity of 0.8 to the bottom of a pond of water 5 m. deep.

225. A mass of 20 kg. is at the bottom of a lake 10 m. deep. The specific gravity of the body is 5.0. Calculate the amount of work required to lift it to the surface, assuming 5 per cent of the force exerted to be [illegible]d to overcome friction.

PROPERTIES OF MATTER

226. A round rod, 2 in. in diameter and 20 ft. long, is stretched ⅛ in. by a force of 10,000 lb. wt. Calculate the value of (*a*) the longitudinal stress, (*b*) the longitudinal strain.

227. A piece of brass wire 0.1066 cm. in diameter and 27.1 cm. long is stretched 0.133 cm. by the addition of a load of 0.454 kg. wt. Calculate the stress and the strain and find Young's modulus for the wire.

228. A load of 10 kg. wt. is to be supported by a wire 2 m. long. Young's modulus for the wire is 8 (10^{11}) dynes per sq. cm. What must be the cross-sectional area of the wire for a maximum stretch of 1 mm. on application of the load?

229. Taking Young's modulus for iron as 19 (10^{11}) dynes per sq. cm., find the increase in length of an iron wire 3 m. long and 1 mm. diameter, when stretched by a force of 5 kg. wt.

230. What is the value of the bulk modulus of water when 2000 cu. in. of water are reduced to 1880 cu. in. by a pressure of 3000 lb. per sq. in.?

231. Find the mean density of a mixture of 8 parts by volume of a substance of density 7, with 19 parts of a substance of density 3.

232. Find the mean density of a mixture of 8 parts by weight of a substance of density 7, with 19 parts of a substance of density 3.

233. Two parts by volume of a liquid of specific gravity 0.8 are mixed with 7 of water, and the mixture shrinks in the ratio of 21 to 20. Find the specific gravity of the mixture.

234. Into an air-tight water tank holding 1000 gal., water is pumped until the tank is three-fourths full of water. Find the pressure in lb. wt. per sq. in. due to the compressed air.

235. A diver in a flexible diving suit and therefore subjected to the full pressure of the water, descends to a depth of 100 ft. in sea water. (*a*) To what pressure greater than that of the atmosphere is the diver subjected? (*b*) If a diver at this depth and breathing air at the resulting pressure requires 250 cu. in. of air per min. how much air per min. must the pumps at the surface which supply him take in? It is assumed that the temperature of the air when it reaches the diver is the same as that of the air taken in by the pumps.

236. In a cylinder whose piston has an area of 20 sq. in., 600 cu. in. of air is under a pressure of 30 lb. per sq. in. The pressure is increased to 120 lb. per sq. in. If the average pressure during the compression can be taken to be 75 lb. per sq. in., how much work must be done?

237. Water is pumped into the bottom of an air-tight tank holding

1000 gal., compressing the air above it until a pressure of 50 lb. per sq. in. is reached. How much water does it then contain? If 200 gal. of water are drawn off, what will the pressure then be?

238. A pneumatic tire is blown up to a pressure of 75 lb. per sq. in. In the pump employed the piston area is 3 sq. in. and its travel is 15 in. At what point in the piston travel will the tire valve open on the last stroke of the pump? What force must be exerted on the piston at that moment?

239. A piston is situated in the middle of a closed cylinder 10 in. long, and there are equal masses of air in the two halves of the cylinder. The piston is pushed until it is within 1 in. of one of the ends. Compare the pressures on the two sides.

240. In rising from the bottom of a lake to its surface, the volume of an air-bubble becomes 10 times as great. If the height of the barometer is 30 in., what is the depth of the lake?

241. A glass tube used for sounding is 38.1 cm. long and is open at the lower end. The inside is coated with a soluble pigment, and the tube is lowered to the bottom of the sea. On raising the tube to the surface, it is found that the water has entered to a depth of 23.6 cm. Find the depth of the sea. Barometer reading is 74 cm.

242. The illuminating power of a gas flame is proportional to the mass of gas burned per hour. A certain gas works supplies at a pressure above that of the atmosphere equal to the pressure of a column of water 3 in. high. When the barometer stands at 30 in. an 18-candle-power gas jet burns 5 cu. ft. per hr. If the gas costs $1.00 per 1000 cu. ft., what will be the cost of an 18-candle-power light for 1 hr. when the barometer is at 28 in.? The temperature of the gas is the same in both cases.

243. At a certain temperature the density of oxygen is 0.00143 g. per cc. when under a pressure of 1 atmosphere. What will be the pressure in a 100 liter tank containing 715 g. of oxygen at the same temperature?

244. A certain balloon bag has a volume of 500 cu. m. when filled with hydrogen gas at the earth's surface where the barometer reading is 76 cm. Assuming that the bag offers no resistance to expansion, (*a*), what will be the volume of the bag after one-sixth of the gas has been let out, and the bag is at a height where the barometer reads 50 cm. but the temperature is the same as at the earth's surface? (*b*) If the weight of the material of the bag is 100 kg., how many additional kg. will the bag lift at each of the two places?

(The densities of air and hydrogen at 76 cm. pressure and at the temperature under consideration are respectively 1.29 and 0.090 kg. per cu. m.)

Simple Harmonic Motion

245. A tooth in the blade of a reaper describes simple harmonic motion of 1.5 in. amplitude in a period of 0.2 sec. Find its speed when at a point 0.5 in. from the center of its path.

246. A point moving with simple harmonic motion has a maximum speed of 10 ft. per sec. and amplitude of 6 ft. Find the time of a complete vibration.

247. The drive wheels of a locomotive whose piston has a stroke of 2 ft. make 185 rev. per min. Assuming that the piston moves with simple harmonic motion, find the speed of the piston relative to the cylinder-head, when at the center of its stroke.

248. A block of iron of mass 100 lb. is caused to vibrate with simple harmonic motion by means of a spring. If the amplitude of vibration is 12 in. and the time of a complete vibration is 0.8 sec., find the maximum kinetic energy of the block.

249. What is the value of g, at a place where the period of a simple pendulum 97.31 cm. long is 1.975 sec.?

250. A pendulum loses 20 sec. per day, where g is 980.3 cm. per sec. per sec. Find its length.

251. A certain pendulum has a period of 2 sec. where g is 982 cm. per sec. per sec. The same pendulum is swung at another station and has a period of 2.003 sec. Find g at the second station.

Sound

252. A certain tuning fork makes 256 vibrations per sec. Taking 1128 ft. per sec. as the speed of sound in air at 68° F., find the number of vibrations which the fork must make before the sound is heard at a distance of 200 ft.

253. A tuning fork makes 256 vibrations per sec. Find the wavelength of the sound that it produces, the velocity of sound being 1135 ft. per sec.

254. A wood chopper makes 20 strokes per min. If the sound of each stroke reaches an observer as the axe makes the next stroke when the temperature of the air is 20° C., what is the distance of the chopper from the observer?

255. A blow struck upon a steel cable was heard through the cable in 0.2 sec. and through the air in 3 sec. The temperature was 0° C. (*a*) How far from the observer was the blow struck? (*b*) What was the speed of sound in the cable?

256. A musical note of 250 vibrations per sec. travels in air with a velocity of 300 m. per sec. What is the length of the sound wave?

257. The vertical walls of a canyon are parallel and 1 mi. apart. A man in the canyon fires a gun and hears the first two echoes, one from each wall, 4 sec. apart. Where is he? (Velocity of sound = 1100 ft. per sec.)

258. A gun is fired and 5 sec. later the gunner hears the echo. If the velocity of sound is 1100 ft. per sec., how far off is the reflecting surface?

259. At a lightship anchored on a reef a bell is struck under water at the same time that a whistle is blown above water. The captain of a vessel receives the underwater sound signal 5 sec. before the sound of the whistle. How far from the reef is he? (The velocity of sound is 1100 ft. per sec. in the air and 4600 ft. per sec. in the water.)

260. A string 80 cm. long vibrates 320 times per sec. when the tension is 40 kilos. (*a*) What will be the frequency when the tension is made 160 kilos? (*b*) If when under the new tension the string is lengthened so as to keep the pitch 320 per sec., what will be the new length?

261. A wire stretched between two rigid supports vibrates 500 times a sec. Another wire of the same material is stretched between supports one-half as far apart as in the first case. If the diameter of the second wire is one-half that of the first, and the tension is one-half that of the first, what will be its frequency?

262. An observer sets his watch by the report of a signal gun 1 mi. away. Find the allowance that he should make on account of the distance of the gun, the temperature of the air being 20° C.

263. How far off is a storm when there is an interval of 10 sec. between a flash of lightning and the thunder, the temperature being 16° C.?

264. Calculate the wave-length in air at 15° C. of the sound produced by a tuning fork which makes 256 vibrations per sec.

265. A tuning fork is held over a resonance tube and resonance occurs when the surface of the water in the tube is 10 cm. below the fork. It next occurs when the water is 26 cm. below the fork. Taking the velocity of sound to be 345 m. per sec., at room temperature, calculate the frequency of the fork.

266. Find the frequency of a tuning fork which produces resonance in a column of air 60 cm. long at 0° C.

267. An open organ pipe is 60 cm. long. If the velocity of sound is taken to be 34,200 cm. per sec., what is the frequency of the tone?

266. Assuming the velocity of sound to be 1120 ft. per sec., find the rate of vibration of an open pipe 2 ft. long.

269. Calculate the number of vibrations in the fundamental note emitted at 13° C. by a closed organ pipe 1 m. long.

270. How many vibrations per sec. in the fundamental note emitted by a closed organ pipe 1 m. long?

271. Find the vibration frequency at 0° C. of the fundamental tone, (*a*) of a 4-ft. open pipe; (*b*) of a 4-ft. closed pipe.

272. Find the vibration frequency at 20° C. of the fundamental tone, (*a*) of an 8-ft. open pipe; (*b*) of an 8-ft. closed pipe.

273. How many beats per sec. will be produced by sounding simultaneously two open organ pipes of lengths 20 in. and 21 in. respectively, (*a*) when the temperature of the air is 0° C., (*b*) when the temperature is 22° C.?

TEMPERATURE AND QUANTITY OF HEAT

274. Under standard atmospheric conditions the mercury level in the open manometer tube of a constant volume hydrogen thermometer is 15 cm. higher than the index when the bulb is in melting ice. It is 48.3 cm. higher when the bulb is in boiling water. Find the centigrade temperatures corresponding to differences in level of 21 cm. and 28 cm. assuming that the temperature of the mercury remains constant.

275. The temperature of water at the maximum density is 4° centigrade. What is the corresponding temperature on the Fahrenheit scale?

276. At what temperature will a centigrade thermometer give the same reading as a Fahrenheit thermometer?

277. At what temperature does a Fahrenheit thermometer read twice as much as a centigrade thermometer?

278. Express on the Fahrenheit scale the following boiling points: Alcohol, 78° C.; Mercury, 357° C.; Ether, 35° C.; Sulphuric acid, 338° C.

279. Express on the centigrade scale the following melting points to the nearest degree: Lead, 630° F.; Brass, 1869° F.; Zinc, 773° F.; Copper, 1906° F.; Silver, 873° F.; Iron, 2786° F.

280. What amount of heat must be given to an iron armor plate 2 m. long, 1 m. broad, and 20 cm. thick, in order to heat it from 10° to 140°?

281. Compare the thermal capacities of equal volumes of water and mercury.

282. A copper kettle weighing 2 lb. and containing 6 lb. of cold water is placed on a fire. Find in what proportion the heat absorbed from the fire is shared between the kettle and its contents.

283. A copper tea-kettle weighing 850 g. contains 2000 g. of water at temperature 20° C. If, of the heat supplied by a gas flame, 16,000 cal.

per min. are utilized in heating the kettle and water, how long a time would be required to raise the temperature of the water to 100° C.?

284. In order to determine, approximately, the temperature of a furnace, a platinum ball weighing 100 g. was placed in the furnace and after a time removed and dropped into 400 g. of water at 0° C. The temperature of water rose to 20° C. Find the temperature of the furnace. (The specific heat of platinum for this range of temperature is 0.035.)

285. If a silver spoon weighing 30 g. and at a temperature of 15° C. be put into a cup weighing 80 g. which contains 150 g. of coffee at 80° C., how much will the coffee be cooled? (Specific heat of the coffee is 1.0. Specific heat of the cup is 0.20.)

286. If the specific heat of air at constant pressure is 0.237, and if a liter of air weighs 1.293 g., how many liters of air would be raised 1° in temperature if all the heat given out by a liter of water in cooling through 1° were imparted to the air?

287. C_1 and C_2 are two equal brass calorimeters each having a mass of 40 g. and containing, respectively, 101.4 g. of water and 86.4 g. of turpentine. Both receive equal quantities of heat from internal current-bearing spirals. The temperature of C_1 and its contents is raised through 4.35°; C_2 and its contents is raised through 11.7° C. What is the specific heat of the turpentine?

288. If you had at your command a supply of boiling water and of tap water at 10°, what volume of each would you take in order to prepare a bath containing 20 gal. of water at 35°?

289. Hot air at 650° is used for superheating steam which is originally at 100°. The air and steam are kept at constant pressure during the operation and they are introduced into the superheater in the proportion of 2 lb. of air to 7 lb. of steam. If the air is allowed to cool to 400°, to what temperature will the steam be raised?

290. A coil of copper wire weighing 45.1 g. was dropped into a calorimeter containing 52.5 g. of water at 10°. The copper before immersion was at 99.6° and the common temperature of copper and water after immersion was 16.7°. Find the specific heat of the copper wire.

291. A piece of silver of specific heat 0.0568 was heated to 102.2° and was then dropped into 75.3 g. of turpentine at 10.98°. The silver weighed 10.205 g., the water equivalent of calorimeter, agitator, and thermometer was 2.91 g., and the final temperature was 12.47°. What was the specific heat of the turpentine?

292. A copper calorimeter contains 300 g. of water at 10°. To this was added 200 g. of water at 80°, giving a resultant temperature of 35°. What is the water equivalent of the calorimeter?

293. Two hundred grams of zinc are heated to the temperature of 99° C. and plunged into 200 g. of water contained in an iron calorimeter at temperature 14°. The water equivalent of the calorimeter is 14 g. Calculate the temperature to which the water rises.

294. How much ice per day of 10 hrs. would be required to cool 20,000 lb. of water per hour from 86° F. to 40° F., supposing there to be no loss in the cooling apparatus?

295. A glass tumbler weighing 150 g. contains 250 g. of water at 20° C. How much ice at temperature 0° C. should be introduced to lower the temperature of the tumbler of water to 10° C., assuming that all the ice is melted?

296. When heat was supplied at a constant rate to a certain block of tin it was found that the temperature rose 2° C. each sec. After the melting point was reached, the temperature remained constant for 130 sec., while all of the tin was being melted. Find heat of fusion of tin.

297. If the heat equivalent of vaporization of ammonia is 341 cal. per g., how much ammonia must be evaporated in an artificial ice plant to make 1000 kg. of ice at 0° C. out of water originally at 10° C.? (Assume that all the heat from the water goes to change the state of the ammonia.)

298. The heat of fusion of zinc is about 28.1 cal. per gram. Find its value in B.t.u. per lb.

299. A piece of copper was heated to 100° and then suddenly dropped into a hole in a piece of ice. The mass of the copper was 500 g. and the mass of ice melted was 58 g. What value does this give for the heat equivalent of fusion of ice?

300. A piece of iron weighing 16 g., is dropped at a temperature of 112.5° C. into a cavity in a block of ice, of which it melts 2.5 g. Find the specific heat of the iron.

301. A piece of ice which has been for some time out of doors in a temperature of −20° is dropped into a calorimeter of water equivalent 5 g. containing 100 g. of water at 35°. The mass of ice is 30 g., and the final temperature that would be reached if there were no radiation is determined to be 7.22°. Find the specific heat of ice.

302. 80 g. of steam at 100° C. are turned into 536 g. of ice at 0° C. What will be the resulting temperature?

303. A porous water jar of water equivalent 500 g. contains 4000 g. of water at the temperature of the surrounding air, which is 30° C. If the heat equivalent of vaporization of water at this temperature is 580 cal.

per g., how many grams of water must evaporate in order to reduce the temperature of the remainder 5°?

304. The last Peary expedition to the North Pole was equipped with a stove that would change 4000 g. of ice, originally at −40° C., into water at 100° C., in 10 min., by burning 140 g. of alcohol. If the heat of combustion of alcohol is 7400 cal. per g., what fraction of the heat produced did the stove utilize?

305. Neglecting loss of heat by radiation, find how much steam at 100° C. is required to raise the temperature of an iron radiator weighing 60 kg. from 10° C. to 100° C.

306. A kettle contains 2 kg. of water at 40° C. How much heat must be supplied in order to boil the water away?

307. A calorimeter contains 316 g. of water at 40° C. Steam at 100° is passed into the water until the mass of water becomes 336 g. What is the temperature of the water?

308. Find the numerical value of the heat of vaporization of water in terms (*a*) of cal. per lb., (*b*) in terms of B.t.u. per lb.

309. How many grams of steam at 100° must be passed into 200 g. of ice-cold water in order to raise it to the boiling point? What will happen if more steam than this is passed in?

310. A mass of 100 g. of copper at 20° is suddenly enveloped in steam at 100°. Find the amount of steam that will condense on the copper.

311. How many lb. of steam at 100° will just melt 50 lb. of ice at 0°?

312. 10 g. of steam at 120° were run into a vessel containing 10 g. of ice at −50°. What was the resulting mass and temperature of the contents of the vessel?

313. The pool in the gymnasium is 9.15 m. by 18.3 m. and the average depth of the water is 1.75 m. During the summer the water was frequently at a temperature of about 17° C. If a rise of 5° C. would have made it comfortable and if this rise were to be produced by condensing steam in the water, how many kg. of steam would be required?

314. The melting point of a certain kind of iron is 1500° C., and the heat equivalent of fusion is 28 cal. per g. Assuming that no heat is lost, find the mass of coal of thermal value 12,000 B.t.u. per lb. that would be necessary to melt 1 lb. of iron starting at 20° C.

315. How many B.t.u. are required to raise 1000 lb. of water from 40° F. and vaporize it at 328° F.? The heat equivalent of vaporization of water at 328° F. is 883 B.t.u. per lb.

316. If coal of heat value 13,500 B.t.u. per lb. can be bought at $8 per ton, what price could one afford to pay per cord for seasoned ash wood of heat value 8480 B.t.u. per lb.? Assume that the density of ash wood is 47 lb. per cu. ft. and that when the wood is piled, the wood occupies 60 per cent of the volume of the pile.

317. If the prices of fuels were proportional to their heat value, and if Pocahontas coal of heat value 14,500 B.t.u. per lb. costs $10 per ton, what would be the price per ton of (*a*) coal of 11,000 B.t.u. per lb.; (*b*) fuel oil of 19,000 B.t.u. per lb.; (*c*) beech wood of 8,600 B.t.u. per lb. (*d*) Also find the price per 1000 cu. ft. of gas of heat value 600 B.t.u. per cu. ft.

Thermal Expansion

318. A wheel is 3 m. in circumference. An iron tire measures 2.993 m. around its inner face. How much must the temperature of the tire be raised in order that it may just slip on the wheel?

319. The Eiffel Tower in Paris is constructed of steel and is about 300 m. high. How much higher is the tower when at 30° C. than when at 5° C.?

320. A steel meter-scale was carefully measured at 10° and its length was found to be 99.981 cm. At 40° its length was found to be 100.015 cm. What was the coefficient of expansion of the steel, and at what temperature was the scale exactly 1 m. long?

321. Assuming the highest summer temperature to be 40°, and the lowest winter temperature −20°, what allowance should be made for expansion of a 1700-ft. steel bridge span?

322. A steam pipe, intended to convey steam at 110°, is formed of iron piping in lengths of 15 ft. Assuming that the temperature of the pipe when it is not conveying steam is 12°, find how much play must be allowed for each joint.

323. The lower end of a vertical steam pipe 50 ft. long is supported rigidly by a hanger attached to a basement ceiling. When the pipe is at 40° F. a steam radiator attached rigidly to the upper end of the pipe rests on the attic floor. Find the distance the radiator is lifted off the floor when the vertical steam pipe is at 220° F.

324. Show that the length of the metal bars of a compensation pendulum should be inversely proportional to the coefficients of expansion of the metals. If the total length of the iron bars is 87 cm., what should be that of the zinc bars?

325. An iron ball, of 5.01 cm. diameter at 0°, rests upon a copper ring, the internal diameter of which is 5.00 cm. at the same temperature. To what temperature must both be heated in order that the ball may just pass through the ring?

326. A copper lightning rod measures 50 ft. in length when the temperature is 0° C. Calculate its length when the temperature is 27° C.

327. Calculate the increase in length of an iron girder 100 ft. long between 30° F. and 68° F.

328. A brass yard measure is correct at 0°, and another is correct at 20°. What is the difference in length when both are at 20°?

329. A platinum wire and a strip of zinc are both measured at 0°, and their lengths are found to be 251 cm. and 250 cm., respectively. At what temperature will their lengths be equal, and what will be their common length at this temperature?

330. Measurements are made at 25° upon a brass tube by a steel meter scale, correct at 0°. The result is 6.425 m. Find the length that would have been obtained if the tube and scale had been at 0°.

331. The height of the barometer appears to be 76.40 cm., according to a brass scale which is correct at 0°. If the temperature at the time of reading is 20°, what is the actual height of the barometer column?

332. A clock which keeps correct time at 25° has a seconds pendulum rod made of brass. How many seconds a day will it gain if the temperature falls to the freezing point?

333. Assuming that the density of silver at 0° is 10.5 g. per cc., and its coefficient of cubical expansion is 0.000057, find its density at 150°.

334. A dealer buys a tank car containing 100,000 gal. of gasolene at 60° F. When received, the temperature was 0° F. Assuming the mean coefficient of expansion of gasolene to be 0.0006 per degree F., find the diminution of volume due to the change of temperature.

335. The height of the barometer is found to be 77.25 cm., the temperature of the air being 25°. What would be the corresponding barometric height reduced to 0°—i.e., what would be the height of the barometric column if the mercury were at 0°?

336. A specific gravity bottle contains just 687 g. of mercury at 70°. What is the internal volume of the bottle at this temperature?

337. Find the volume at 100° of a glass flask that at 0° has a volume of 100 cc.

338. Calculate the increase in area produced by a rise of 40° in a plate of sheet iron which is 5 ft. long and 3 ft. broad at 0°.

339. If the pressure of a gas is 8726 dynes per sq. cm. when its volume is 7375 cc., what will be its pressure at the same temperature if the volume is diminished to 1586 cc.?

340. What would be the volume at 0° and constant pressure of a mass of gas which at 78° occupies a volume of 9 liters?

341. At what temperature will the volume of a given mass of gas be exactly twice what it is at 17°, the pressure remaining constant?

342. A caisson is lowered into a lake until the surface of the water is 690 cm. below the surface of the lake. What does 1 cc. of air within the caisson weigh if, at the surface of the lake where the barometric reading is 76 cm., 1 cc. of air weighs 0.0012 g.?

343. A liter of air at 0° C. and under a pressure of 76 cm. of mercury weighs 1.29 g. What will be the mass of the same volume at the same temperature, at a pressure of 10 atmospheres?

344. The volume of a body of gas at 27° C. is 100 cc. If the pressure on the gas is doubled, to what temperature must it be heated in order to maintain the volume constant?

345. A bottle, previously open to the air, is closed when the barometer reads 75 cm. and the thermometer reads 20° C. If this bottle is heated to 40° C. what will be the new pressure in the bottle?

346. A 50-gal. tank containing air at atmospheric pressure (15 lb. per sq. in.) is connected by a pipe to the city water mains, the pressure in which is 60 lb. per sq. in. Assuming that the air remains at constant temperature, how many gallons of water will flow into the tank?

347. If 800 cc. of air at normal temperature and pressure is to be compressed to 650 cc at constant temperature, what pressure would be required? If this pressure is to be applied by means of a piston having an area of 2.5 sq. cm., what force would have to be exerted on it?

348. A sounding tube open at the lower end was lowered into the sea until the water rose to a height of two-thirds of the length of the tube. If the barometric pressure was 29 in. of mercury, how far down into the water was the sounding tube lowered?

349. A man requires the same mass of air at each breath on top of a mountain that he requires when at the bottom. On top of a certain mountain the pressure is 40 cm. of mercury and the temperature is 3° C., whereas at the bottom the pressure is 70 cm. of mercury and the temperature is 17° C. Find the change in the volume of each inhalation.

350. An automobile tire is pumped up under a pressure of 80 lb. per sq. in. when the air is at 17° C. The car is driven until the temperature of the air in the tube is 57° C. Assuming that the tube does not stretch, what will the pressure become?

351. Six liters of air at 10° are enclosed in the cylinder of an air engine, the cross-section of which is 200 sq. cm. The piston moves through a distance of 5 cm. toward the other end of the cylinder. What elevation of temperature is required to keep the pressure constant?

352. If the pressure remains constant calculate the volume of gas at 572° C. which occupies 2579 cc. at 198°.

353. A certain volume of gas at 13° C. increases to 3 liters at 20°. Calculate the original volume.

354. A cylinder open at one end and closed at the other is fitted with a piston which is loaded with 2 kg. weight. If the volume of the gas enclosed is 100 cc. and the area of the piston 10 sq. cm., find the increase in temperature necessary to raise the piston through 5 cm. The temperature of the gas is 18° C., and the atmospheric pressure is 760 mm.

355. When the barometer reads 76 cm. and the temperature is 0° C., the volume of a certain mass of gas is 50 cc. When the temperature is increased to 100°, the volume becomes 68.3 cc., the pressure remaining the same. Find the coefficient of cubical expansion of the gas.

356. A liter flask contains 1.293 g. of air at 0°. How much air will a liter flask contain at 100° at the same pressure?

357. On heating a certain quantity of mercuric oxide it was found to give off 380 cc. of oxygen. The temperature was 23° and the pressure 74 cm. of mercury. What would be the volume of the oxygen if it were at normal pressure and temperature?

358. A quantity of air at the atmospheric pressure and at a temperature of 7° was compressed until its volume was reduced to one-seventh, during which process the temperature rose to 20°. Find the pressure at the end of the operation.

359. The cubical content of a certain room is 750 cu. m. Calculate the mass of air contained in it at 17° and 77 cm. pressure.

360. Compare the densities of the air at the bottom and at the top of a mine shaft, when the temperatures and barometric pressures are respectively 20° and 31 in., and 5° and 30 in.

361. A cylindrical test-tube 10 in. in length and containing air at 0° is inverted over a mercury bath and forced downward until its closed upper end is level with the surface of the mercury in the bath, the barometric height at the time being 30 in. To what temperature must the bath be raised in order that the air may fill the test-tube?

362. At the sea-level the barometer stands at 750 mm., and the temperature is 7°, while on top of a mountain the barometer stands at 400 mm. and the temperature is −13°. Compare the masses of a cubic meter of air at the two places.

363. If the volume of a gas at 0° C., is 2560 cc. under a pressure of 2.14 million dynes per sq. cm., what will be its volume at 95° under a pressure of 1.013 million dynes per sq. cm.?

364. What will be the mass of a cubic meter of air at 50° under a pressure of 50 cm. of mercury?

365. The pressure on a given mass of gas is doubled and at the same time the temperature is raised from 0° to 91°. How is the volume changed?

366. Compare the masses of a cu. m. of air at the bottom of a lake 300 m. deep, and at the surface. Barometric pressure is 746 mm., and the temperatures are 19° and 28° respectively.

367. Ten million cu. ft. of gas are supplied to a town per week under a pressure of 3 in. of water over the atmospheric pressure at $1.00 per 1000 cu. ft. When the barometer is 31 in., the company neither makes nor loses. Find the profit in a week when the average height of the barometer is 29 in.

368. A town is supplied with gas at a pressure of 3 in. of water above the atmospheric pressure. When the barometric pressure is 31 in. and the gas temperature is 50° F., the town uses 10,000,000 cu. ft. per week and the company neither gains nor loses. Assuming that the same volume of gas is used during a week when the average barometric pressure is 30 in., and the average gas temperature is 70° F. find the profit or loss of the company for the week.

THERMAL CONDUCTION

369. A kettle whose base is 400 sq. cm. in area contains 500 g. of ice at 0° C. If the kettle is set on a stove whose temperature is maintained at 200° C., how long will it take for all the ice to be melted? The kettle bottom is 8 mm. thick and made of metal whose conductivity is .02.

370. An iron boiler is 1 cm. thick, and has a heating surface of 2 sq. m. The water in the boiler is at 100°, and the heating surface is kept at 280°. Find how much water can be evaporated each hour.

371. How much water is evaporated per hour per sq. ft. of boiler plate if the plate is 0.5 in. thick, the difference between the temperature on the two sides is 5° F., and the temperature of the water in the boiler is 311° F.? (The heat equivalent of vaporization of water at 311° F. is 896 B.t.u. per lb.)

372. A pond 400 sq. m. in area is covered with a sheet of ice 5 cm. thick. If the temperature of the air is $-5°$, how much heat will pass in an hour from the water through the ice? The thermal conductivity of ice is 0.00586.

373. How many calories of heat will be conducted in an hour through an iron bar 2 sq. cm. in cross-section and 4 cm. long, its two ends being kept at the respective temperatures of 100° C. and 178° C.?

374. Calculate how much heat is conducted in half an hour through an iron plate 2 cm. thick and 1000 sq. cm. in area, the temperature of the two sides being kept at 0° C. and 20° C.

375. The walls of a cottage are 3 dm. thick, and are built of materials having a thermal conductivity of 0.0035. The temperature inside the cottage is kept at 15°, while the outside temperature is 5°. The area of the walls is 1000 sq. m. Find how much heat is lost by conduction each hour, and what is the minimum quantity of coal of calorific power 8400 cal. per g. that must be burned in order to keep the temperature constant.

376. A frame house has an outside wall area of 3000 sq. ft. of which the windows occupy 20 per cent. Assuming that 1 sq. ft. of glass loses as much heat as 4 sq. ft. of wall, and that when the outside is at 0° F. and the inside is at 70° F., 1 sq. ft. of glass loses 85 B.t.u. per hour, find the number of B.t.u. per hour required to maintain the inside temperature at 70° F. (*b*) Assuming that coal of heat value 13,000 B.t.u. per lb. costs $9.00 per ton, and that the furnace utilizes one-half of the heat value of the coal, find the cost of heating the above house for 1000 hrs.

377. From the hottest part of a steam boiler 50 lb. of water are being evaporated per hour at a temperature of 347° F. from 1 sq. ft. of heating surface. At this temperature, the heat equivalent of vaporization is 872 B.t.u. per lb. If the iron has a thickness of 0.25 in. and a thermal conductivity of 0.0009 B.t.u. per sec., per sq. in., per ° F., difference in temperature, find the difference of temperature between the two sides of the boiler plate.

378. An iron pipe 1.1 in. outside diameter and 0.2 in. thick is surrounded by steam at 330.2° F. When water at a mean temperature of 69° F. was flowing through the pipe at a speed of 17.13 ft. per sec., the outer surface of the pipe was found to have a temperature of 220° F., and the heat conducted per min. through 1 sq. ft. area was found to be 3995 B.t.u. Assuming the conductivity of iron to be 0.0009 B.t.u. per sec., per sq. in., per inch thickness, for a temperature difference of 1° F., find the temperature of the inside wall of the pipe.

379. When water flowed through the pipe of the above problem at a speed of 2.30 ft. per sec., the temperature of the outer wall was found to be 267.1° F., and the heat conducted per sq. ft. area, per min., was found to be 2370 B.t.u. Find the temperature of the inside wall of the pipe.

380. How much heat would be lost per square meter per minute by a man clothed in a fabric 0.3 cm. thick, having a conductivity 1.22 (10^{-4}), if the temperature of the air be 5° C., and the temperature of the body be 30° C.?

381. A concave mirror 1 sq. ft. in area is placed in the sunshine and the sun's rays are by it brought to a focus upon a copper calorimeter containing water. The mass of the copper is 1 oz. and it contains 2 oz. of water. The temperature of the water is found to rise 30° F. in 5 min. Find the amount of heat received by the earth per square yard per minute, and find the equivalent horse-power.

THERMODYNAMICS

382. How much will a mass of copper be heated by striking a hard surface after a fall of 400 ft., if half of the energy is used in heating the copper?

383. The water at Niagara Falls drops 160 ft. and is heated 0.12° C. If lead fell the same distance, how much would its temperature be raised?

384. A small leaden bullet shot horizontally against an iron target is just melted by the impact. Find the velocity with which it strikes the target, assuming that the temperature of the bullet before striking was 30° C., that the melting point of lead is 330° C. and that no heat is lost to the target.

385. How much heat is developed in drawing an iron nail that requires a force of $2(10^6)$ dynes through a distance of 6 cm.? If the nail has a mass of 3 g., and two-thirds of the heat developed goes into the nail, find the temperature rise.

386. What is the horse-power required to raise the temperature of 100 lb. of water at 40° F. to the boiling point in 30 min.?

387. Find the number of (*a*) ft.-lbs.; B.t.u. and (*c*) calories in 1 joule.

388. A cannon ball moving at the rate of 800 ft. per sec. strikes against a target, and the heat produced is equally divided between the target and the ball. Supposing the latter to be made of iron, how much will its temperature be raised?

389. A boy eats 1 lb. of ice in 10 min. What horse-power is required to melt the ice and raise it to the temperature of the body, 98° F.?

390. If a horse does 60 kg. m. of work per sec. for 5 hrs. each day, how much, at least, of oats per week of 7 days must he eat to supply energy for this work, if the combustion of 1 gr. of oats would warm 10 kg. of water 1°?

391. How much will a mass of copper be heated by striking a hard non-conducting surface after a fall of 368 ft.?

392. A block of ice falls into a well of water, both ice and water being at zero. From what height must the ice fall in order that one-fiftieth of it shall be melted?

393. Find the amount of heat developed in drilling a hole in a block of iron if 0.4 H.P. is applied for 3 min.

394. To what height would the energy obtained by burning 1 ton of coal of heat value 13,000 B.t.u. per lb. raise a body of 10 tons if the over-all efficiency of the apparatus is 20 per cent?

395. What horse-power is required to change 100 lb. of water at 50° F. to steam at 212° F. in 30 min.? How much coal of heat value 14,000 B.t.u. per lb. will be required if 30 per cent of the heat value of the coal is utilized?

396. Find the number of B.t.u. and of calories, equivalent to 1 kilowatt-hour.

397. Find (*a*) the number of B.t.u. per hour, and (*b*) the calories per hour, equivalent to 1 H.P.

398. Find (*a*) the number of horse-power, and (*b*) the B.t.u. per hour, equivalent to 1 kilowatt.

399. If electric energy costs 10 cents per kilowatt-hour, how many calories of heat will be produced for 1 cent?

400. Compare the cost of cooking with a gas stove with the cost of cooking with an electric stove. Assume that gas costs 80 cents a thousand cu. ft., that each cu. ft., when burned yields 140,000 cal. and that electric power costs 10 cents a kilowatt-hour. Assume that only 40 per cent of the heat generated in the gas stove is useful, while in the electric stove 70 per cent is useful.

401. In a certain city electric energy for domestic purposes costs 9 cents per kilowatt-hour and illuminating gas of heating value 550 B.t.u. per cu. ft. costs $1.00 per thousand cu. ft. An electric range delivers to a kettle 70 per cent of the energy supplied to the range, whereas a gas range delivers to a kettle 25 per cent of the energy of the gas consumed. Find the cost of bringing 10 lb. of water from 50° F. to the boiling point and then evaporating 5 lb., (*a*) by gas, (*b*) by electricity.

402. 100,000 B.t.u. per day are required to heat a certain residence. Coal of heat value 12,000 B.t.u. per lb. costs $6.00 per ton, and illuminating gas of heat value 550 B.t.u. per cu. ft. costs $1.00 per thousand cu. ft. The efficiency of the furnace with either fuel is 60 per cent. Electric energy can be obtained at 4 cents per kilowatt-hour and the heating device has an efficiency of 100 per cent. Find the cost of heating by each method.

403. What is the theoretical efficiency of a steam engine whose boiler is at 150° C. and its condenser at 40° C.?

404. A heat engine works between the temperatures 127° C., and 52° C. Its actual efficiency is one-third of the theoretical efficiency. What per cent of the total amount of heat supplied is usefully employed?

405. A locomotive burning 1271 lb. of coal per hr. and running 30 mi. per hr. exerts a draw-bar pull of 4065 lb. wt. The coal has a thermal value of 14,500 B.t.u. per lb. The temperature of the steam entering the engine is 370° F., and the temperature of the exhaust is 212° F. Find the maximum efficiency of the engine, the actual efficiency, and the ratio of the maximum to the actual efficiency of the combined boiler and engine.

406. A locomotive burning 1361 lb. coal per hr. and running 30 mi. per hr. exerts a draw-bar pull of 4154 lb. wt. The coal has a thermal value of 14,500 B.t.u. per lb. Steam enters the engine at 388° F. and escapes at 212° F. Find the horse-power developed, the maximum efficiency of the engine, the actual efficiency, and the ratio of the maximum to the actual efficiency of the combined boiler and engine.

407. A locomotive burning 1595 lb. coal per hr. and running 30 mi. per hr. exerts a draw-bar pull of 5258 lb. wt. The coal has a thermal value of 14,500 B.t.u. per lb. Steam enters the engine at 403° F. and escapes at 212° F. Find the horse-power developed, the maximum efficiency of the engine, the actual efficiency, and the ratio of the maximum to the actual efficiency of the combined boiler and engine.

408. A locomotive burning 1834 lb. coal per hr. and running 40 mi. per hr. exerts a draw-bar pull of 4596 lb. wt. The coal has a thermal value of 14,500 B.t.u. per lb. Steam enters the engine at 403° F. and escapes at 212° F. Find the horse-power developed, the maximum efficiency of the engine, the actual efficiency of the combined boiler and engine.

Resistivity

409. Find the resistance at 20° C., of five miles of copper wire 400 mils in diameter.

410. The resistance of 500 meters of copper wire with a cross-section of 0.001 sq. cm. is 79.5 ohms at 0° C. What is the resistivity of copper in centimeter-ohm measure?

411. A column of mercury 106.3 cm. long with a section 1 sq. mm. has a resistance of 1 ohm at 0° C. What is the resistance between opposite faces of an inch cube?

412. No. 30 wire has a diameter of 0.01 in. Calculate the resistance of 50 ft. of this wire.

413. No. 36 wire has a diameter of 0.005 in. How many feet of German silver wire of this number will there be in a 500-ohm coil?

414. An iron wire has an area of cross-section of 3 sq. mm. and the same resistance as a copper wire 1000 m. long and cross-section of 0.5 sq. mm. Supposing the conductivity of iron to be one-seventh that of copper, what would be the length of the iron wire?

415. The secondary circuit of an induction coil is a copper wire 0.2 mm. thick and 100,000 ohms resistance. What is the length?

416. A cable 23 km. long was placed in a river; it had a copper resistance of 306.8 ohms; after a rupture of the cable the resistance from one of the stations was found to be 75.3 ohms. What is the distance of the rupture from the station?

417. The resistance of 18.12 yds. of No. 30 B. W. G. copper wire was found to be 3.02 ohms. Another coil of the same wire had a resistance of 22.65 ohms. What length of wire was there in the coil?

418. From the resistivity of copper calculate the resistance of a double line of copper wire, 6.25 km. long and 0.7 cm. in diameter, allowing 4.5 per cent. for "sag" and waste.

419. Of two platinum wires the first has a length of 70 m. and a thickness of 1.2 mm.; the second has a thickness of 0.3 mm. and a resistance half that of the first. What is the length of the second?

420. One of the trans-Atlantic cables has a length of 3000 km., and the copper core a radius of 2.5 mm. What is its resistance?

421. The bobbins of the electro-magnet of a certain telegraph relay carry 940 m. of copper wire 0.2 mm. thick. What is the resistance of the bobbin?

422. A telegraph line 5 mi. long is made of iron wire 0.204 in. in diameter. Find its resistance. The line failed to work and the operator at one end found that the part with which he was connected had a resistance of 2 ohms. How far from his end was the "ground"?

423. The resistance of an iron wire at 20° C. is 1106 ohms. What is the resistance at 0° C.? at 40° C.?

424. Assuming the temperature coefficient to be constant, find the temperature at which copper would have no resistance.

425. The temperature coefficient for a carbon filament is −0.0003. How many ohms of copper resistance must be joined in series with a carbon filament of 100 ohms resistance so that the combined resistance may be constant?

426. What is the temperature of a furnace in which the coil of a platinum thermometer has a resistance of 1020 ohms if the resistance at 0° C. is 300 ohms?

427. It is found that a certain wire has a resistance of 10 ohms at 15° C., and 16.8 ohms at 215° C. Find the temperature coefficient of resistance of the material.

428. Find the resistance at 25° C. of copper wire 10 m. long and 1 mm. in diameter.

429. A coil on a dynamo has a resistance of 43 ohms at 20°. After the machine has been in operation for some time the resistance is 52 ohms. Find the temperature of the coil.

Joule's Law

430. A current of 10 amp. flows through a conductor having a resistance of 4 ohms. How much heat is generated in the conductor per minute?

431. A current of 0.5 amp. in a glow lamp generates 15 cal. of heat in 10 sec. Required the resistance of the lamp in ohms and the power expended in the lamp in watts.

432. What horse-power is required to maintain a current of 4 amp. through a resistance of 37.3 ohms?

433. Each of 38 arc lamps has a resistance of 6 ohms. They are arranged in series on a circuit 6 km. long, and it is desired that the line absorb only one-thirtieth of the available energy. What must be the diameter of the wire?

434. Suppose a German silver wire, the resistance of which is 3.5 ohms, to be immersed in 3 liters of water at 20° C. What will be the temperature of the water after a current of 4 amp. has been passing through it for 20 min.?

435. In order to determine the strength of a current, it was made to pass through a coil of wire of 5 ohms resistance placed in a calorimeter. A steady stream of water was kept flowing through the calorimeter at the rate of 15 cc. per min., and the heating effect of the current was such that the water was 4° warmer on leaving the calorimeter than it was on entering. Find the strength of the current.

436. When a vessel containing 1500 g. of hot water is allowed to cool, its temperature when it passes 90° C. is falling at the rate of 12° C. per min. A wire of 6 ohms resistance is submerged in the water. How much current must be sent through this wire to keep the temperature of the vessel at 90° C.?

437. In the same series circuit there are a platinum wire 20 cm. long and 0.4 mm. in diameter, and a silver wire 400 cm. long and 0.6

mm. in diameter. What is the relation between the quantities of heat evolved in the wires by the same current?

438. If 20 per cent of the heat developed is lost by radiation, how much will the temperature of a copper conductor 200 m. long and 0.4 cm. in diameter rise in 10 min. if it carries 100 amp.?

439. The armature of a dynamo has a resistance of 0.5 ohm, the leads 1.2 ohm, and each of 5 arc lamps in series 2 ohms. What is the fraction of the total energy utilized in the lamps?

440. If electric energy costs 10 cents per kilowatt-hour, how many calories of heat will be produced for one cent?

441. Find the number of calories developed in 2 min. by a current of 10 amp. supplied to an electric heater whose resistance is 7 ohms.

442. By means of an electric heating coil, 250 g. of water at 12° C. are to be vaporized per sec. The resistance of the coil is 50 ohms. Find the current and power required.

443. A current of 5 amp. traverses an electrolytic bath of 0.6 ohm resistance. What is the energy expended per minute in consequence of this resistance, and what quantity of heat is supplied to the bath per min.?

444. Find the number of (*a*) joules, (*b*) calories, (*c*) ft. lb., (*d*) B.t.u. in 1 watt-hour.

OHM'S LAW

445. What current flows through an electric heater of 100 ohms resistance placed across a 550-volt circuit?

446. On connecting a 10,000-ohm voltmeter, which is in series with an unknown resistance, to 110-volt mains, the voltmeter reading is 5 volts. Find the unknown resistance.

447. A battery has an E.M.F. of 1.3 volts and a resistance of 18 ohms. What is the maximum current it can give? What current will it give when the external resistance is 10 ohms?

448. A galvanic cell of E.M.F. 1 volt and internal resistance 0.8 ohm is connected to a galvanometer which indicates a current of 0.016 amp. What is the resistance of the galvanometer?

449. A certain galvanic cell has an E.M.F. of 2 volts and an internal resistance of 0.4 ohm. What external resistance must be placed in circuit that the current may be 2 amp.?

450. A galvanometer of 240 ohms resistance when connected in series with a cell of 5 ohms resistance and a rheostat of 100 ohms resistance deflects 50 scale divisions. If the deflection is proportional to the current, what resistance in series with the galvanometer and cell would reduce the deflection to 5 scale divisions?

451. With an external resistance of 9 ohms a certain battery gives a current of 0.43 amp. When the external resistance is increased to 32 ohms the current falls to 0.2 amp. What is the resistance of the battery?

452. Ten cells in series, each having an E.M.F. of 1.5 volts and an internal resistance of 1 ohm, will give what current through an external resistance of 30 ohms?

453. The resistance of telegraph wire being taken as 13 ohms per mile and the E.M.F. of each cell as 1.4 volts and the resistance of each cell as 5 ohms, calculate how many cells are needed to send a current of 13 milliamperes through a line 120 miles long, assuming that the instruments in circuit offer as much resistance as 20 miles of wire would do and that the return current through earth meets with no appreciable resistance.

454. A galvanic cell indicates a certain current when connected to a galvanometer of negligible resistance. When a resistance of No. 20 German silver wire 5 ft. long is inserted into the circuit it is found to reduce the current to one-half its former value. If No. 20 German silver wire has a resistance of 190.2 ohms per 1000 ft., find the resistance of the cell in ohms.

455. What is the internal resistance of a cell whose E.M.F. on open circuit is 1.12 volts when the circuit is closed, if with an external resistance of 20 ohms the voltage falls to 1 volt?

456. A telegraph line has a resistance of 1 ohm per mile. Find how many cells each of an E.M.F. 1.5 volts and internal resistance 0.1 ohm are required to send a current of 0.2 amp. through a line 50 mi. long, neglecting the instrument resistance.

457. An inclosed arc lamp requires 80 volts and 4.5 amp. to run it properly. (*a*) What is its resistance? (*b*) How much resistance must be added to it so that it shall carry the proper current when it is coupled to a 110-volt circuit?

458. Two 50-volt lamps that require 8 amp. each are put in series across the terminals of a 110-volt incandescent circuit. How much resistance must be put in series with them?

459. A current is sent by a battery of constant E.M.F. (*a*) through a resistance of 20 ohms, (*b*) through a wire of unknown resistance, and (*c*) through a resistance of 40 ohms. The currents produced are in the ratios of 10:9 and 10:8. What are the resistances of the battery and of the wire tested?

460. Two cells, each of E.M.F. 1.1 volts, are connected separately to resistances of 4 and 5 ohms. If the cells have equal internal resistance of 1.5 ohms, what will be the P.D. between their terminals?

461. A voltmeter that has a resistance of 26,000 ohms indicates 37 volts. (*a*) What is the strength of the current? (*b*) What voltage would such an instrument indicate with a current of 3 milliamperes?

462. Two voltmeters, one of which has a resistance of 25,000 ohms, and the other a resistance of 15,000 ohms, are connected in series across 110 volts. (*a*) What current flows through the system? (*b*) What voltage does the first instrument indicate? (*c*) What voltage does the second instrument indicate?

463. A uniform copper wire 500 ft. long, and having a resistance of 2 ohms, is connected to the poles of a dynamo of E.M.F. 50 volts. If the resistance of the dynamo is 10 ohms, and if the positive pole is electrically connected through a voltmeter to a point 100 ft. along the wire, what will be the voltmeter reading?

464. In a machine we find the P.D. at the terminals to be 75 volts, the resistance of the armature 0.52 ohm, the resistance of the external circuit 15 ohms. What is the total E.M.F. when running?

465. The E.M.F. of a battery is 5 volts. When the external resistance is 100 ohms the P.D. at the terminals is 4 volts. What is the internal resistance?

466. Ten lamps, each of 50 ohms resistance, are arranged in series. The leads have a resistance of 5 ohms, the generator 1 ohm, and the current is 1.087 amp. Find the E.M.F. and P.D. of the generator.

467. In a given circuit, the P.D. across the terminals of a resistance of 19 ohms is found to be 3 volts. What is the P.D. across the terminals of a 3-ohm wire in the same circuit?

468. A trolley wire of No. 0 B.S.G. has a resistance of 0.519 ohm per mile. What is the drop of potential between the station and a car taking 20 amp. 2 mi. out on the line.

469. A power station maintains a difference of potential of 550 volts between trolley and ground at the station. The resistance of the trolley wire is 0.519 ohm per mile and that of the rail 0.04 ohm per mile. If there is only one car on the line, how far from the station must it be to have the potential drop to 500 volts with a current of 35 amp.?

470. A current of 10 amp. develops 144 (10^4) cal. of heat per minute. What is the resistance of the circuit? What quantity passes per minute? What potential difference is required to maintain the current?

471. To find the terminal P.D. of a dynamo you connect to its brushes the ends of a wire 120 ft. long, and find that when one terminal of a galvanic cell of 1.05 volts is joined to a point on the wire, and the other terminal in series with a galvanometer is connected to another point 1 ft. from the first, no deflection is observed. What is the terminal P.D. of the dynamo?

472. How many lead accumulators each of a E.M.F. of 2.1 volts and an internal resistance of 0.005 ohm can be charged in series at the rate of 15 amp. by a dynamo of terminal potential difference 115 volts?

473. A battery of 40 secondary cells in series, each of E.M.F. 2.1 volts and internal resistance 0.02 ohm is to be charged by a current of 10 amperes. The battery is to be connected to a dynamo by conductors of resistance 0.1 ohm. What must be the potential difference at the terminals of the dynamo?

474. A battery of 20 accumulators each of E.M.F. 2.1 volts and internal resistance 0.01 ohm is to be charged at 4 amp. from a 110-volt supply line. Find the resistance that must be placed in circuit.

475. If a current of 5 amp. passes through a wire having a resistance of 2 ohms, (*a*) what E.M.F. is required to maintain the current? (*b*) how much heat is developed in the wire in 10 min.?

476. An electric pocket flash-lamp has a resistance of 12 ohms and is lighted by a battery of 3 cells in series, each having an E.M.F. of 1.5 volts and an internal resistance of 2 ohms. The output of the battery is 0.5 ampere-hour. Find the number of flashes the lamp will give, supposing each to be 5 sec. long.

477. How much is generated in 10 min. in an electric lamp which takes a current of 0.45 amp. with an applied E.M.F. of 110 volts?

478. A generator having a brush potential difference of 220 volts furnishes power to a factory 10,000 ft. distant through wires having a resistance of 0.08 ohm per 1000 ft. What will be the line voltage at the factory when using 10 amp.? (*b*) When using 20 amp.?

Electric Energy and Power

479. Find the amount of heat generated per minute by an incandescent lamp that takes 0.4 amp. when the potential difference at its terminals is 110 volts.

480. A motor is rated to take 9 amp. and consume 2000 watts. For what voltage is it built?

481. A 50-candle-power carbon lamp, rated as 3.1 watts per candle, is to take 2.58 amp. On what voltage should it be run?

482. An incandescent lamp of 16 candle-power takes a current of 0.75 amp. with a difference of potential of 60 volts between its terminals. Find the number of watts per candle-power absorbed.

483. How many 110-volt lamps can be operated by a 12 kilowatt generator if the current in each lamp is 0.5 amp.?

484. An arc lamp has a current of 10 amp. and a potential difference of 60 volts between the carbon points. Find (*a*) the power required to operate the lamp; (*b*) the cost per hour of operating the lamp at 6 cents per kilowatt-hour.

485. A street car, running on a system in which the difference of potential between trolley wire and track is 500 volts, requires 15 amp. to run it on a certain grade. What is the number of watts used? What is the horse-power used?

486. An electric motor delivers to the belt 10 horse-power when taking 80 amp. from a 110-volt circuit. Find the efficiency.

487. A certain motor takes 8 amp. from a 500-volt circuit, and delivers 5 horse-power. What is its efficiency?

488. An electric motor found by a brake test to develop 2 horse-power, requires 8.25 amp. at an electromotive force of 220 volts. What is the efficiency of the motor.

489. A certain motor on a 110-volt line is traversed by a current of 80 amp. when developing 9.7 H.P. Find the efficiency.

490. A motor of 90 per cent efficiency on a 110-volt line is developing 10 H.P. Find the current through the motor.

491. How many calories of heat are developed per second by a 110-volt incandescent lamp carrying a current of 0.6 amp.?

492. The resistivity of a certain kind of wire is 50 microhms per centimeter cube. Calculate the length of this wire, of diameter 2 mm., such that when the current strength in it is 5 amp., the power consumption will be 1000 watts.

493. A current of 6 amp. from a 110-volt circuit flows through an electric flat-iron for 1 hr. Find the heat energy developed in kilowatt hours and in calories.

494. Electric energy costs 8 cents per kw.-hr. A 110-volt motor takes 2 amp. How much will it cost to run the motor for 30 days, 10 hours per day?

495. If electric energy costs 8 cents per kilowatt-hour, what will be the cost per horse-power-hour?

496. An electric flat-iron of 27.5 ohms resistance is connected to a 110-volt circuit. At 10 cents per kilowatt-hour how much would it cost per hour to heat the flat-iron?

497. An electric stove having a resistance when hot of 15 ohms is connected to a 110-volt circuit. Find the power absorbed, in kilowatts and in horse-power.

498. If an incandescent lamp requires a current of 0.5 amp. and a difference of potential of 110 volts at its terminals, (*a*) how great is its resistance? (*b*) How many such lamps may be supplied with power by a 2 kilowatt electric generator?

499. An elevator weighing 2 metric tons is to be lifted at the rate of 5 m. per sec. If the output of the driving motor is 80 per cent of its intake of energy, (*a*) how many kilowatts must be supplied to the motor? (*b*) How many joules of work must the motor do per second?

500. A 32-candle-power tungsten lamp is operating on a 112-volt circuit. It requires 1.25 watts per candle. Find current, resistance and cost per hour at 8.1 cents per kilowatt-hour.

501. If a cell has an E.M.F. of 2 volts, and furnishes a current of 5 amp. what is the rate of expenditure of energy in watts? If the resistance of the external circuit is 0.1 ohm, what is the ratio of energy spent in the internal to that in the external circuit?

502. The same amount of power is to be transmitted over two lines from a power plant to a distant city. If the heat losses in the two lines are to be the same, what must be the ratio of the cross-sections of the two lines if one current is transmitted at 100 volts and the other at 10,000 volts?

503. How many 16-candle-power lamps, each requiring 3.5 watts per candle, can be run by a four kilowatt dynamo? If the lamps are 50-volt lamps, what current does each take?

504. A 220-volt lamp has a resistance, when hot, of about 750 ohms How many calories will be developed in it in 10 min.?

505. Given a transmission line of 0.5 ohm resistance, the power required at the farther end is 4000 kw. at a potential difference of 100 volts. (*a*) What is the current flowing? (*b*) What is the line drop? (*c*) What is the initial voltage?

506. A dynamo receives 525 H.P. of mechanical energy and delivers 350,000 watts at a P.D. of 10,000 volts. The line that completes the circuit has a resistance of 14 ohms. (*a*) Determine the current strength. (*b*) What is the line loss in volts? (*c*) In watts? (*d*) What is the efficiency of the dynamo?

507. An electric motor is supplied at a brush P.D. of 100 volts; the armature resistance is 0.01 ohm. When it is supplying 20 H.P., what is its electrical efficiency?

508. A motor was tested with a current of 25.5 amp. and an E.M.F. of 40 volts, and was found to develop 0.98 H.P. Find its efficiency, and the power supplied to it.

509. Calculate the efficiency of a long-distance line of 10.8 ohms resistance, when 50 amp. at 3600 volts are supplied to it.

510. Two pipes each 4 in. in diameter and 4 ft. long, one covered with an insulating material, the other bare, are filled with oil into which is immersed a coil of wire by which the oil may be heated electrically. When the temperature of the oil in each pipe was constant and 100° C., the following data were taken: In the coil of the covered pipe the current was 4.2 amp. and the potential difference of its terminals was 23.0 volts. In the coil of the uncovered pipe the current was 5.5 amp. and the potential difference of its terminals was 38.5 volts. Find the saving in one month on 1000 sq. ft. of pipe under similar conditions when energy is supplied at 4 cents per kilowatt-hour.

511. Compare the cost of purchase and operation for 2000 hr. of the two following lamps.

	Life.	Watts per C.P.	First Cost.	Cost per Kw. Hr.	C.P
(*a*)	1000 hrs.	3.5	25 cts.	9 cts.	32
(*b*)	500 hrs.	2.2	140 cts.	9 cts.	32

512. In a brake test, a force of 800 g. wt. was applied tangentially to the pulley of a motor. The speed was 1100 r.p.m. and the pulley was 25 cm. in diameter. The motor had an efficiency of 80 per cent and the applied E.M.F. was 100 volts. What was the current supplied?

513. An 8-ton car requires a certain current to run on a level track at a speed of 22 ft. per sec. At 500 volts what additional current will be required to pull the car at the same speed up a grade of 5.6 per cent, assuming the resistance to motion on the hill equal to that on the level?

514. A generator receives power at the pulley at the rate of 106 H.P. The terminal P.D. of this machine is 6000 volts, and the current 9.8 amp. What is the rate at which energy is available at the terminals of the generator? What is the rate at which energy is absorbed by the machine?

515. A dynamo is connected to an engine. If 15 per cent of the power of the engine is wasted by friction, etc., what must be its horse-power to run the dynamo when lighting 550 incandescent lamps, each carrying 1 amp. of current and having a resistance of 110 ohms?

516. A generator having a resistance of 0.02 ohm is used to charge a battery of 30 storage cells in series. The resistance of the cells is negligible and of the leads is 0.5 ohm. The E.M.F. of the generator is 75.7 volts, and the current is 10 amp. Where is energy used in the circuit? How is it distributed?

517. A dynamo of E.M.F. 115 volts is to deliver 100 amp. at a point 1000 ft. distant. The allowable loss of energy in the line wire is 5 per cent. Find the required diameter of the line wire, expressed in mils.

518. Assuming a power plant which is 20 per cent efficient using coal of heat value 14,000 B.t.u. per pound, calculate the number of pounds of coal which must be burned per hour to maintain a current strength of 50 amp. in a line of 20 ohms resistance.

519. If electric energy costs 9 cents per kilowatt-hour, find the cost of energy required to operate for 100 hrs. a lamp which takes 0.54 amp. from a 110-volt circuit.

520. If electric energy costs 9 cents per kilowatt-hour, find the cost per hour of operation of an electric flat-iron of 50 ohms on a 110-volt line. Find the number of calories of heat developed per hour.

521. (*a*) Compare the cost for electric power of running a 10-ohm heating coil on a 110-volt circuit, with the cost of running the same heating coil for the same length of time on a 220-volt circuit. (*b*) If the coil, when running on the 110-volt circuit, takes 10 min. to bring to a boil a certain quantity of water in which it is immersed, how long will it take the same coil, when running on the 220-volt circuit, to bring the same quantity of water to a boil?

522. By means of an electric heating coil, 250 g. of water at 12° C. are to be vaporized per sec. The resistance of the coil is 50 ohms. Find the power and current required.

Divided Circuits

523. Find the resistance between two points in a circuit when they are joined by:

(*a*) Three wires in multiple, resistance 2, 5, 7 ohms respectively.
(*b*) Three wires in series, resistance 2, 5, 7 ohms respectively.
(*c*) Four wires in multiple, resistance 40, 20, 30, and 50 ohms respectively.
(*d*) Four wires in series, resistance 40, 20, 30, and 50 ohms respectively.

524. The resistance between two points in a circuit is 60 ohms. What resistance must be placed in multiple with this to reduce the resistance to 22 ohms?

525. Five incandescent lamps, each having a resistance when hot, of 220 ohms, are arranged in series. If 0.5 amp. is needed to bring each lamp to its proper candle power, what E.M.F. is needed for the group? If the five lamps were arranged in parallel what current would be necessary?

526. Two incandescent lamps have resistances, when hot, of 120 and 240 ohms, respectively. What current will flow through each when they are joined (*a*) in series, (*b*) in parallel, between two points maintained at a constant difference of potential of 120 volts?

527. Five 16-candle-power incandescent lamps each having a resistance when hot, of 220 ohms, are arranged in parallel on a 110-volt circuit. Calculate (*a*) the total current taken by the lamps; (*b*) the watts consumed by one lamp.

528. At 10 cents per kilowatt-hour how much would it cost per hour to light 20 incandescent lamps connected to a 100-volt circuit, each lamp having a resistance, when hot, of 250 ohms?

529. What is the horse-power of a 110-volt generator which can just supply the current for 440 incandescent lamps which are joined in parallel. Each lamp having a resistance, when hot, of 220 ohms.

530. If 10,000 incandescent lamps are arranged in parallel and each requires a current of 0.5 amp., what is the total current furnished by the dynamo? What is the activity of the machine in kilowatts and in horsepower?

531. A copper wire of length l is divided in the ratio of 3 to 5, and the pieces joined in multiple. What length of the same wire might have been taken to get the same resistance?

532. Two wires have resistances of 36 and 45 ohms respectively. They are connected in parallel so that the total current in both branches is 9 amp. What is the joint resistance and what is the current in each branch?

533. The hot resistance of a 110-volt incandescent lamp is 220 ohms. Find the resistance of 2, 3, 4, and 5 lamps in parallel. Find the current in each case.

534. A coil of copper wire, of resistance 500 ohms, is placed in shunt with a resistance of 1000 ohms. How much must the temperature of the copper be varied to change the multiple resistance 1 per cent?

535. Three electric bells are connected in parallel. The resistance of each bell, including the wire connecting it with the line wires, is 3.3 ohms. The resistance of the line wire is 0.5 ohm and the internal resistance of the battery is 3 ohms. What is the total resistance of the circuit?

536. If the poles of a battery are connected by means of two wires in parallel, one having a resistance of 6 ohms and the other of 8 ohms, what will be the resistance of the external circuit? If the internal resistance of the cell is 1 ohm, what is the E.M.F. if a current of 1.5 amp. flows between the terminals?

537. Two electric mains are connected by four conductors in parallel, having respectively resistances of 3, 5, 7, and 8 ohms. Find the combined resistance of the four. What current flows through the system if the mains are at 10 volts potential difference?

538. To the poles of a storage battery having 10 volts difference of potential between its terminals are connected in parallel a motor having a resistance of 12 ohms, a resistance box with a resistance of 15 ohms, and a plating bath having a resistance of 5 ohms. Find (*a*) the amount of current flowing through the system; (*b*) through each branch of the circuit.

539. A resistance of 80 ohms joins the terminals of a battery of E.M.F. 100 volts and resistance 20 ohms. A shunt of 5 ohms is placed around 40 ohms of the external resistance. What will be (*a*) the increase in the total current, (*b*) the decrease in the P.D. at the points joined?

540. Two points A and B are connected by three wires; the first contains a cell of E.M.F. 1.734 volts and a total resistance of 0.66 ohms; the two others have resistances of 16 ohms and 2 ohms. What are the currents in the three branches?

541. Four similar cells each with an E.M.F. of 1.5 volts are joined in series through a resistance and found to give a current of 1 amp., and when joined in parallel through the same resistance the current is a third less. What is the resistance of each cell?

542. A 110-volt current flows through two motors in series having resistances of 10 and 15 ohms respectively. (*a*) What is the fall of potential through the first motor; the second? (*b*) What is the flow of current through the first; the second?

543. If the motors mentioned above are connected in parallel to the same circuit, what is the fall of potential through each? What is the current through each? What is the current through the dynamo?

544. A 220-volt current passes through four lamps in series, each having a resistance of 55 ohms. What is the current through the lamps? The fall of potential through each? If the lamps were in parallel, what difference of potential between the mains would give the same current through the lamps?

545. A battery consists of five cells, each having an E.M.F. of 1.08 volts and an internal resistance of 4 ohms. What current will the battery produce with an external resistance of 7 ohms, (*a*) when connected in series, (*b*) in multiple?

546. A dynamo has a resistance of 10 ohms, and an E.M.F. of 100 volts. The current flows through a resistance of 10 ohms in series with a set of three resistances of 2, 4, and 6 ohms, respectively, in parallel. What is the total resistance of the circuit? What current flows through the dynamo?

547. Two wires of resistance 2 and 5 ohms, respectively, are connected in parallel and included in a circuit in which a current of 12 amp. is flowing. Calculate the current in each coil, the P.D. between the branch points, and the combined resistance of the two coils.

548. Six conductors have the following resistances: 10, 15, 16, 20, 24, and 30 ohms, respectively. They are connected to an E.M.F. of 15 volts, in the following order: The first two are in parallel, the third in series, and the last three in parallel. What is the current in the first two?

549. Three resistances of 4, 4 and 2 ohms, respectively, are connected in parallel, and two resistances, of 6 and 3 ohms are in parallel. The first combination is connected in series with the second, and with a battery of 3 volts and negligible resistance. What is the current in the 2-ohm and in the 3-ohm resistance?

550. Two cells have internal resistances each equal to 3 ohms. In one case they are joined in series by a wire of 3 ohms resistance. In another case they are joined in parallel by the same wire. Compare the total heats evolved per second in the two cases.

551. In circuit with a dynamo of 0.01 ohm resistance are placed 600 incandescent lamps in parallel, each lamp having 100 ohms resistance and requiring 0.9 amp. to bring it to proper incandescence. What must be the E.M.F. of the dynamo?

552. Two points A and B are connected by three wires whose resistances are 1, 3 and 6 ohms. Find the total current which passes through the multiple arc when the difference of potential between A and B is 3 volts.

553. A divided circuit consists of two equal and similar wires. If the wires are made to touch so that a point one-quarter the length from an end of one wire is in contact with a point three-quarters the length from the corresponding end of the other, what will be the final resistance compared with the original?

554. If a certain type of electrical heating coil takes 500 watts from a 100-volt line, how many watts will two such heaters take from the same line, (*a*) if connected in series, (*b*) if connected in parallel?

555. If a battery of E.M.F. 10 volts and resistance 0.5 ohm is joined to an external circuit consisting of two wires of 10 ohms and 1 ohm connected in parallel, find the current in each wire.

556. A lighting system consists of 10 groups of lamps in multiple between the line wires, the groups being 100 ft. apart and the nearest group 500 ft. from the generator. Each group of lamps takes 5 amp., and the resistance of the line is 0.102 ohm per 1000 ft. What is the difference in voltage between the generator terminals and the terminals of the tenth group of lamps?

557. Six cars a mile apart, the first car 1 mi. from the plant, are each taking 20 amp. of current. The voltage between trolley and earth at the plant is 550 volts. Find the drop of potential at each car, assuming line resistance to be 0.5 ohm per mile, and earth return 0.04 ohm per mile.

ELECTROLYSIS

NOTE.—One gram of hydrogen is deposited by 96,530 coulombs of electricity, or the electro-chemical equivalent of hydrogen = 0.000010358 g. per coulomb.

Substance	Atomic Mass	Valence
Copper	63.6	2
Gold	197.2	2 or 3
Hydrogen	1.0	1
Iron	55.8	2 or 3
Nickel	58.7	2
Oxygen	16.0	2
Silver	108.0	1
Zinc	65.4	2

558. How long will it take a current of 1 amp. to deposit 1 g. of silver from a solution of silver nitrate?

559. If the same current used in the above problem were passed through a solution containing a zinc salt, how much zinc would be deposited in the same time?

560. In calibrating an ammeter, the current which produces a certain deflection is found to deposit 0.5 g. of silver in 50 min. What is the strength of the current?

561. Determine the current necessary to deposit 0.1557 g. of silver per hour.

562. A current of 2 amp. passes through a copper sulphate solution for 1 hr. If the anode is a copper wire, how much copper will be deposited on the cathode?

563. How many ampere-hours should be developed by the consumption in a voltaic cell of 1 lb. of zinc?

564. Ten grams of silver are to be deposited upon a certain surface. How long will it take a current of 8 amp. to do it?

565. A current passes by platinum electrodes through three cells, the first containing a solution of $CuSO_4$, the second containing a solution of $FeSO_4$, the third containing a solution of $Fe_2(SO_4)_3$. State the amounts of the different substances evolved at each electrode by the passage of 1000 coulombs of electricity.

566. In a plating workshop the same current is made to pass for the same time through baths of copper, silver, gold and nickel. What are the relations between the masses of the different metals deposited?

567. How much Zn is consumed in a battery which deposits 60 gms. of silver from a bath of $AgNO_3$, supposing 20 per cent of the Zn is wasted through local action?

568. An ammeter indicates 10 amp. when traversed by a current which deposits 12.4 g. copper from a solution of $CuSO_4$ in 1 hr. Find the error of the ammeter reading.

569. A current which gives a reading of 0.27 amp. on a milliammeter deposits 0.2008 g. of silver in 10 min. 42 sec. What is the error in the ammeter readings?

570. A dynamo is capable of depositing 6 kg. of copper each hour. What is the strength of the current produced by it?

571. A battery of three cells is connected in series with a copper voltameter in which 31.7 g. of copper are deposited in 1 hr. How much copper is deposited and zinc dissolved in the whole battery in the same time?

572. How long a time is required for 100 amp. to refine 2000 lb. of copper?

573. If 200 g. of copper are deposited by a certain current, what mass of hydrogen will be produced?

574. An object whose surface was 4 sq. cm. was silver plated by a current of 0.1 amp., continued for 12 hrs. What was the average thickness of the plating?

575. Assuming the electrochemical equivalent of hydrogen find the mass of copper deposited per hour by a current of 1 amp., (*a*) from a solution of $CuNO_3$; (*b*) from a solution of $Cu(NO_3)_2$.

GALVANOMETERS

576. A galvanometer of 100 ohms resistance is to be provided with a shunt such that one-fifth of the whole current shall pass through the galvanometer. Compute the resistance of the shunt.

577. A direct reading ammeter has a resistance of 0.03 ohm. The instrument is to be shunted so that the total current passing through the instrument and shunt is ten times the ammeter reading. What is the resistance of the shunt?

578. The maximum scale reading of a voltmeter whose resistance is 300 ohms is 3 volts. What must be the resistance of the multiplier in order that the maximum reading shall correspond to 150 volts?

579. An ammeter whose resistance is 0.1 ohm and maximum scale reading is 0.15 amp. is to be used as a voltmeter. What resistance must be used as a multiplier in order that the maximum scale reading shall be 15 volts?

580. A voltmeter measures up to 15 volts, and has a resistance of 3500 ohms. It is to be used on a 115-volt circuit and is therefore put in series with a resistance of 24,000 ohms. With the two thus connected and with a P.D. of 110 volts, (*a*) what is the current strength? (*b*) what is the indication of the voltmeter? (*c*) by what must the reading of the voltmeter be multiplied to get the actual voltage?

581. A certain galvanometer has a resistance of 25 ohms and gives unit deflection when 0.002 amp. traverses it. What resistance, and how placed, will change this instrument into a voltmeter reading 1 volt per scale division?

582. A certain galvanometer has a resistance of 25 ohms and gives unit deflection when 0.002 amp. traverses it. What resistance, and how placed, will change this instrument into an ammeter reading 1 amp. per scale division?

583. A battery is connected by wires of inappreciable resistance with a galvanometer and the current is read off. The galvanometer is now shunted with a shunt having one-ninth of its own resistance, and it is found that the current through the galvanometer is now reduced to one-half. Find the resistance of the galvanometer relative to the resistance of the battery.

584. An ammeter indicates milliamperes up to 100. It has a resistance of 6 ohms. It is to be used on a circuit that is known to have a current of 6 or 7 amp. What must be the resistance of the shunt used so that the instrument shall have a multiplying factor of 100?

585. A galvanometer of 240 ohms resistance, has its terminals joined by a shunt of 10 ohms resistance. The galvanometer is connected to two points whose P.D. is 0.2 volt. What current flows, (*a*) through the galvanometer coil? (*b*) through the shunt?

586. A voltmeter is to be used to measure the P.D. between the terminals of a resistance of 100 ohms, which is in series with another resistance of 500 ohms and a battery of internal resistance 0.1 ohm. What must be the resistance of the voltmeter such that its application will not alter the P.D. across the 100-ohm resistance by more than one-tenth of 1 per cent?

Dynamos and Motors

587. Find the electromotive force induced in a wire 100 cm. long that is moved at the uniform rate of 200 cm. per sec. across a uniform magnetic field of intensity 5000 gauss.

588. A railway train runs south on a straight track with a velocity of 15 m. per sec. If the vertical component of the earth's magnetic field is 0.55 gauss, find the E.M.F. induced in a car axle 160 cm. long. Which end is at the higher potential?

589. When a horizontal circular coil of 100 turns of wire 1 m. in diameter is turned over in 2 sec., at a place where the vertical component of the earth's magnetic field is 0.55 gauss, what is the average E.M.F.?

590. Each of the wires on the surface of the armature of a dynamo is 50 cm. long and is traversed by a current of 40 amp. If the average magnetic intensity of the field in which the wires are situated is 5000 gauss, find the force acting on each wire.

591. The armature of a certain bipolar direct current dynamo is 20 cm. in diameter, has 120 conductors, and rotates with a speed of 1000 r.p.m. in a magnetic field of mean field strength 8000 gauss between pole faces of area 600 sq. cm. each. Find the E.M.F.

592. A certain two-pole direct current dynamo has 560 conductors on its periphery. When running at a speed of 1000 r.p.m., it generates an E.M.F. of 120 volts. Find the magnetic flux through the armature.

593. A two-pole dynamo field magnet has a flux of 15,000 "lines" per sq. cm. The radius of the armature is 15 cm. How many inducing wires of 20 cm. length must there be on the armature in order that the dynamo may generate 108 volts when driven at a speed of 600 r.p.m?

594. The resistance of the armature of a generator is 0.02 ohm and the shunt fields have a resistance of 25 ohms. The generator absorbs 10 H.P. and delivers 50 amp. to the line while the brush P.D. is 118 volts. Calculate the various power losses.

595. Find the force exerted upon a wire 100 cm. in length at right angles to a magnetic field of 50 gauss when the wire carries a current of 10 amp.

596. A series-wound direct current motor of armature resistance 0.4 ohm and field resistance of 6 ohms takes 5 amp. on a 110-volt circuit. Compute the total watts absorbed and the watts transformed into heat.

597. A shunt-wound, direct current motor has an armature resistance of 1 ohm and a field resistance of 200 ohms. When connected to a 110-volt circuit, there is developed a back electromotive force of 105 volts.

Find the power absorbed by the motor, and the electrical efficiency of the machine.

598. A direct current motor armature of 0.05 ohm resistance is traversed by a current of 70 amp. when connected to a 110-volt line. Find (*a*) the back E.M.F. in the armature, (*b*) the power delivered to the motor armature, (*c*) the power absorbed in heating the armature, (*d*) the power absorbed due to the back E.M.F.

599. A motor that is to receive 150 kilowatts is connected to a power plant distant 20,000 ft. by transmission wires 200 mils in diameter. Find the E.M.F. of the generator required for an allowable line loss of 10 per cent.

600. A dynamo of E.M.F. 120 volts is to deliver electric energy at 110 volts to a motor of 85 per cent efficiency distant 2500 ft. The motor develops 10 H.P. at the belt. Find, in mils, the diameter of the line wire.

INTERFERENCE OF LIGHT

601. Find the breadth of a bright band of light of wave-length 0.0000589 cm., when the distance between the slits is 1.24 mm. and the distance from the screen to the diaphragm is 2.01 m.

602. In a certain experiment with Young's double slit interference apparatus the breadth of one bright band was 0.0655 cm.; for another source of light the width was 0.124 cm. Assuming the data in the preceding problem, find the wave-lengths of the two light sources.

603. In a particular interference experiment, the distance between the slits was 1.23 cm.; the distance from slits to screen was 143.5 cm., and the width of 15 bands was 0.103 cm. Find the wave-length of the light used.

LIGHT QUANTITIES

604. Find the luminous flux of a 20-candle-power lamp.

605. Find the distance from a 20-candle-power lamp at which the illumination is, (*a*) 6 foot-candles, (*b*) 20-meter-candles.

606. Find the distance from a 16-candle-power lamp at which the illumination is (*a*) 8 lumens per sq. ft., (*b*) 25 lux.

607. Compare the illumination from a 16 candle-power lamp 2 m. away, with that from a 50 candle-power lamp 3 m. away.

608. A body placed 40 ft. from an arc light is brought 10 ft. nearer the light. Compare the illuminations received by the body in the two cases.

609. At what distance from a 40-candle-power mantle burner would a newspaper receive the same illumination as it would receive from an 8-candle-power incandescent lamp 2 ft. distant from it?

610. A 20-candle-power lamp is 5 ft. above and 5 ft. to one side of the center of a horizontal table top. Find the illumination at the center of the table expressed in foot-candles and in lux.

611. A lamp of 10 candle power is placed 5 ft. from a screen, and a second lamp of 20 candle power is placed 10 ft. from the screen. Compare the intensities of illumination. Find the amount of illumination in candle-feet in each case.

612. A hefner lamp and a lamp of 10 candle power are placed 50 cm. and 4 m. respectively from a screen. Find the value of the illumination on the screen due to each source.

613. How far from a screen must a hefner lamp be placed to give the same illumination as a 16 candle power electric light 3 m. away?

614. A source of light equal to five hefner units and a 30 candle power source of light are 2 m. apart. Where must the photometer be placed that it may be equally illuminated on the two sides?

615. A luminous source equal to 10 hefner units is placed 2 m. from a screen. Find the value of the illumination on the screen in hefner-meters, in candle-feet, and candle-meters.

616. What is the quantity of light per second that passes through an aperture of 1 sq. m. placed 4 m. from a 10-candle-power lamp?

617. A gas flame having a luminous intensity of 16 candle power has an area of 50 sq. cms. What is the value of the intrinsic brilliancy of the source in hefners per sq. mm. and in candles per sq. in.?

618. Twelve lamps each of luminous flux 300 lumens are used to light a room 20 ft. by 30 ft. Find the mean illumination of the floor, expressed in foot-candles and in meter-candles.

619. At the focus of a parabolic mirror is the crater of an electric arc of 5000 candle power. The angle subtended at the crater by the mirror is one steradian. If no light were lost by reflection or transmission, and if the emergent reflected beam were a cylinder of 1 sq. ft. cross-section, what would be the equivalent candle power of the search light at a point 1000 ft. distant?

620. At the focus of a parabolic mirror is a concentrated filament lamp of 100 candle power. The angle subtended at the point source by the mirror is 4 steradians. If no light were lost by reflection at the mirror or by transmission through the air, and if the emergent reflected beam were a cylinder of cross-section 0.5 sq. ft., what would be the illumination due to the reflected light on a screen placed perpendicular to the axis of the beam?

SIMPLE REFRACTION OF LIGHT

621. The index of refraction of water is 1.33. What is the critical angle of incidence?

622. The index of refraction of a certain kind of glass is 1.66. Determine its critical angle of incidence.

623. Find the distance that a fish is below the surface of still water when, to an observer in air, above the fish, the distance seems to be 4 ft. (Index of refraction of water relative to air is 1.33.)

624. The wave-length of a certain radiation from sodium vapor in free ether is 0.00005896 cm. What is the wave-length of the same radiation in glass whose index of refraction is 1.66?

SINGLE LENS

625. A camera which is used for objects from 10 to 100 ft. distant has a lens of 8 in. focal length. What must be the range of the bellows?

626. What should be the focal length of a lens placed 8.5 in. from the sensitive plate in order to photograph an object 3 ft. distant?

627. What focal length lens must be used in a camera in order to take a 10 cm. picture of a 20 m. building at a distance of 30 m.?

628. In the preceding problem what would be the size of the picture if a lens of 10 cm. focal length were used? 20 cm. focal length?

629. A tree 5 m. high and 40 m. distant is photographed by means of a camera provided with an objective of 20 cm. focal length. What will be the height of the image?

630. What is the largest size object at a distance of 30 ft. that can be photographed on a 4×5 in. plate with a lens of 7 in. focal length? What would be the effect of using a lens of larger diameter and the same focal length?

631. The focal length of a certain lens is 10 cm. and the index of refraction with reference to air, of the glass from which it is made, is $\frac{3}{2}$. Determine the focal length of the same lens immersed in water whose index of refraction with respect to air is $\frac{4}{3}$.

632. What must be the focal length of a camera lens to take a picture on a 4×5-in. plate of an object 60 ft. square at a distance of 50 ft.?

633. A camera of 8 in. focal length is focalized on a distant object. How much must the bellows be lengthened to make a picture of the same size as a small object?

634. A convex lens of 20 cm. focal length is placed 15 cm. from an object. Where will the image be formed? Will it be real or virtual? Illustrate by diagram.

635. A distinct image of an object placed 12 in. from a convex lens is projected on a screen 4 in. from the lens. At what distance from the lens would the image be formed if the object were placed 6 in. farther away from the lens?

636. The lens in a camera has a focal length of 15 in. How far from the lens must an object be placed in order that a clearly defined image of it may be thrown on a sensitive plate which is 16 in. from the lens.

637. A sharp image is formed on the ground glass of a camera when the object is 500 ft. distant from the camera and the ground glass is 6 in. from the lens. What must be the distance between the ground glass and the lens for a sharp image to be formed of an object 20 ft. distant?

638. The focal length of a convex lens is 15 cm. An object 2 cm. long is situated 10 cm. from the lens. (*a*) Is the image real or imaginary? (*b*) How far is the image from the lens? (*c*) What is the size of the image?

639. The image of an object 5 cm. long, placed 20 cm. from a lens, is found to be inverted at a distance of 15 cm. from the lens. Is the image real or virtual? Is this lens convex or concave? Compute the focal length of the lens and the length of the image.

640. It is desired to focalize the image of an electric arc on a screen 12 ft. from the light and the image is to be three times as large as the object. State the kind and find the focal length of a lens which will accomplish the result.

641. The principal focal length of the objective of the great telescope of the Lick Observatory is 1500 cm. Find the focal length of an eyepiece that will make the instrument have a magnifying power of 500 when the observer's eye is accommodated for most easy vision.

642. The objective of the great telescope of the Lick Observatory is 1500 cm. in focal length. The sun subtends at the earth an angle of half a degree. Find the linear diameter of the image of the sun formed by light that has traversed the Lick telescope objective.

643. When a certain camera is focalized on a very distant object, it is found that the plate has to be placed 9 in. from the lens. (*a*) Must the distance between the plate and the lens be increased or decreased, and by how much, to give a sharp image of an object 6 ft. from the lens? (*b*) If the object, which is 6 ft. away, is 14 in. high, what will be the height of the picture?

644. The focal length of a camera lens is 20 cm. How far from the lens must a sensitized plate be placed when the object is 200 cm. from the lens? Make a diagram locating positions of object, image, and lens.

RAY DIAGRAMS FOR LENSES AND LENS SYSTEMS

Directions for Graphical Constructions.—All diagrams are to be carefully constructed with instruments and the various points and distances clearly indicated.

Unless otherwise required, light is to proceed from left to right, and the individual lenses constituting a system are to be numbered consecutively. For the first lens, the principal foci, the equivalent points, the principal focal length, the distance between object and lens, and the distance between lens and image, are to be represented by the symbols F_1 F'_1, E_1 E'_1, f_1, u_1 and v_1, respectively. For the second lens, the same quantities are to be represented by F_2 F'_2, E_2 E'_2, f_2, u_2 and v_2, respectively. Similarly for the succeeding lenses of the system.

The equivalent points and principal foci of a lens system are to be represented by the symbols E_s E'_s and F_s F'_s, respectively.

Unless otherwise directed, for diagrams on paper make the distance between the equivalent planes of each lens three-sixteenths inch, and for diagrams on the blackboard make the distance two inches.

Represent an object by a heavy arrow, an aerial object by a heavy dotted arrow, a real image by a light arrow, and a virtual image by a light dotted arrow.

When an instrument is focalized for most easy vision, all rays from a point source will emerge from the eye lens parallel to one another. In the following problems when an instrument is said to be focalized, it is to be understood that it is focalized for most easy vision.

645. Knowing the equivalent planes and the principal focal length of a positive lens, trace through the lens two parallel rays inclined to the axis. Also, trace through the lens two rays from a point beyond the principal focus. Also, trace two rays from a point nearer the lens than the principal focus.

646. Knowing the equivalent planes and the principal focal length of a negative lens, trace through the lens two parallel rays inclined to the axis. Also, trace through the lens two rays from a point beyond the principal focus. Also, trace two rays from a point nearer the lens than the principal focus.

647. A certain microscopic eyepiece consists of a field lens of $f_1 = 2\frac{1}{4}$ in. and an eye lens of $f_2 = 1\frac{3}{8}$ in., the distance between the lenses being $2\frac{1}{8}$ in. Locate graphically the equivalent points and the principal foci of the combination. (Draw to scale. The drawing will be about 10 in. long.)

648. The principal focal lengths of the objective and of the eye lens of a certain microscope are 1 and 2½ in. respectively; the distance between the lenses is 5¾ in. From a point ⅛ in. above the principal axis and 1¼ in. in front of the objective, trace two rays through the system.

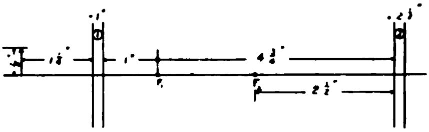

FIG. 592.

649. Through a Ramsden eyepiece trace two rays incident on the field lens parallel to the principal axis. Locate graphically the principal foci of the system.

650. The components of a certain Ramsden eyepiece have a principal focal length of 6 in. Determine graphically the equivalent points and also the principal focal length of the combination.

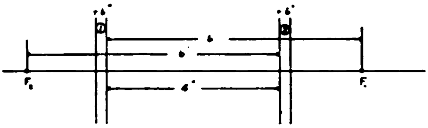

FIG. 593.

651. The principal focal length of the field lens and of the eye-lens of a certain Huyghens eyepiece are 3 in. and 1 in. respectively. Determine graphically the equivalent points and also the principal focal length of the combination.

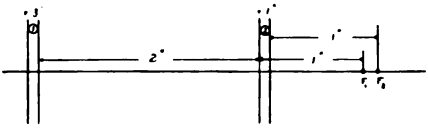

FIG. 594.

652. The principal focal lengths of the objective and the eye-lens of a certain opera glass are 20 cm. and 5 cm., respectively. Draw a diagram giving dimensions, showing how the lenses must be placed so that rays from a star will emerge parallel. Trace through the combination two rays that are parallel to one another and inclined to the principal axis of the system.

653. The principal focal lengths of the objective and of the eye-lens of a certain Galilean telescope are 2 and ⅝ in. respectively; the distance between the lenses is 2⅜ in. From a point 1 in. above the principal axis and 6 in. in front of the objective, trace two rays through the system.

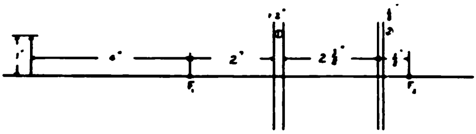

FIG. 505.

654. Construct a ray diagram of a simple astronomical telescope focalized for most easy vision. On the diagram indicate the limit of the field of view.

655. The principal focal lengths of the objective and of the eye-lens of a certain simple astronomical telescope are 2 and ⅝ in., respectively; the distance between the lenses is 3⅝ in. From a point 1 in. above the principal axis and 6 in. in front of the objective, trace two rays through the system.

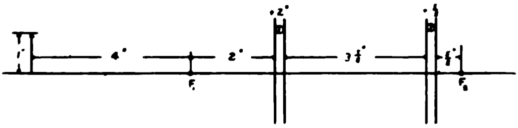

FIG. 596.

656. In a two-lens astronomical telescope, when focalized for a plane wave, the positive objective and the positive ocular are separated by a distance equal to the sum of their focal lengths. Assuming the position of the equivalent points and principal foci of objective and ocular, trace rays of light through such a telescope showing the position of the image of an object at an infinite distance.

657. Convex lenses of principal focal lengths 25 cm. and 1.5 cm., respectively, are to be used in the construction of a simple astronomical telescope. Draw a diagram, giving dimensions, which will show the relative positions of these lenses and the cross wires when the telescope is focalized for infinity. Also diagram the passage of a pencil of parallel rays through the lenses and calculate the magnifying power of the telescope.

658. Diagram a simple astronomical telescope focalized for an object at a finite distance, and trace through the telescope two rays from a point on the object. Show the position that cross-wires would need to be placed. Show the position that a stop would need to be placed to diminish the field of view the least amount. What is the length of this telescope when focalized for an object at infinite distance?

659. Construct a ray diagram of a Huyghens eyepiece focalized for most easy vision.

660. The principal focal lengths of the individual lenses of an astronomical telescope with Huyghens eyepiece are, beginning at the objective, 2, $1\frac{7}{8}$, and $\frac{5}{8}$ in., respectively; the distances between the lenses, beginning at the objective, are $2\frac{1}{16}$ and $1\frac{1}{4}$ in., respectively. From a point 1 in. above the principal axis and 6 in. in front of the objective, trace two rays through the system.

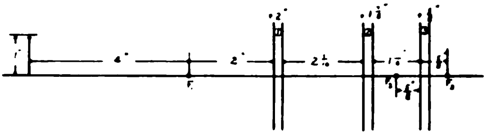

FIG. 597.

661. Through a Huyghens eyepiece in which the principal focal length of the field lens is three times that of the eye lens, trace two rays that are parallel on emergence. Find analytically the position of the field lens with respect to the aerial object due to light that has traversed an objective lens.

662. Through a Huyghens eyepiece in which the principal focal length of the field lens is two times that of the eye lens, trace two rays that are parallel on emergence. Find analytically the position of the field lens with respect to the aerial object due to light that has traversed an objective lens.

663. Diagram an astronomical telescope provided with a Huyghens eyepiece when focalized for an object at a finite distance, and trace through the telescope two rays from a point on the object. Show the position that cross-wires would need to be placed. Show the position that a stop would need to be placed to diminish the field of view the least amount. What is the length of the telescope when focalized on an object at infinite distance?

664. The principal focal lengths of the individual lenses of an astronomical telescope with Ramsden's eyepiece are, beginning at the objec-

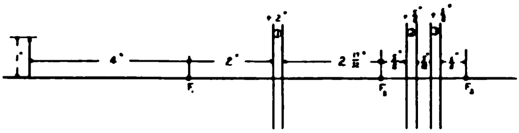

FIG. 598.

tive, 2, $\frac{5}{8}$ and $\frac{5}{8}$ in., respectively; the distances between the lenses, beginning at the objective, are $3\frac{5}{32}$ and $\frac{5}{12}$ in. From a point 1 in. above the principal axis and 6 in. in front of the objective, trace two rays through the system.

665. Diagram an astronomical telescope provided with a Ramsden eyepiece when focalized for an object at a finite distance, and trace through the telescope two rays from a point on the object. Show the position that cross-wires would need to be placed. Show the position that a stop would need to be placed in order to diminish the field of view the least amount. What is the length of this telescope when focalized for an object at infinite distance?

666. A simple astronomical telescope having an objective of principal focal length 30 cm. is to be provided with a Ramsden eyepiece, the field lens of which is of principal focal length 6 cm. What must be the principal focal length of the eye lens? Draw a diagrammatic sketch giving dimensions, which will show the relative positions of lenses, cross-wires, rays and images for a distant object.

667. An astronomical telescope, the objective of which has a focal length of 25 cm., is to be provided with a Ramsden eyepiece. A lens of focal length 5 cm. is available for use as the field lens. What must be the focal length of a lens that could be employed as the eye-lens? Draw a diagram showing the position of lenses and cross-wires. Also trace through the telescope parallel rays incident on the objective in a direction inclined to the principal axis at an angle of about 10°.

668. Dray a ray diagram of a four-lens erecting eyepiece for most easy vision.

669. The principal focal lengths of the individual lenses of a terrestrial telescope with a Huyghens eyepiece are, beginning at the objective, 2, 2, 2, $1\frac{7}{8}$ and $\frac{5}{8}$ in., respectively. The distance between the lenses, beginning at the objective, are 5, 2, $1\frac{3}{32}$ and $1\frac{1}{4}$ in. respectively. From a point 1 in. above the principal axis and 6 in. in front of the objective, trace two rays through the system. (If drawn full scale, the diagram will be about 18 in. long.)

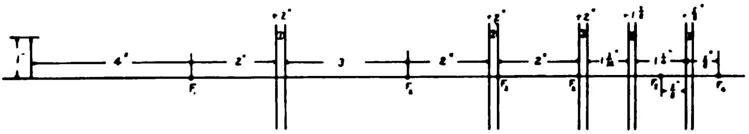

FIG. 599.

670. The principal focal lengths of the individual lenses of a terrestrial telescope with Ramsden's eyepiece are, beginning at the objective, 2, 2, 2, $\frac{5}{8}$, and $\frac{5}{8}$ in., respectively; the distance between the lenses, beginning at the objective, are 5, 2, $2\frac{5}{12}$ and $\frac{5}{12}$ in., respectively. From a point 1 in. above the principal axis and 6 in. in front of the objective, trace two rays through the system. (If drawn full scale, the diagram will be a out 18 in. long.)

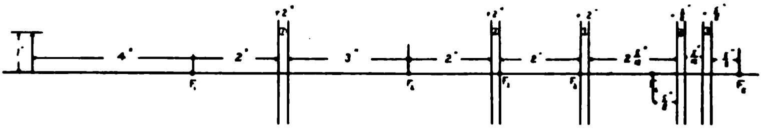

FIG. 600.

671. Diagram a terrestrial telescope consisting of an objective, a pair of erecting lenses and a simple eye lens when focalized for an object at a finite distance. Trace through the system two rays of light from a point on the object. Show the position of the cross hairs, and the position that a stop would need to be placed to reduce the field of view by the least amount.

672. Diagram a terrestrial telescope consisting of an objective, a pair of erecting lenses and a Ramsden eyepiece, when focalized for an object at a finite distance. Trace through the system two rays of light from a point on the object. Show the position of the cross-hairs, and the position that a stop would need to be placed to reduce the field of view by the least amount.

673. Diagram a terrestrial telescope consisting of an objective, a pair of erecting lenses and a Huyghens eyepiece (3 to 1 combination), when focalized on an object at a finite distance. Trace through the system two rays of light from a point on the object. Show the position of the cross-hairs, and the position that a stop would be placed to reduce the field of view by the least amount.

674. Diagram a terrestrial telescope consisting of an objective, a pair of erecting lenses and a Huyghens eyepiece (2 to 1 combination), when focalized on an object at a finite distance. Trace through the system two rays of light from a point on the object. Show the position of cross-hairs, and the position that a stop would need to be placed to reduce the field view by the least amount.

OPTICAL INSTRUMENTS

675. A certain near-sighted person can see distinctly an object when it is not more than 15 cm. from the eye. Find the power, in diopters, of a spectacle lens which will enable him to see distinctly an object 300 cm. from the eye.

676. A certain far-sighted person can see distinctly an object when it is not less than 100 cm. from the eye. Find the power, in diopters, of a spectacle lens which will enable him to see distinctly an object 25 cm. from the eye.

677. If a photographic print can be made in 30 sec. when held 3 ft. from a 32-candle-power lamp, how long will it take at a distance of 2 ft. from a 16-candle-power lamp?

678. If with a stop of *f*: 11.3 the time of exposure is 0.01 sec., what will be the time of exposure for a stop of *f*: 32?

679. If a certain plate requires an exposure of 0.02 sec., at stop *f* : 16 what would be the time of exposure under the same light conditions for (*a*) *f* : 32, (*b*) *U.S.* 32?

680. For existing light conditions an exposure table gives 0.04 sec., as the time of exposure at stop *f* : 32. The object is moving so that to prevent blurring the exposure must not exceed 0.01 sec. Find the required stop in the *f* system and in the uniform system.

681. Assume a Galilean telescope composed of two symmetrical lenses of principal focal lengths 25 and 5 cm., respectively, and of thickness 1.8 and 0.9, respectively. Find the distance between the center of these lenses when the telescope is focalized on an object distant 150.3 cm. from the center of the objective. Sketch the optical system indicating the various distances.

682. The principal focal lengths of the objective and of the eyelens of a simple astronomical telescope are 25 and 5 cm., respectively. The lenses are symmetrical double convex and of thickness 1.8 and 0.9 cm., respectively. Find analytically the relative position of the lenses and cross-wires when the instrument is focalized for a distance of 100.3 cm. from the center of the objective. Sketch the optical system indicating the various distances.

683. An astronomical telescope has an objective of principal focal length 35 cm. and thickness 2.4 cm., and a Huyghens eyepiece the eyelens of which has a principal focal length of 6 cm. and thickness of 0.9 cm., and the field lens of which has a principal focal length of 18 cm. and thickness 1.5 cm. Compute the distance between the centers of the lenses when the telescope is focalized for an object 200.4 cm. from the center of the objective.

684. An astronomical telescope has an objective of principal focal length 30 cm. and thickness 2.4 cm., and a Ramsden eyepiece each lens of which has a principal focal length of 5 cm. and thickness of 0.9 cm. Find analytically the distance between the center of the objective and the field lens when the telescope is focalized for an object 300.4 cm. from the center of the objective.

685. A telescope has an objective of $23\frac{3}{8}$ in. principal focal length, and an erecting eyepiece composed of four lenses, as follows, reading toward the eye-lens; 2, $1\frac{7}{8}$, $1\frac{7}{8}$, and $1\frac{3}{8}$ in., with separations of $2\frac{1}{4}$, 4, and 2 in. Compute the distance from the objective to the first lens of the eyepiece, and trace through the system two rays from a point at infinity when the telescope is focalized for most easy vision.

Suggestion:—As the eyepiece moves as a unit relative to the objective, find the focus of the eyepiece analytically, beginning at the eye-lens. The drawing will be about 12 in. long if drawn to one-quarter scale.

686. A certain terrestrial telescope comprises five positive lenses of principal focal lengths 30, 15, 15, 9 and 3 cm. respectively, and of thickness 1.8, 1.5, 1.5, 1.2, and 0.6, respectively. Compute the distance from the objective to each of the other lenses, and to the real image, when the instrument is focalized for an object distant 500 cm. from the objective. Construct a diagram (not necessarily to scale) of the optical system with distances indicated.

687. The principal focal lengths of the lenses of a certain telescope, beginning with the objective, are 48 cm., 3.8 cm., 4.75 cm., 4.6 cm., 3.0 cm., respectively. The distances between the second and third lens is 5.8 cm., between the third and fourth is 10.2 cm., and between the fourth and fifth is 5 cm. Compute the distance between the first and second lens when the instrument is focalized for most easy vision on an object at infinity. Trace through the instrument two rays from a point at infinity when the instrument is adjusted for most easy vision.

Suggestion:—As the eyepiece moves as a unit relative to the objective, find the focus of the eyepiece analytically, beginning at the eye-lens. The drawing will be about 20 in. long if drawn to one-half scale.

ANSWERS

2. 250 lb. wt.
4. 2000 lb. wt.
5. 300 ft.-lb.
6. 61.5 per cent.
7. 12 ft.
8. 17,400,000 ft.-lb.
9. 720,000 ft.-lb.
10. 109,800,000 ft.-lb.
11. 119,500,000 ft.-lb.
12. 625,000 ft.-lb.
13. 12,124 ft.-lb.
14. 2165 ft.-lb.
15. (*a*) 20,000 ft.-lb.; (*b*) 80,000 ft.-lb.
16. (*a*) 20,000 ft.-lb.; (*b*) 6000 ft.-lb.
17. (*a*) 8; (*b*) 7.1; (*c*) 0.89.
18. 3600 lb. wt.
19. 6 lb. wt.; 125 ft.
20. 19.8 kg. wt.
21. 81° 50′.
22. 37.5 lb. wt.
23. $\frac{1}{4}$ the length of the beam.
24. 31.25 cm. from the end.
25. 70 lb. wt.; 30 lb. wt.
26. 3 ft.
27. 150 lb. wt.; 250 lb. wt.
28. 30 kg. wt.
29. $2\frac{2}{3}$ ft. from B; 600 lb. wt.
30. 150 lb. wt.; 3.7 ft.
31. 150 lb. wt.
32. 4.8 ft. from B.
33. 18 ft. from overhanging end.
34. 97.5 lb. wt.
35. (*a*) $8\frac{1}{3}$ lb. wt. upward; (*b*) 50 lb. wt. upward at the center.
36. 3 ft. from A.
37. (*a*) 2 ft.; (*b*) $7\frac{1}{3}$ ft.
38. 8 lb. wt.; 16 lb. wt.
39. $96\frac{2}{3}$ lb. wt.; $73\frac{1}{3}$ lb. wt.
40. 54 kg. wt.
41. 30.4 kg. wt.; 42.4 kg. wt.
42. 1 ft.
43. 0.33 in. from junction.
44. 128 lb. wt.; 132 lb. wt.
45. 95.3 lb. wt.; about 27°.
46. 1500 lb. wt.
47. $2\sqrt{3}$ lb. wt. toward center of triangle.
48. 1500 lb. wt.
49. 76.9 lb. wt.
50. 10 lb. wt.
51. 3.06 lb. wt.
52. 181 lb. wt.
53. 115 lb. wt.
54. 31.9 lb. wt.
55. 43.3 lb. wt.
56. 800 lb. wt. up the incline.
57. 0.8.
58. 14 ft.
59. 231 kg. wt.; 115 kg. wt.
60. 14.4 lb. wt.; 12.9 kg. wt.
61. 82.7 lb. wt.; 117 lb. wt.
62. 144 kg. wt.; 72.1 kg. wt.
63. 75 kg. wt.; 53 kg. wt.; 106 kg. wt.; 53 kg. wt.
64. 75 kg. wt.
65. 22.5 lb. wt.; 22.5 lb. wt.
66. 62 lb. wt.
67. 78.1 lb. wt.; 46.9 lb. wt.

68. 31.2 kg. wt.; 18.7 kg. wt.; 25 kg. wt.
69. (*a*) 368 lb. wt.; (*b*) 260 lb. wt.; (*c*) 100 lb. wt.
70. (*a*) 1572 lb. wt.; (*b*) 1572 lb. wt. 11°; (*c*) 1543 lb. wt., 300 lb. wt.
71. 29 kg. wt.; 10.5 kg. wt.; 25.1 kg. wt.
72. 130 lb. wt.; 150 lb. wt.; 130 lb. wt.
73. 14 lb. wt.; 100 lb. wt.; 14 lb. wt.
74. 2.8 lb. wt.; 2.8 lb. wt.
75. 975 lb. wt.; 562 lb. wt.; 844 lb. wt.
76. (*a*) 8100 lb. wt.; (*b*) 6525 lb. wt.
77. 200 lb. wt.
78. 404 kg.-m.
79. 25 lb. wt.; 43 lb. wt.
80. 60 kg. wt.; 17.3 kg. wt.; 0.288.
81. 45 kg. wt.; 0.5.
82. 13,300 lb. wt.
83. 3840 lb. wt.
84. 92 lb. wt.
85. 2 triple pulley blocks; 355 lb. wt.
86. 5.9 sec.
87. 968 ft.
88. 101 ft.; 80.5 ft. per sec.
89. 0.002 sec.; 375,000 m. per sec. per sec.
90. 0.14 sec.
91. 0.05 sec.
92. (*a*) 1176 cm. per sec.; (*b*) 176.4 m.
93. 62,200 lb. wt.
94. 20 ft. per sec.; 100 ft.
95. 2,940,000 dynes.
96. 32 lb.
97. 1558 ft.
98. 8 ft. per sec.
99. 12.5 sec.
100. 1200 lb. wt.
101. 25 tons wt.
102. 3 tons wt.
103. 8° 40′.
104. 196 lb. wt.
105. 9156 lb. wt.
106. 3.8 tons wt.; 13.9 tons wt.
107. 914 lb. wt.; 1554 lb. wt.; 640 lb. wt.
108. 108,000 lb. wt.
109. (*a*) 1.2 sec.; (*b*) 565 cm. per sec.
110. 8 ft. per sec. per sec. downward.
111. 20.8 lb. wt.
112. 8 ft. per sec. per sec. downward.
113. 8 ft. per sec. per sec. upward; 36 ft.
114. 52 kg. wt.; 98 kg. wt.
115. 115 lb. wt.; 0.
116. 5.6 ft. per sec. per sec.
117. 3000 kg. wt.; 6061 kg. wt.
118. (*a*) 140 cm. per sec. per sec.; (*b*) 136 (10^3) dynes.
119. (*a*) 1.5 ft. per sec. per sec.; (*b*) 10.5 lb. wt.
120. 329 g.; 314 g. wt.
121. 6.29 tons wt.; 419 tons wt.
122. 300 lb. wt.; 0.015.
123. 127 ft.
124. 3280 g. wt.
125. 0.09 radians per sec.
126. 1270 lb. wt.
127. 24.8 cm.
128. 18.7 lb. wt.
129. 1.79 radians per sec.
130. 18.4 lb. wt.
131. 18 ft. per sec.
132. 2 sec.
133. 60.5 tons wt.; 7°.
134. 3.8 in.
135. 4°.
136. 968 lb. wt.; 25° 51′.
137. 10.5 radians per sec.
138. 80 cm. per sec.; 13° 36′ 27″ east of north.
139. 15,000 lb. wt.
140. 15 mi. per hr.
141. 1 H.P.

142. 77 H.P.
143. 1.44 H.P.
144. 88 gals.
145. 0.28 ft.
146. 75 lb. wt.
147. 496,300 ft.-lbs.; 1 H.P.
148. 2200 lb. wt.
149. 62,500 lb. wt.
150. 1320 lb.
151. 0.04 H.P.
152. 1.88 kg.-m. per sec.; 0.025 H.P.; 18.4 watts.
153. 249,000 ft.-lb.; 1.5 H.P.
154. 198,000 lb.
155. 100 lb. per sec.
156. 1,600,000 ft.-lb.; 0.2 H.P.
157. 2.9 H.P.
158. 49.5 H.P.
159. 21,000 ft.-lb.; 7.6 H.P.
160. 1705 H.P.
161. 6.8 (10^6) H.P.
162. 12,960 ft.-lb.; 118 H.P.
163. 861 kg. wt.
164. 4088 ft.-lb.
165. 0.17 H.P.
166. 3.75 lb. wt.
167. 2,412,000 ft.-lb.; 4825 lb. wt.
168. (*a*) 12,500 ft.-lb; (*b*) 1300 ft.
169. 312 ft.
170. 102 H.P.
171. (*a*) 64 H.P.; (*b*) 224 H.P.; (*c*) 384 H.P.
172. 23 H.P.
173. 0.7 H.P.
174. 0.2 H.P.
175. 15 H.P.
176. 0.14 H.P.
177. 100 H.P.
178. (*a*) 5.3 H.P.; (*b*) 10.6 H.P.
179. 2100 lb.-ft.
180. 186 ft.-lb. per min.
181. (*a*) 720,000 ft.-lb. per min.; (*b*) 22 H.P.
182. 15,360 kg. wt.
183. 480 lb. wt.
184. 12,800 lb. wt.
185. (*a*) 6910 g. wt.; (*b*) 12,400 g. wt.
186. 3180 lb. wt.
187. 429 ft.
188. 2.8 mm.
189. 16.6 cm.
190. 62,500 lb. wt.
191. (*a*) 20.6 mm.; (*b*) 38.9 mm.
192. 3200 lb. wt. per sq. ft.
193. 17.3 lb. wt. per sq. in.
194. 10,080 lb. wt.
195. 8 to 5.
196. 8064 lb. wt.; 0.1 in
197. 0.49 in.
198. 20.1 m.
199. $\frac{3}{7}$ of volume of sphere.
200. 0.8; 413 lb. wt.
201. 0.228 g.
202. 1.12.
203. 180 lb. wt.
204. 10 g.
205. 215 lb. wt.
206. 1210 kg. wt.
207. 5 cc.
208. 20 g. wt.; 0.13.
209. 0.55.
210. 6.4 oz. wt.
211. 36.4 cc.; 7.56 g. per cc.
212. 21.1 g.
213. 10.5 cc.
214. 6440 cu. yd.
215. 1.4.
216. 1030 g.
217. 133,000 cu. ft.
218. 2.8 g. per cc.
219. 0.11.
220. 194 kg. wt.
221. 260 kg. wt.
222. 70.2 g., 6.6 g.
223. 1315 lb.
224. 784 joules.
225. 168 kg. m.

226. (*a*) 3183 lb. per sq. in.; (*b*) 0.001 in.
227. 1.016 (10^{10}) dynes per sq. cm.
228. 0.0245 sq. cm.
229. 0.098 cm.
230. 50,000 lb. per sq. in.
231. 4.18 g. per cc.
232. 3.61 g. per cc.
233. 1.003.
234. 58.8 lb. per sq. in.
235. 44.7 lb. per sq. in.; 1010 cu. in.
236. 2813 ft.-lb.
237. 700 gal.; 30 lb. per sq. in.
238. 3 in from the valve.
239. 9 to 1.
240. 306 ft.
241. 15.9 m.
242. 0.54 cents.
243. 5 atmos.
244. (*a*) 633 cu. m.; (*b*) 500 kg. wt.; 400 kg. wt.
245. 44.4 in. per sec.
246. 3.8 sec.
247. 19.4 ft. per sec.
248. 3084 ft.-lb.
249. 985 cm. per sec. per sec.
250. 99.4 cm.
251. 979 cm. per sec. per sec.
252. 45 vib.
253. 4.43 ft.
254. 3380 ft.
255. (*a*) 3260 ft.; (*b*) 16,300 ft. per sec.
256. 120 cm.
257. 1540 ft. from nearer wall.
258. 2750 ft.
259. 7228 ft.
260. (*a*) 640 vib. per sec.; (*b*) 160 cm.
261. 1410 vib. per sec.
262. 4.7 sec.
263. 2.12 mi.
264. 133 cm.
265. 1078 vib. per sec.
266. 138 vib. per sec.
267. 285 vib. per sec.
268. 280 vib. per sec.
269. 85 vib .per sec.
270. 75 vib. per sec.
271. (*a*) 136 vib. per sec.; (*b*) 68 vib. per sec.
272. (*a*) 70.4 vib. per sec.; (*b*) 35.2 vib. per sec.
273. (*a*) 15; (*b*) 16.
274. 18° C.; 39° C.
275. 39.2° F.
276. −40°.
277. 320° F.
278. 172°; 675°; 95°; 640°.
279. 320°; 1021°; 412°; 1041°; 467°; 1530°.
280. 44.8 (10^6) cal.
281. 1 to 0.452.
282. 0.03 to 1.
283. 10.4 min.
284. 2125° C.
285. 0.55° C.
286. 3263 l.
287. 0.41 g.
288. 5.56 gal. hot; 14.44 gal. cold.
289. 135°.
290. 0.094.
291. 0.425.
292. 60 g.
293. 21°.
294. 30.3 tons.
295. 30.8 g.
296. 14 cal. per g.
297. 264 kg.
298. 50.6 B.t.u. per lb.
299. 80.2 cal. per g.
300. 0.11.
301. 0.505.
302. 13.4°.
303. 38.5 g.
304. 0.77.
305. 1102 g.
306. 1,198,000 cal.
307. 75.7°.

308. 244,490 cal. per lb.; 970 B.t.u. per lb.
309. 37.1 g.
310. 1.38 g.
311. 6.26 lb.
312. 13.62 g.; 100°.
313. 2375 kg.
314. 0.028 lb.
315. 1,170,000 B.t.u.
316. $9.07.
317. (*a*) $7.58; (*b*) $13.10; (*c*) $5.93; (*d*) $.21.
318. 195° C.
319. 8.25 cm.
320. 0.0000113; 26.8°.
321. 1.22 ft.
322. 0.018 ft.
323. 0.06 ft.
324. 37 cm.
325. 402°.
326. 50.023 ft.
327. 0.025 ft.
328. 0.014 in.
329. 196.5°; 251.4 cm.
330. 6.424 m.
331. 76.43 cm.
332. 20.5 sec.
333. 10.4 g. per cc.
334. 3470 gal.
335. 76.9 cm.
336. 151.16 cc.
337. 100.24 cc.
338. 0.0144 sq. ft.
339. 40,580 dynes per sq. cm.
340. 7 liters.
341. 307°.
342. 0.0019 g.
343. 12.9 g.
344. 327° C.
345. 80.1 cm. of mercury.
346. 37.5 gal.
347. 93.5 cm. of mercury; 3180 g. wt.
348. 65.7 ft.
349. 1.66 v.
350. 91 lb. per sq. in.
351. to 57.2°.
352. 4627 cc.
353. 2.93 l.
354. 145.5°.
355. 0.00366.
356. 0.947 g.
357. 341 cc.
358. 7.3 atmos.
359. 925 kg.
360. 1 to 1.02
361. 91°.
362. 1 to 0.57.
363. 7290 cc.
364. 719 g.
365. Final volume = ¾ initial volume.
366. 31.5 to 1.
367. $640.60.
368. Profit 7 per cent.
369. 20 sec.
370. 3870 kg.
371. 5.2 lb.
372. 84.4 (10^6) cal.
373. 22,500 cal.
374. 3 (10^6) cal.
375. 42 (10^6) cal. per hr.; 5 kg. per hr.
376. (*a*) 102,000 B.t.u. per hr.; (*b*) $70.60.
377. 23.4° F.
378. 117° F.
379. 206° F.
380. 6100 cal.
381. 7.05 B.t.u.; 0.166 H.P.
382. 1.55°.
383. 3.6° C.
384. 353 m. per sec.
385. 0.197 cal.; 0.6° C.
386. 13.5 H.P.
387. (*a*) 0.738 ft.-lb.; (*b*) 0.00095 B.t.u.; (*c*) 0.238 cal.
388. 57° F.
389. 0.6 H.P.
390. 1770 g.
391. 5° F.

392. 656 m.
393. 51 B.t.u.
394. 202,300 ft.
395. 55.6 H.P.; 26.5 lb.
396. 857,000 cal.; 3400 B.t.u.
397. (a) 2545 B.t.u. per hr.; (b) 641,000 cal. per hr.
398. (a) 1.34 H.P.; (b) 3410 B.t.u. per hr.
399. 85,900 cal.
400. Electricity costs 11.7 times as much as gas.
401. (a) 4.7 cents; (b) 24.3 cents.
402. Coal, $0.04; gas, $0.30; electricity, $1.18.
403. 26%.
404. 6¼%.
405. 19%; 4¼%; 4¼.
406. 333 H.P.; 21%; 4.3%; 4.8.
407. 421 H.P.; 22%; 4.6%; 4.8.
408. 490 H.P.; 22%; 4.7%; 4.7.
409. 1.6 ohms.
410. 0.00000159 ohm per cm. cube.
411. 0.000037 ohm.
412. 4.9 ohms.
413. 99 ft.
414. 857 m.
415. 19.8 (10^6) m.
416. 5.64 km.
417. 136 yds.
418. 5.41 ohms.
419. 2.19 m.
420. 2445 ohms.
421. 479 ohms.
422. 36.8 ohms; 0.26 mil.
423. 1000 ohms; 1212 ohms.
424. −256° C.
425. 77 ohms.
426. 656° C.
427. 0.0036 ohm per ° C.
428. 0.22 ohm.
429. 78° C.
430. 5714 cal.
431. 25.2 ohms; 6.3 watts.
432. 0.8 H.P.
433. 0.39 cm.
434. 25.4° C.
435. 0.91 amp.
436. 14.5 amp.
437. 63 to 100.
438. 131° C.
439. 85.5%.
440. 85,900 cal.
441. 20,000 cal.
442. 115 amp.; 656 kw.
443. 900 joules; 216 cal. per min.
444. 3600 joules; 859 cal.; 2655 ft.-lb.; 3.4 B.t.u.
445. 5.5 amp.
446. 210,000 ohms.
447. 0.072 amp.; 0.046 amp.
448. 61.7 ohms.
449. 0.6 ohm.
450. 3160 ohms.
451. 11 ohms.
452. 0.375 amp.
453. 17 cells.
454. 0.95 ohm.
455. 2.4 ohms.
456. 7 cells.
457. (a) 17.8 ohms; (b) 6.67 ohms.
458. 1.25 ohms.
459. 60 ohms; 28.9 ohms.
460. 0.80 volt; 0.85 volt.
461. (a) 0.00147 amp.; (b) 78 volts.
462. (a) 0.00275 amp.; (b) 68.7 volts; 41.3 volts.
463. 1.68 volts.
464. 77.6 volts.
465. 25 ohms.
466. 550 volts; 549 volts.
467. 0.474 volts.
468. 20.8 volts.
469. 2.55 mi.
470. 1008 ohms; 600 coulombs per min.; 10,080 volts.
471. 126 volts.
472. 52 cells.

473. 93 volts.
474. 16.8 ohms.
475. (*a*) 10 volts; (*b*) 714 cal.
476. 1440 flashes.
477. 707 cal.
478. (*a*) 204 volts; (*b*) 188 volts.
479. 639 cal.
480. 222 volts.
481. 60 volts.
482. 2.81 watts per c.p.
483. 218 lamps.
484. 600 watts; $0.036.
485. (*a*) 7500 watts; (*b*) 10 H.P.
486. 84.8%.
487. 93%.
488. 82.2%.
489. 82.2%.
490. 75.4 amp.
491. 15.7 cal.
492. 251 m.
493. (*a*) 0.66 kw.-hr.; 566,000 cal.
494. $5.28.
495. 6 cents.
496. 4.4 cents.
497. (*a*) 0.81 kw.; (*b*) 1.1 H.P.
498. (*a*) 220 ohms; (*b*) 36 lamps.
499. (*a*) 122.5 kw.; (*b*) 98,000 joules per sec.
500. 0.357 amp.; 314 ohms; 0.32 ct.
501. 10 watts per sec.; 3 to 1.
502. 10,000 to 1.
503. 72 lamps; 1.12 amp.
504. 9219 cal.
505. (*a*) 40 amp.; (*b*) 20 volts; (*c*) 120 volts.
506. (*a*) 35 amp.; (*b*) 490 volts; (*c*) 17,150 watts; (*d*) 89.4%.
507. 98.5%.
508. 72%; 1.37 H.P.
509. 85%.
510. $791.77.
511. (*a*) 20.66; (*b*) $18.27.
512. 1.41 amp.
513. 53.4 amp.
514. 58.8 kw.; 20.3 kw.
515. 95.4 H.P.
516. Generator, 2 watts; line, 50 watts; charging cells, 705 watts.
517. 604 mils.
518. 61 lb. per hr.
519. 53.5 cents.
520. 2.18 cents; 208,000 cal.
521. (*a*) 1 to 4; 2.5 min.
522. 655 kw.; 114 amp.
523. (*a*) 1.19 ohms; (*b*) 14.0 ohms; (*c*) 7.79 ohms; (*d*) 140 ohms.
524. 34.7 ohms.
525. (*a*) 550 volts; (*b*) 2.5 amp.
526. (*a*) 0.33 amp.; (*b*) 1.5 amp.
527. (*a*) 2.5 amp.; (*b*) 5.5 watts.
528. 8 cents.
529. 32.4 H.P.
530. 5000 amp.; 550 kw.; 737 H.P.
531. 0.234 l.
532. 20 ohms; 5 amp. in 36-ohm wire; 4 amp. in 45-ohm wire.
533. 110, 73.3, 55.0, 44.0 ohms; 1.0, 1.5, 2.0, 2.5 amp.
534. 3.8° C.
535. 4.6 ohms.
536. 3.43 ohms; 6.64 volts.
537. 1.25 ohms; 8 amp.
538. 3.5 amp.; 0.833 amp.; 0.667 amp.; 2 amp.
539. (*a*) 0.55 amp.; (*b*) 33.2 volts.
540. 0.71 amp.; 0.079 amp.; 0.63 amp.
541. 1 ohm.
542. (*a*) 44 volts, 66 volts; (*b*) 4.4 amp., 4.4 amp.
543. 110 volts, 110 volts; 11 amp., 7.33 amp; 18.3 amp.
544. 1 amp.; 55 volts; 13.7 volts.
545. 0.2 amp.; 0.14 amp.
546. 21.1 ohms; 4.74 amp.
547. 8.57 amp, 3.43 amp.; 17.1 volts; 1.43 ohms.

548. 0.3 amp. in 10-ohm wire; 0.2 amp. in 15-ohm wire.
549. 0.5 amp.; 0.66 amp.
550. 2 to 1.
551. 95.4 volts.
552. 4.5 amp.
553. 0.75.
554. (*a*) 250 watts; (*b*) 1000 watts.
555. 0.645 amp.; 6.45 amp.
556. 9.69 volts.
557. $E_1=485.2$ volts; $E_2=431.2$ volts; $E_3=388$ volts; $E_4=355.6$ volts; $E_5=334$ volts; $E_6=323.2$ volts.
558. 14.9 min.
559. 0.303 g.
560 0.149 amp.
561. 0.03868 amp.
562. 2.373 g.
563. 372 amp.-hrs.
564. 18.63 min.
565. 1st cell, anode, 0.08286 g. of O; cathode, 0.3296 g. of Cu; 2d cell, anode, 0.08286 g. O; cathode, 0.2902 g. Fe; 3d cell, anode, 0.08286 g. of O; cathode, 0.1935 g. Fe.
566. 100 to 341; 100 to 207; 100 to 92.
567. 22.7 g.
568. 0.47 amp.
569. 0.008 amp
570. 5060 amp.
571. 95.1 g. Cu; 97.8 g. Zn.
572. 7670 hr.
573. 6.3 g.
574. 0.114 cm.
575. 2.37 g.; 1.19 g.
576. 25 ohms.
577. 0.0033 ohm.
578. 14,700 ohms.
579. 99.9 ohms.
580. (*a*) 0.004 amp.; (*b*) 14 volts; (*c*) 7.86.
581. 475 ohms.
582. 0.0501 ohm.
583. $R_g=8R_b$.
584. 0.0606 ohm.
585. (*a*) 0.000833 amp.; (*b*) 0.02 amp.
586. 83,270 ohms.
587. 1 volt.
588. 0.00132 volt.
589. 0.0173 volt.
590. 1040 g. wt.
591. 96 volts.
592. 1 286 000 maxwells.
593. 60 conductors.
594. Armature, 60 watts; field, 557 watts; friction 943 watts.
595. 5000 dynes.
596. 550 watts; 160 watts.
597. 85.5 watts; 86%.
598. (*a*) 106.5 volts; (*b*) 7700 watts; (*c*) 245 watts; (*d*) 1 H.P.
599. 4190 volts.
600. 707 mils.
601. 0.0954 cm.
602. 0.0000404 cm.; 0.0000765 cm.
603. 0.0000589 cm.
604. 251 lumens.
605. (*a*) 1.85 ft.; (*b*) 1 m.
606. (*a*) 1.4 ft.; (*b*) 2.66 m.
607. 1.39 to 1.
608. 1.77 to 1.
609. 4.47 ft.
610. 0.4 ft.-candles; 0.9 lux.
611. 2 to 1; 0.4 candle-ft.; 0.2 candle-ft.
612. 4 hefner-meters; 0.69 hefner-meters.
613. 71 cm.
614. 0.56 m. from the hefner lamp.
615. 2.5 hef.-m.; 0.209 c.-ft.; 2.25 cm.
616. 0.694 lumen.
617. 0.00288 hefner per sq. mm.; 2.06 c.p. per sq. in.

618. 6 ft.-candles; 64.6 m.-candles.

619. 5 (10^9) candle power

620. 800 lumens per sq. ft.

621. 48° 50′.

622. 37°.

623. 5.33 ft.

624. 0.0000355 cm.

625. 8.07 in. to 8.50 in.

626. 6.87 in.

627. 14.9 cm.

628. 10.03 cm.; 20.135 cm.

629. 2.51 cm.

630. 16.8 ft.

631. 40 cm.

632. 3.31 in.

633. 8 in.

634. $v = 60$ cm.

635. (*a*) 3 in.; (*b*) 3.6 in.

636. 20 ft.

637. 6.15 in.

638. (*b*) 30 cm.; (*c*) 6 cm.

639. 8.6 cm.; 3.75 cm.

640. 3 ft.

641. 3 cm.

642. 13.1 cm.

643. (*a*) 1.3 in.; (*b*), 2 in.

644. 22 cm.

645 to **674.** Graphical Constructions.

675. 6.33 diopters.

676. 3 diopters.

677. 26.7 sec.

678. 0.08 sec.

679. (*a*) 0.08 sec.; (*b*) 0.04 sec.

680. (*a*) 16; (*b*) 16.

681. 25.4 cm.

682. 38.6 cm. from center of objective to center of eye-lens; 33.9 cm. from center of objective to cross wires.

683. (1) to (2) = 34.0 cm.; (1) to (3) = 46.4 cm.

684. 35.1 cm.

685. 23.9 in.

686. (1) to (2) = 46.95 cm.; (1) to (3) = 62.45 cm.; (1) to (4) = 73.45 cm.; (1) to (5) = 79.85 cm.. (1) to (I_1) = 31.95 cm.; (1) to (I_2) = 76.85 cm.

687. 49 cm.

TABLE OF TRIGONOMETRIC FUNCTIONS

The Natural Values of Sines, Cosines, Tangents and Cotangents

1°—13°

°	′	sin	tan	cot	cos	′	°
1	0	0.0175	0.0175	57.2900	0.9998	0	**89**
	10	0.0204	0.0204	49.1039	0.9998	50	
	20	0.0233	0.0233	42.9641	0.9997	40	
	30	0.0262	0.0262	38.1885	0.9997	30	
	40	0.0291	0.0291	34.3678	0.9996	20	
	50	0.0320	0.0320	31.2416	0.9995	10	
2	0	0.0349	0.0349	28.6363	0.9994	0	**88**
	10	0.0378	0.0378	26.4316	0.9993	50	
	20	0.0407	0.0407	24.5418	0.9992	40	
	30	0.0436	0.0437	22.9038	0.9990	30	
	40	0.0465	0.0466	21.4704	0.9989	20	
	50	0.0494	0.0495	20.2056	0.9988	10	
3	0	0.0523	0.0524	19.0811	0.9986	0	**87**
	10	0.0552	0.0553	18.0750	0.9985	50	
	20	0.0581	0.0582	17.1693	0.9983	40	
	30	0.0610	0.0612	16.3499	0.9981	30	
	40	0.0640	0.0641	15.6048	0.9980	20	
	50	0.0669	0.0670	14.9244	0.9978	10	
4	0	0.0698	0.0699	14.3007	0.9976	0	**86**
	10	0.0727	0.0729	13.7267	0.9974	50	
	20	0.0756	0.0758	13.1969	0.9971	40	
	30	0.0785	0.0787	12.7062	0.9969	30	
	40	0.0814	0.0816	12.2505	0.9967	20	
	50	0.0843	0.0846	11.8262	0.9964	10	
5	0	0.0872	0.0875	11.4301	0.9962	0	**85**
	10	0.0901	0.0904	11.0594	0.9959	50	
	20	0.0929	0.0934	10.7119	0.9957	40	
	30	0.0958	0.0963	10.3854	0.9954	30	
	40	0.0987	0.0992	10.0780	0.9951	20	
	50	0.1016	0.1022	9.7882	0.9948	10	
6	0	0.1045	0.1051	9.5144	0.9945	0	**84**
	10	0.1074	0.1080	9.2553	0.9942	50	
	20	0.1103	0.1110	9.0098	0.9939	40	
	30	0.1132	0.1139	8.7769	0.9936	30	
	40	0.1161	0.1169	8.5555	0.9932	20	
	50	0.1190	0.1198	8.3450	0.9929	10	
7	0	0.1219	0.1228	8.1443	0.9925	0	**83**
7	0	0.1219	0.1228	8.1443	0.9925	0	**83**
	10	0.1248	0.1257	7.9530	0.9922	50	
	20	0.1276	0.1287	7.7704	0.9918	40	
	30	0.1305	0.1317	7.5958	0.9914	30	
	40	0.1334	0.1346	7.4287	0.9911	20	
	50	0.1363	0.1376	7.2687	0.9907	10	
8	0	0.1392	0.1405	7.1154	0.9903	0	**82**
	10	0.1421	0.1435	6.9682	0.9899	50	
	20	0.1449	0.1465	6.8269	0.9894	40	
	30	0.1478	0.1495	6.6912	0.9890	30	
	40	0.1507	0.1524	6.5606	0.9886	20	
	50	0.1536	0.1554	6.4348	0.9881	10	
9	0	0.1564	0.1584	6.3138	0.9877	0	**81**
	10	0.1593	0.1614	6.1970	0.9872	50	
	20	0.1622	0.1644	6.0844	0.9868	40	
	30	0.1650	0.1673	5.9758	0.9863	30	
	40	0.1679	0.1703	5.8708	0.9858	20	
	50	0.1708	0.1733	5.7694	0.9853	10	
10	0	0.1736	0.1763	5.6713	0.9848	0	**80**
	10	0.1765	0.1793	5.5764	0.9843	50	
	20	0.1794	0.1823	5.4845	0.9838	40	
	30	0.1822	0.1853	5.3955	0.9833	30	
	40	0.1851	0.1883	5.3093	0.9827	20	
	50	0.1880	0.1914	5.2257	0.9822	10	
11	0	0.1908	0.1944	5.1446	0.9816	0	**79**
	10	0.1937	0.1974	5.0658	0.9811	50	
	20	0.1965	0.2004	4.9894	0.9805	40	
	30	0.1994	0.2035	4.9152	0.9799	30	
	40	0.2022	0.2065	4.8430	0.9793	20	
	50	0.2051	0.2095	4.7729	0.9787	10	
12	0	0.2079	0.2126	4.7046	0.9781	0	**78**
	10	0.2108	0.2156	4.6382	0.9775	50	
	20	0.2136	0.2186	4.5736	0.9769	40	
	30	0.2164	0.2217	4.5107	0.9763	30	
	40	0.2193	0.2247	4.4494	0.9757	20	
	50	0.2221	0.2278	4.3897	0.9750	10	
13	0	0.2250	0.2309	4.3315	0.9744	0	**77**
°	′	cos	cot	tan	sin	′	°

77°—89°

Note. $\sin\phi = \cos(90° - \phi) = \sin(180° - \phi)$

$\cos\phi = \sin(90° - \phi) = -\cos(180° - \phi).$

13°—29°

° ′	sin	tan	cot	cos	′ °	° ′	sin	tan	cot	cos	′ °
13 0	0.2250	0.2309	4.3315	0.9744	0 **77**	**21** 0	0.3584	0.3839	2.6051	0.9336	0 **69**
10	0.2278	0.2339	4.2747	0.9737	50	10	0.3611	0.3872	2.5826	0.9325	50
20	0.2306	0.2370	4.2193	0.9730	40	20	0.3638	0.3906	2.5605	0.9315	40
30	0.2334	0.2401	4.1653	0.9724	30	30	0.3665	0.3939	2.5386	0.9304	30
40	0.2363	0.2432	4.1126	0.9717	20	40	0.3692	0.3973	2.5172	0.9293	20
50	0.2391	0.2462	4.0611	0.9710	10	50	0.3719	0.4006	2.4960	0.9283	10
14 0	0.2419	0.2493	4.0108	0.9703	0 **76**	**22** 0	0.3746	0.4040	2.4751	0.9272	0 **68**
10	0.2447	0.2524	3.9617	0.9696	50	10	0.3773	0.4074	2.4545	0.9261	50
20	0.2476	0.2555	3.9136	0.9689	40	20	0.3800	0.4108	2.4342	0.9250	40
30	0.2504	0.2586	3.8667	0.9681	30	30	0.3827	0.4142	2.4142	0.9239	30
40	0.2532	0.2617	3.8208	0.9674	20	40	0.3854	0.4176	2.3945	0.9228	20
50	0.2560	0.2648	3.7760	0.9667	10	50	0.3881	0.4210	2.3750	0.9216	10
15 0	0.2588	0.2679	3.7321	0.9659	0 **75**	**23** 0	0.3907	0.4245	2.3559	0.9205	0 **67**
10	0.2616	0.2711	3.6891	0.9652	50	10	0.3934	0.4279	2.3369	0.9194	50
20	0.2644	0.2742	3.6470	0.9644	40	20	0.3961	0.4314	2.3183	0.9182	40
30	0.2672	0.2773	3.6059	0.9636	30	30	0.3987	0.4348	2.2998	0.9171	30
40	0.2700	0.2805	3.5656	0.9628	20	40	0.4014	0.4383	2.2817	0.9159	20
50	0.2728	0.2836	3.5261	0.9621	10	50	0.4041	0.4417	2.2637	0.9147	10
16 0	0.2756	0.2867	3.4874	0.9613	0 **74**	**24** 0	0.4067	0.4452	2.2460	0.9135	0 **66**
10	0.2784	0.2899	3.4495	0.9605	50	10	0.4094	0.4487	2.2286	0.9124	50
20	0.2812	0.2931	3.4124	0.9596	40	20	0.4120	0.4522	2.2113	0.9112	40
30	0.2840	0.2962	3.3759	0.9588	30	30	0.4147	0.4557	2.1943	0.9100	30
40	0.2868	0.2994	3.3402	0.9580	20	40	0.4173	0.4592	2.1775	0.9088	20
50	0.2896	0.3026	3.3052	0.9572	10	50	0.4200	0.4628	2.1609	0.9075	10
17 0	0.2924	0.3057	3.2709	0.9563	0 **73**	**25** 0	0.4226	0.4663	2.1445	0.9063	0 **65**
10	0.2952	0.3089	3.2371	0.9555	50	10	0.4253	0.4699	2.1283	0.9051	50
20	0.2979	0.3121	3.2041	0.9546	40	20	0.4279	0.4734	2.1123	0.9038	40
30	0.3007	0.3153	3.1716	0.9537	30	30	0.4305	0.4770	2.0965	0.9026	30
40	0.3035	0.3185	3.1397	0.9528	20	40	0.4331	0.4806	2.0809	0.9013	20
50	0.3062	0.3217	3.1084	0.9520	10	50	0.4358	0.4841	2.0655	0.9001	10
18 0	0.3090	0.3249	3.0777	0.9511	0 **72**	**26** 0	0.4384	0.4877	2.0503	0.8988	0 **64**
10	0.3118	0.3281	3.0475	0.9502	50	10	0.4410	0.4913	2.0353	0.8975	50
20	0.3145	0.3314	3.0178	0.9492	40	20	0.4436	0.4950	2.0204	0.8962	40
30	0.3173	0.3346	2.9887	0.9483	30	30	0.4462	0.4986	2.0057	0.8949	30
40	0.3201	0.3378	2.9600	0.9474	20	40	0.4488	0.5022	1.9912	0.8936	20
50	0.3228	0.3411	2.9319	0.9465	10	50	0.4514	0.5059	1.9768	0.8923	10
19 0	0.3256	0.3443	2.9042	0.9455	0 **71**	**27** 0	0.4540	0.5095	1.9626	0.8910	0 **63**
10	0.3283	0.3476	2.8770	0.9446	50	10	0.4566	0.5132	1.9486	0.8897	50
20	0.3311	0.3508	2.8502	0.9436	40	20	0.4592	0.5169	1.9347	0.8884	40
30	0.3338	0.3541	2.8239	0.9426	30	30	0.4617	0.5206	1.9210	0.8870	30
40	0.3365	0.3574	2.7980	0.9417	20	40	0.4643	0.5243	1.9074	0.8857	20
50	0.3393	0.3607	2.7725	0.9407	10	50	0.4669	0.5280	1.8940	0.8843	10
20 0	0.3420	0.3640	2.7475	0.9397	0 **70**	**28** 0	0.4695	0.5317	1.8807	0.8829	0 **62**
10	0.3448	0.3673	2.7228	0.9387	50	10	0.4720	0.5354	1.8676	0.8816	50
20	0.3475	0.3706	2.6985	0.9377	40	20	0.4746	0.5392	1.8546	0.8802	40
30	0.3502	0.3739	2.6746	0.9367	30	30	0.4772	0.5430	1.8418	0.8788	30
40	0.3529	0.3772	2.6511	0.9356	20	40	0.4797	0.5467	1.8291	0.8774	20
50	0.3557	0.3805	2.6279	0.9346	10	50	0.4823	0.5505	1.8165	0.8760	10
21 0	0.3584	0.3839	2.6051	0.9336	0 **69**	**29** 0	0.4848	0.5543	1.8040	0.8746	0 **61**
° ′	cos	cot	tan	sin	′ °	° ′	cos	cot	tan	sin	′ °

61°—77°

29°—45°

° ′	sin	tan	cot	cos	′ °
29 0	0.4848	0.5543	1.8040	0.8746	0 **61**
10	0.4874	0.5581	1.7917	0.8732	50
20	0.4899	0.5619	1.7796	0.8718	40
30	0.4924	0.5658	1.7675	0.8704	30
40	0.4950	0.5696	1.7556	0.8689	20
50	0.4975	0.5735	1.7437	0.8675	10
30 0	0.5000	0.5774	1.7321	0.8660	0 **60**
10	0.5025	0.5812	1.7205	0.8646	50
20	0.5050	0.5851	1.7090	0.8631	40
30	0.5075	0.5890	1.6977	0.8616	30
40	0.5100	0.5930	1.6864	0.8601	20
50	0.5125	0.5969	1.6753	0.8587	10
31 0	0.5150	0.6000	1.6643	0.8572	0 **59**
10	0.5175	0.6048	1.6534	0.8557	50
20	0.5200	0.6088	1.6426	0.8542	40
30	0.5225	0.6128	1.6319	0.8526	30
40	0.5250	0.6168	1.6212	0.8511	20
50	0.5275	0.6208	1.6107	0.8496	10
32 0	0.5299	0.6249	1.6003	0.8480	0 **58**
10	0.5324	0.6289	1.5900	0.8465	50
20	0.5348	0.6330	1.5798	0.8450	40
30	0.5373	0.6371	1.5697	0.8434	30
40	0.5398	0.6412	1.5597	0.8418	20
50	0.5422	0.6453	1.5497	0.8403	10
33 0	0.5446	0.6494	1.5399	0.8387	0 **57**
10	0.5471	0.6536	1.5301	0.8371	50
20	0.5495	0.6577	1.5204	0.8355	40
30	0.5519	0.6619	1.5108	0.8339	30
40	0.5544	0.6661	1.5013	0.8323	20
50	0.5568	0.6703	1.4919	0.8307	10
34 0	0.5592	0.6745	1.4826	0.8290	0 **56**
10	0.5616	0.6787	1.4733	0.8274	50
20	0.5640	0.6830	1.4641	0.8258	40
30	0.5664	0.6873	1.4550	0.8241	30
40	0.5688	0.6916	1.4460	0.8225	20
50	0.5712	0.6959	1.4370	0.8208	10
35 0	0.5736	0.7002	1.4281	0.8192	0 **55**
10	0.5760	0.7046	1.4193	0.8175	50
20	0.5783	0.7089	1.4106	0.8158	40
30	0.5807	0.7133	1.4019	0.8141	30
40	0.5831	0.7177	1.3934	0.8124	20
50	0.5854	0.7221	1.3848	0.8107	10
36 0	0.5878	0.7265	1.3764	0.8090	0 **54**
10	0.5901	0.7310	1.3680	0.8073	50
20	0.5925	0.7355	1.3597	0.8056	40
30	0.5948	0.7400	1.3514	0.8039	30
40	0.5972	0.7445	1.3432	0.8021	20
50	0.5995	0.7490	1.3351	0.8004	10
37 0	0.6018	0.7536	1.3270	0.7986	0 **53**
° ′	cos	cot	tan	sin	′ °

° ′	sin	tan	cot	cos	′ °
37 0	0.6018	0.7536	1.3270	0.7986	0 **53**
10	0.6041	0.7581	1.3190	0.7969	50
20	0.6065	0.7627	1.3111	0.7951	40
30	0.6088	0.7673	1.3032	0.7934	30
40	0.6111	0.7720	1.2954	0.7916	20
50	0.6134	0.7766	1.2876	0.7898	10
38 0	0.6157	0.7813	1.2799	0.7880	0 **52**
10	0.6180	0.7860	1.2723	0.7862	50
20	0.6202	0.7907	1.2647	0.7844	40
30	0.6225	0.7954	1.2572	0.7826	30
40	0.6248	0.8002	1.2497	0.7808	20
50	0.6271	0.8050	1.2423	0.7790	10
39 0	0.6293	0.8098	1.2349	0.7771	0 **51**
10	0.6316	0.8146	1.2276	0.7753	50
20	0.6338	0.8195	1.2203	0.7735	40
30	0.6361	0.8243	1.2131	0.7716	30
40	0.6383	0.8292	1.2059	0.7698	20
50	0.6406	0.8342	1.1988	0.7679	10
40 0	0.6428	0.8391	1.1918	0.7660	0 **50**
10	0.6450	0.8441	1.1847	0.7642	50
20	0.6472	0.8491	1.1778	0.7623	40
30	0.6494	0.8541	1.1708	0.7604	30
40	0.6517	0.8591	1.1640	0.7585	20
50	0.6539	0.8642	1.1571	0.7566	10
41 0	0.6561	0.8693	1.1504	0.7547	0 **49**
10	0.6583	0.8744	1.1436	0.7528	50
20	0.6604	0.8796	1.1369	0.7509	40
30	0.6626	0.8847	1.1303	0.7490	30
40	0.6648	0.8899	1.1237	0.7470	20
50	0.6670	0.8952	1.1171	0.7451	10
42 0	0.6691	0.9004	1.1106	0.7431	0 **48**
10	0.6713	0.9057	1.1041	0.7412	50
20	0.6734	0.9110	1.0977	0.7392	40
30	0.6756	0.9163	1.0913	0.7373	30
40	0.6777	0.9217	1.0850	0.7353	20
50	0.6799	0.9271	1.0786	0.7333	10
43 0	0.6820	0.9325	1.0724	0.7314	0 **47**
10	0.6841	0.9380	1.0661	0.7294	50
20	0.6862	0.9435	1.0599	0.7274	40
30	0.6884	0.9490	1.0538	0.7254	30
40	0.6905	0.9545	1.0477	0.7234	20
50	0.6926	0.9601	1.0416	0.7214	10
44 0	0.6947	0.9657	1.0355	0.7193	0 **46**
10	0.6967	0.9713	1.0295	0.7173	50
20	0.6988	0.9770	1.0235	0.7153	40
30	0.7009	0.9827	1.0176	0.7133	30
40	0.7030	0.9884	1.0117	0.7112	20
50	0.7050	0.9942	1.0058	0.7092	10
45 0	0.7071	1.0000	1.0000	0.7071	0 **45**
° ′	cos	cot	tan	sin	′ °

45°—61°

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